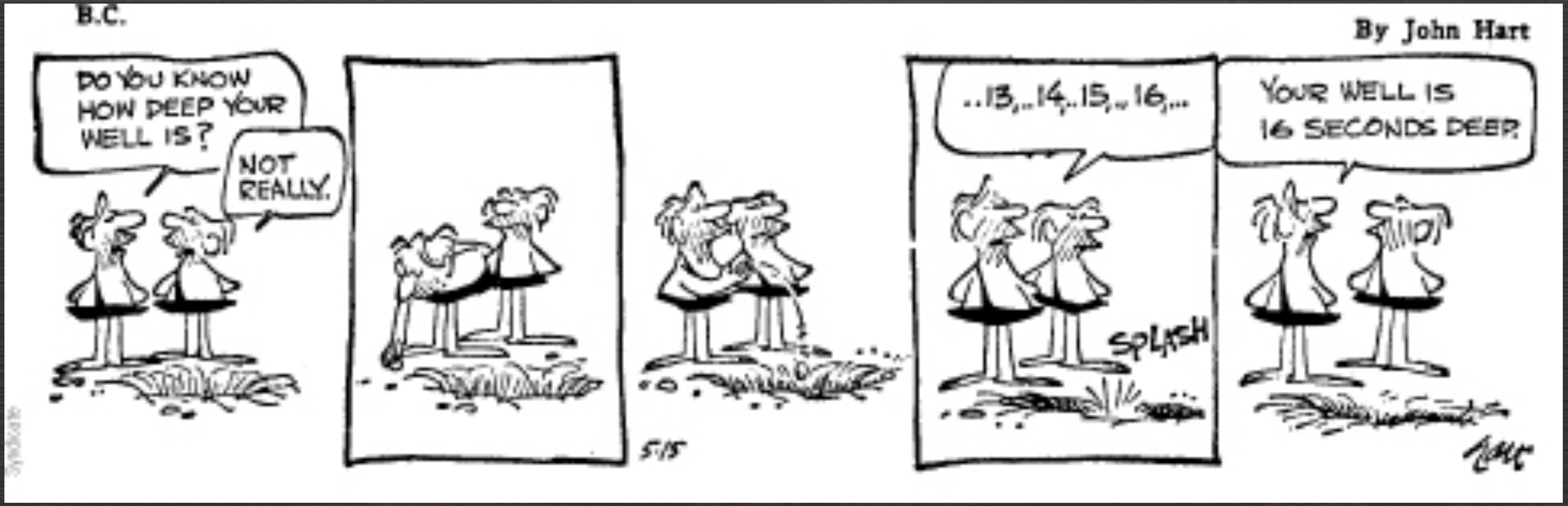
Jednoliko i ubrzano gibanje
Složena gibanja



Mehanika - sile i Newtonovi zakoni

Sila

↓

Gibanje

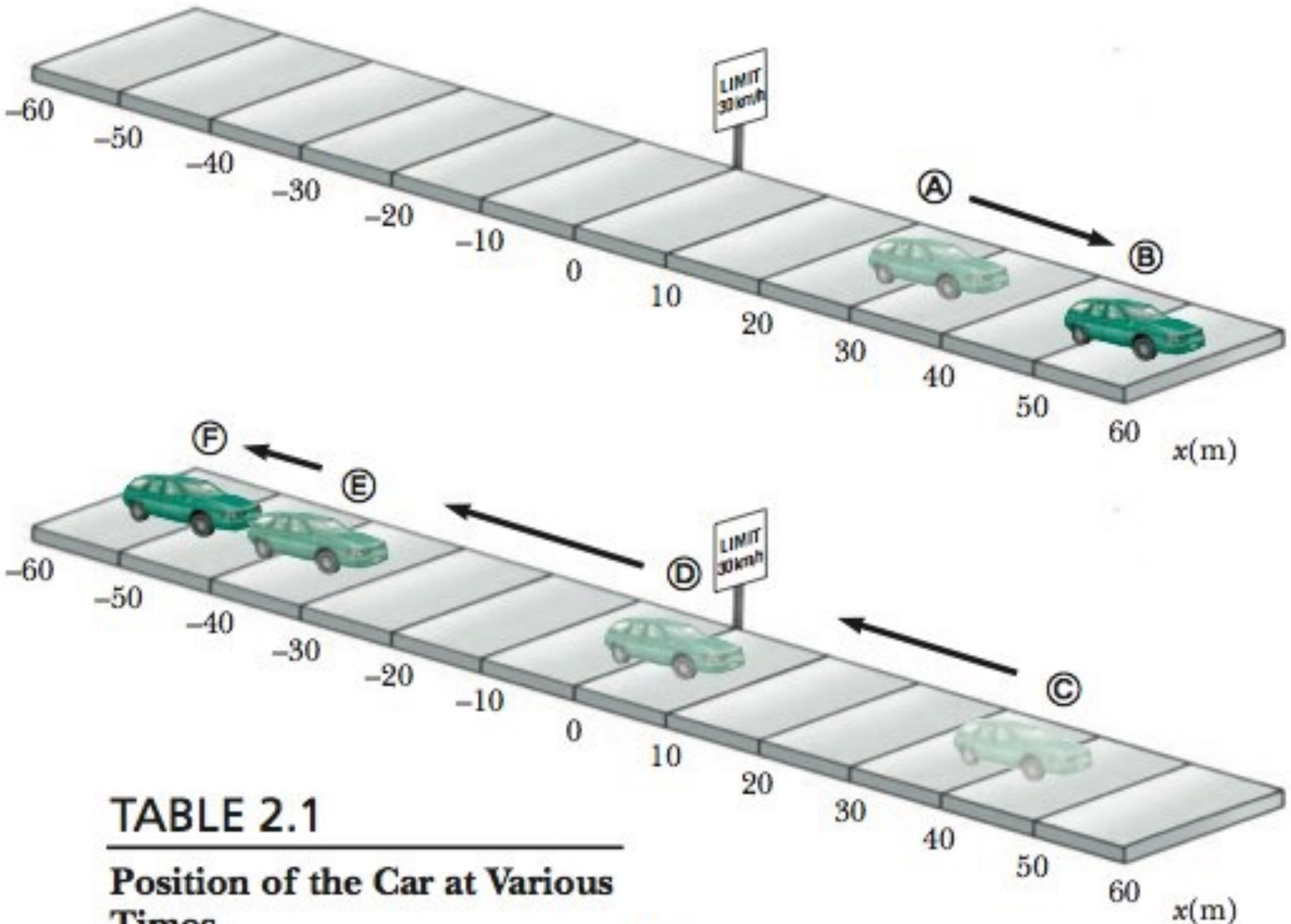
koje opisujemo jednadžbama gibanja

↓

Položaj, brzina, ubrzanje...

su rješenja jednadžbi gibanja koja općenito ovise o vremenu

Pomak, brzina i ubrzanje

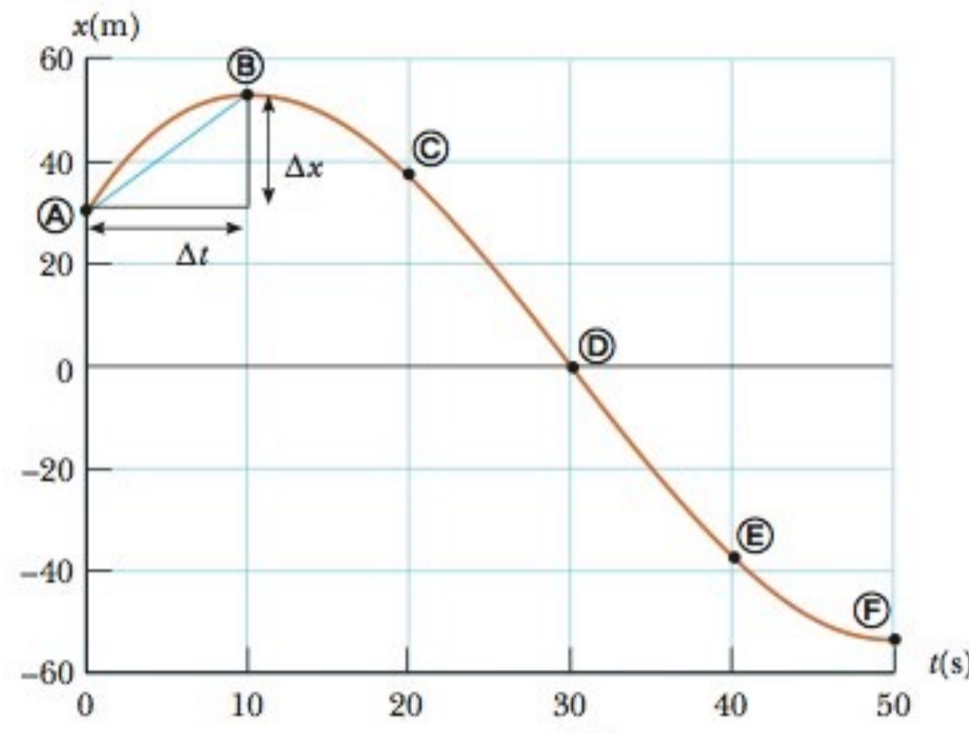


(a)

TABLE 2.1

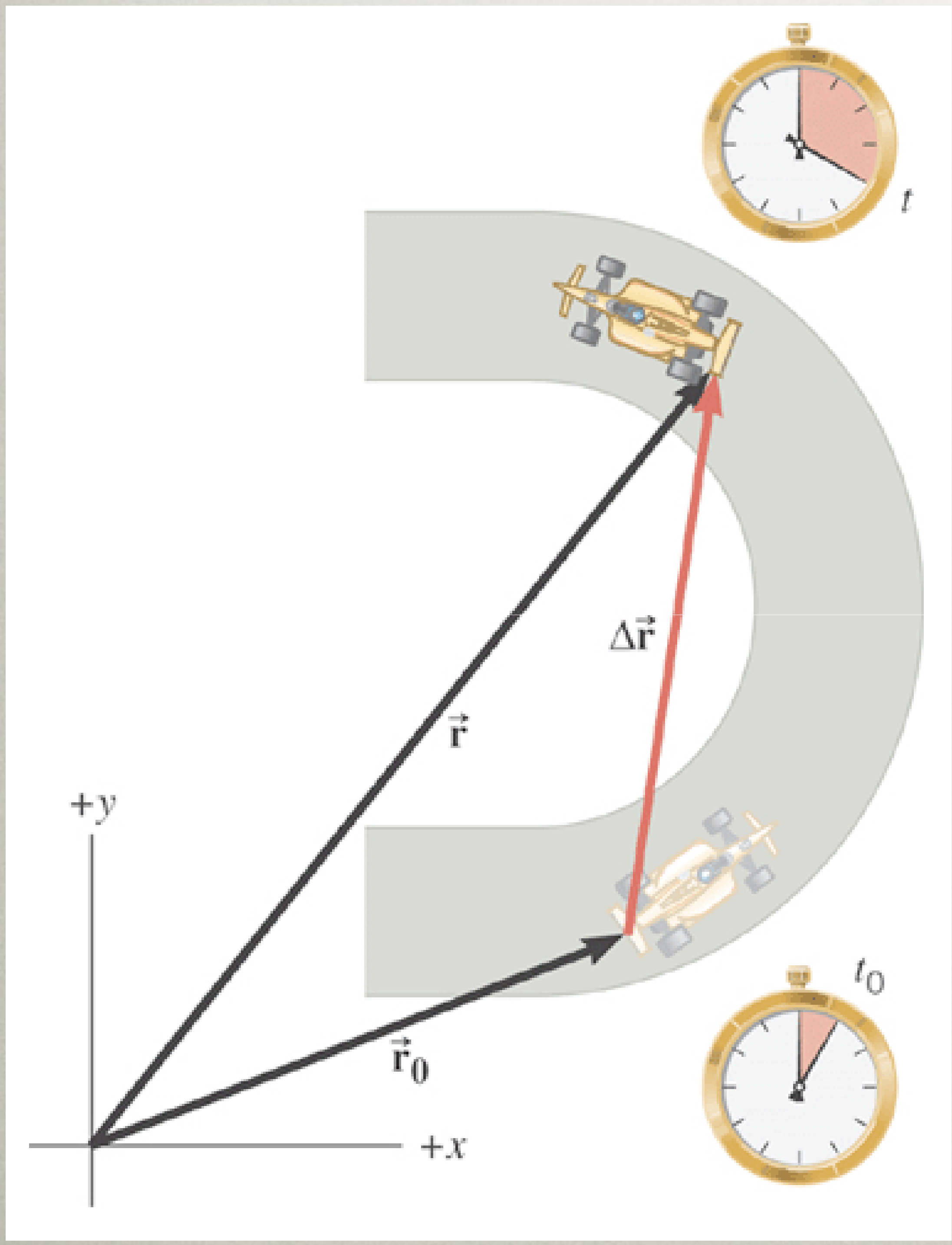
Position of the Car at Various Times

Position	t (s)	x (m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	− 37
Ⓕ	50	− 53



(b)

Pomak, brzina i ubrzanje



$$\Delta\vec{r} = \vec{r} - \vec{r}_0$$

Važno!

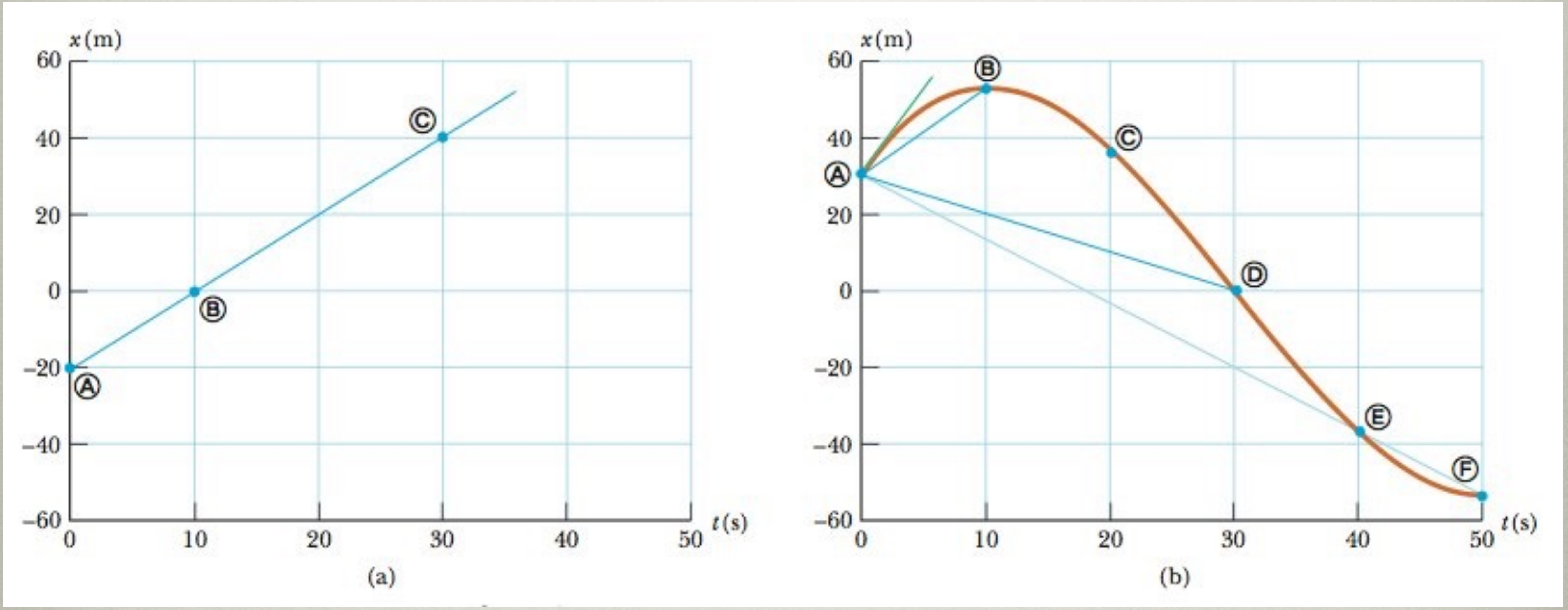
$$\bar{\vec{v}} = \frac{\vec{r} - \vec{r}_0}{t - t_0} = \frac{\Delta\vec{r}}{\Delta t}$$

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t}$$

$$\bar{\vec{a}} = \frac{\vec{v} - \vec{v}_0}{t - t_0} = \frac{\Delta\vec{v}}{\Delta t}$$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$$

Pomak, brzina i ubrzanje



konstantna brzina

promjenjiva brzina

Pomak, brzina i ubrzanje

TABLE 2.2

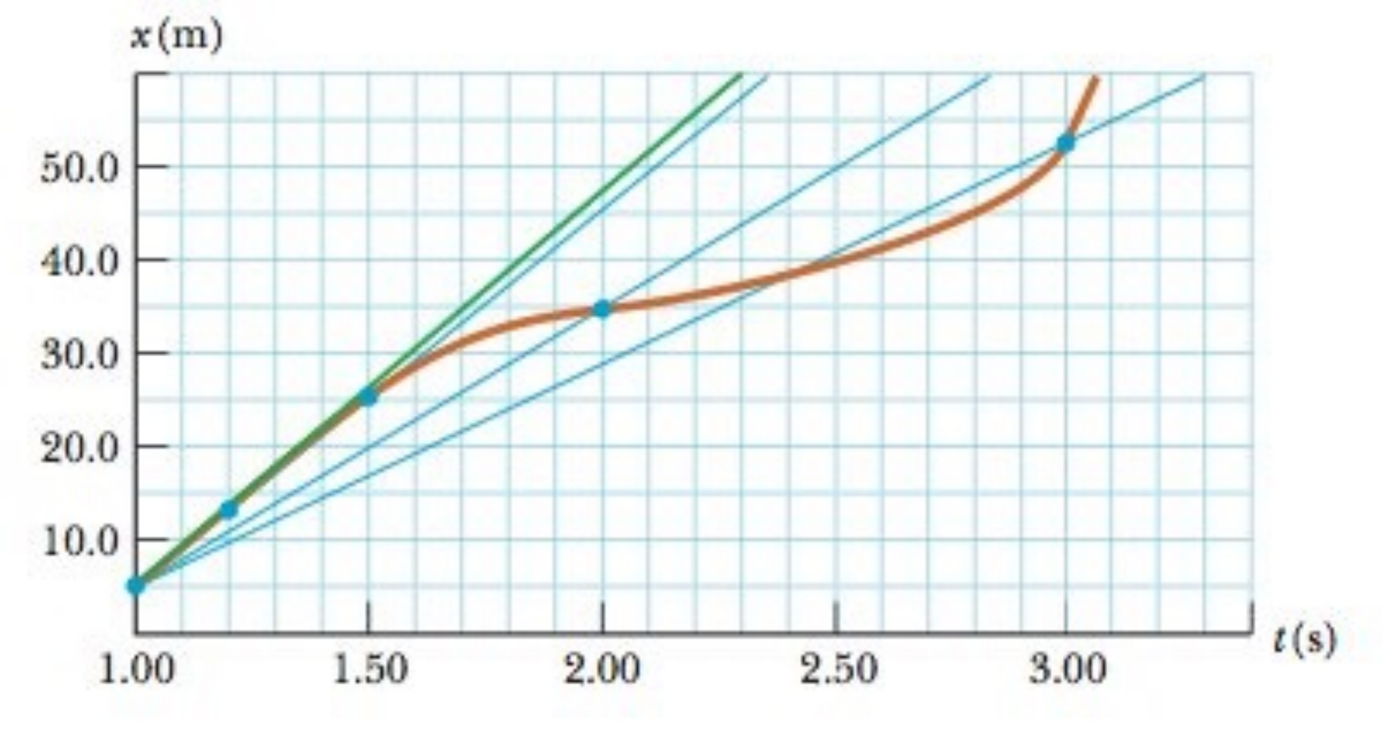
Positions of a Car at Specific Instants of Time

t (s)	x (m)
1.00	5.00
1.01	5.47
1.10	9.67
1.20	14.3
1.50	26.3
2.00	34.7
3.00	52.5

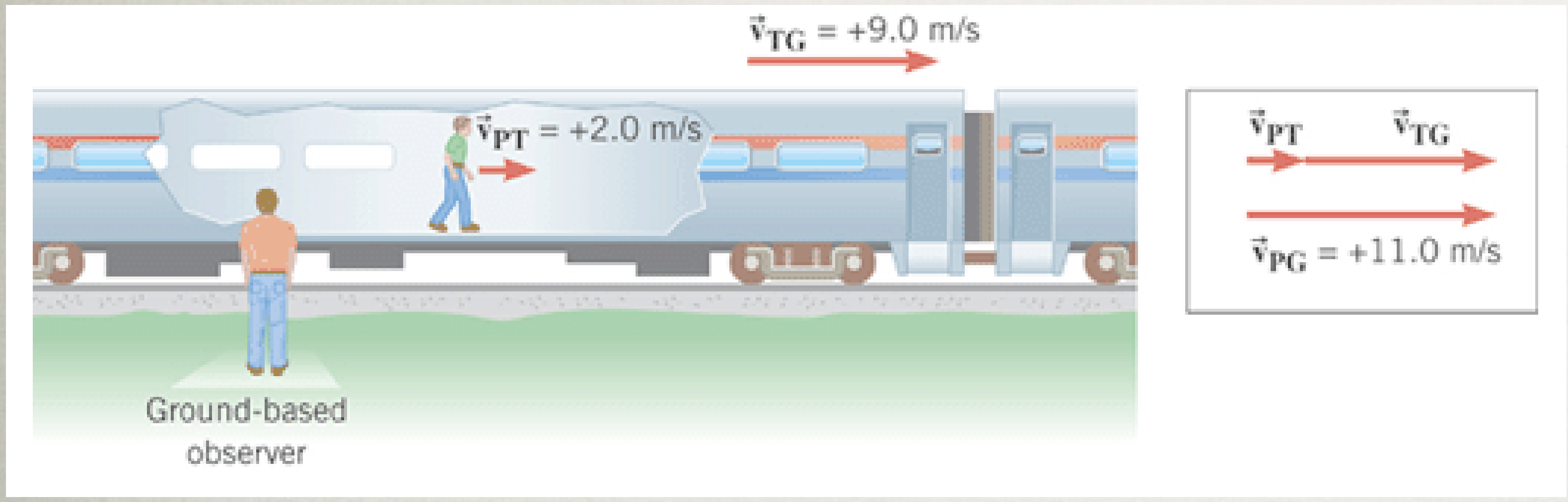
TABLE 2.3

Calculated Values of the Time Intervals, Displacements, and Average Velocities for the Car of Table 2.2

Time Interval (s)	Δt (s)	Δx (m)	$\bar{v}$ (m/s)
1.00 to 3.00	2.00	47.5	23.8
1.00 to 2.00	1.00	29.7	29.7
1.00 to 1.50	0.50	21.3	42.6
1.00 to 1.20	0.20	9.30	46.5
1.00 to 1.10	0.10	4.67	46.7
1.00 to 1.01	0.01	0.470	47.0



Relativna brzina

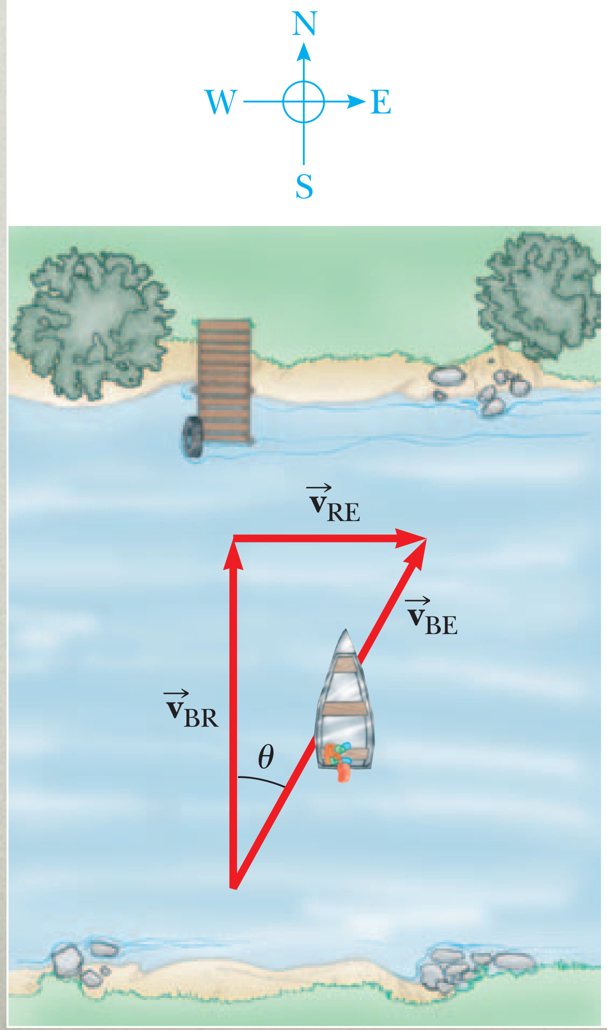


$$\vec{v}_{PG} = \vec{v}_{PT} + \vec{v}_{TG}$$

$\vec{v}_{PG}$ – brzina putnika u odnosu na tlo

$\vec{v}_{PT}$ – brzina putnika u odnosu na vlak

$\vec{v}_{TG}$ – brzina vlaka u odnosu na tlo



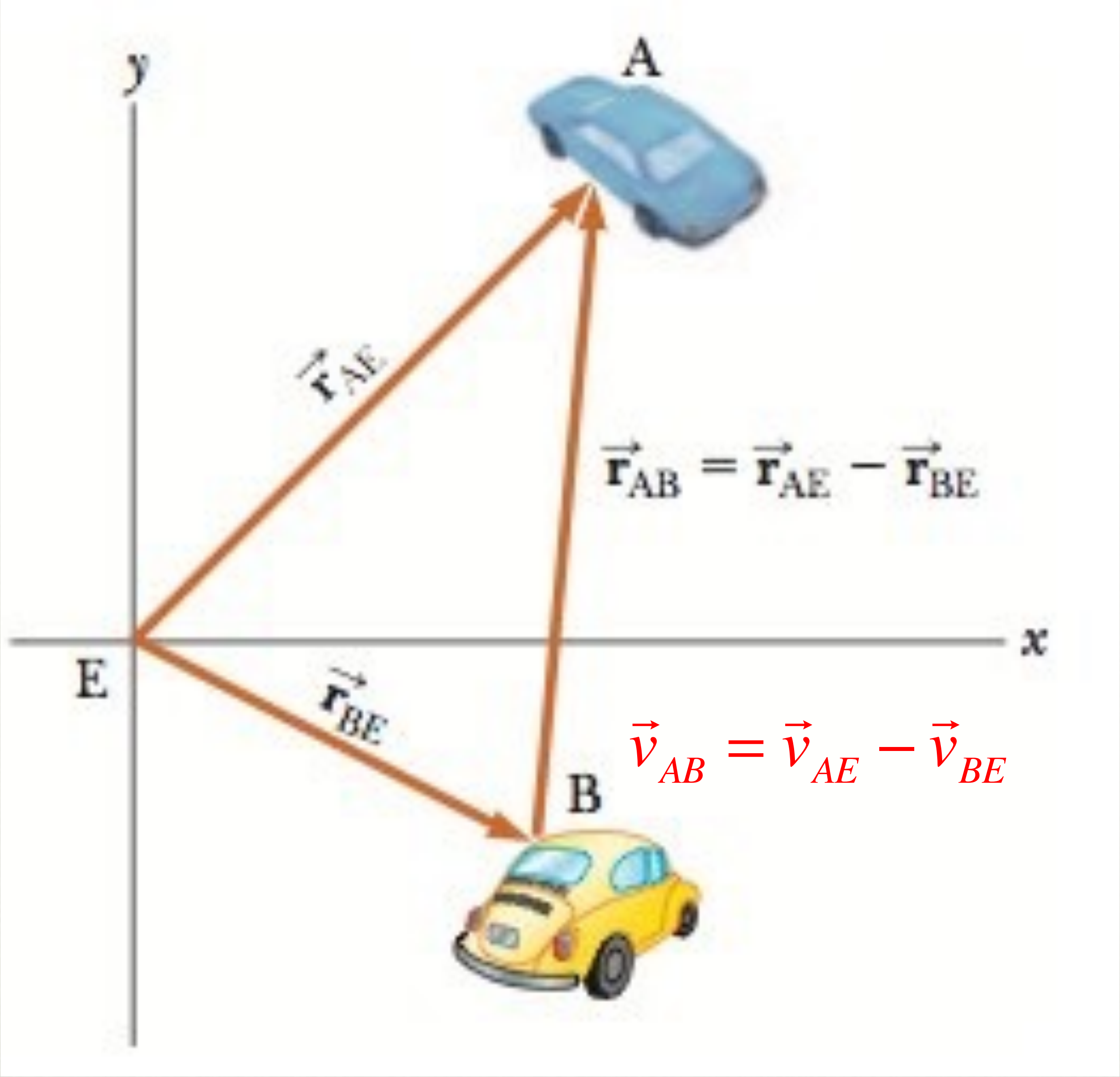
$$\vec{v}_{BE} = \vec{v}_{BR} + \vec{v}_{RE}$$

$\vec{v}_{BE}$ – brzina čamca u odnosu na obalu

$\vec{v}_{BR}$ – brzina čamca u odnosu na vodu

$\vec{v}_{RE}$ – brzina vode u odnosu na obalu

Relativna brzina



$\vec{v}_{AB} = \vec{v}_{AE} - \vec{v}_{BE}$

PROSJEČNA I TRENUTNA BRZINA

$$\text{PROSJEČNA BRZINA} = \frac{\text{POMAK}}{\text{UKUPNO VRIJEME GIBANJA}}$$

$$\bar{\vec{v}} = \frac{\vec{x} - \vec{x}_0}{t - t_0} = \frac{\Delta \vec{x}}{\Delta t}$$

SI [m/s]

TRENUTNA BRZINA

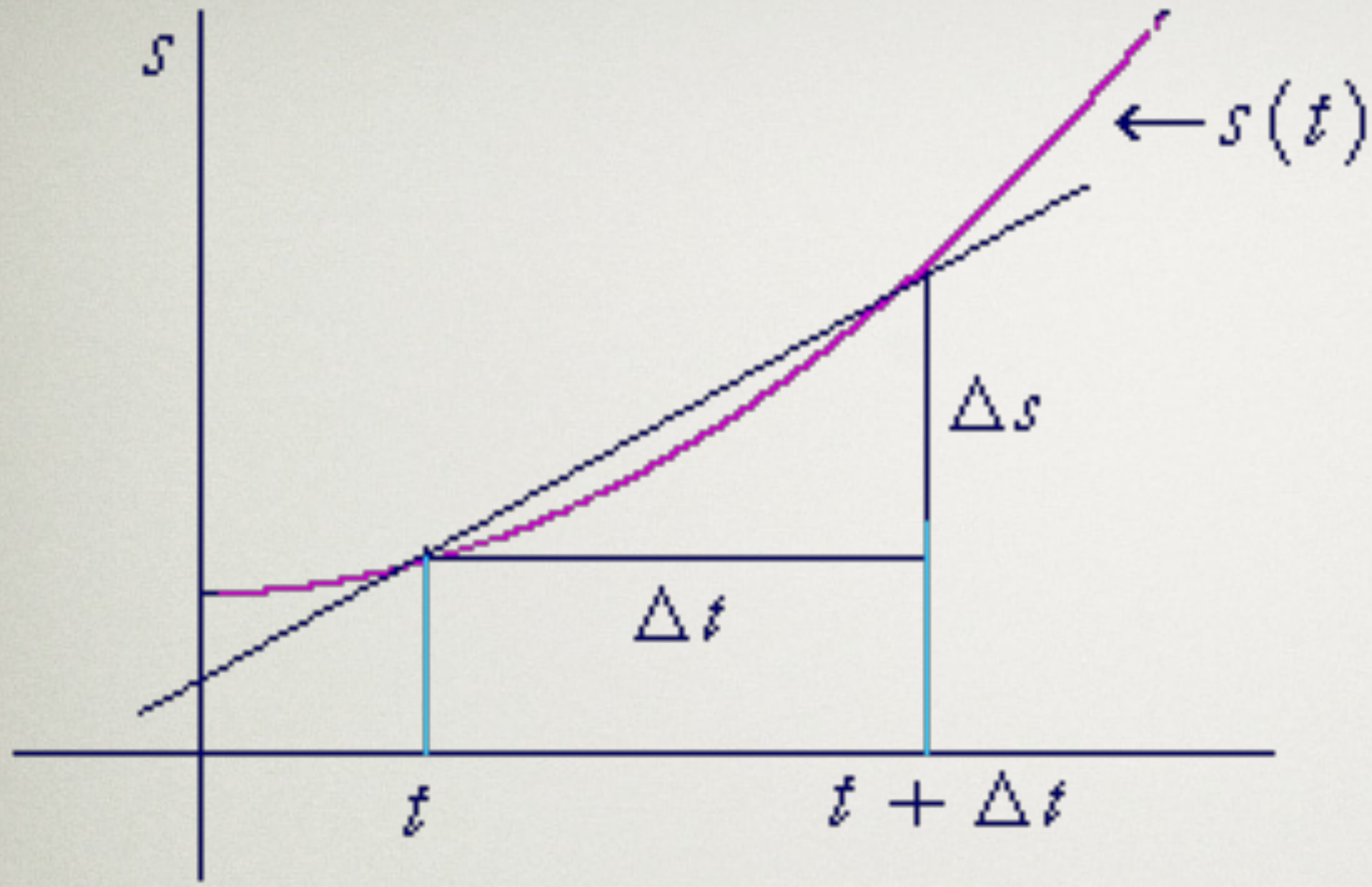
$$\vec{v} = \lim_{\Delta t \to 0} \frac{\vec{x}(t + \Delta t) - \vec{x}(t)}{\Delta t} = \frac{\mathrm{d}\vec{x}}{\mathrm{d}t}$$

PROSJEČNO UBRZANJE

$$\bar{\vec{a}} = \frac{\vec{v} - \vec{v}_0}{t - t_0} = \frac{\Delta \vec{v}}{\Delta t}$$

TRENUTNO UBRZANJE

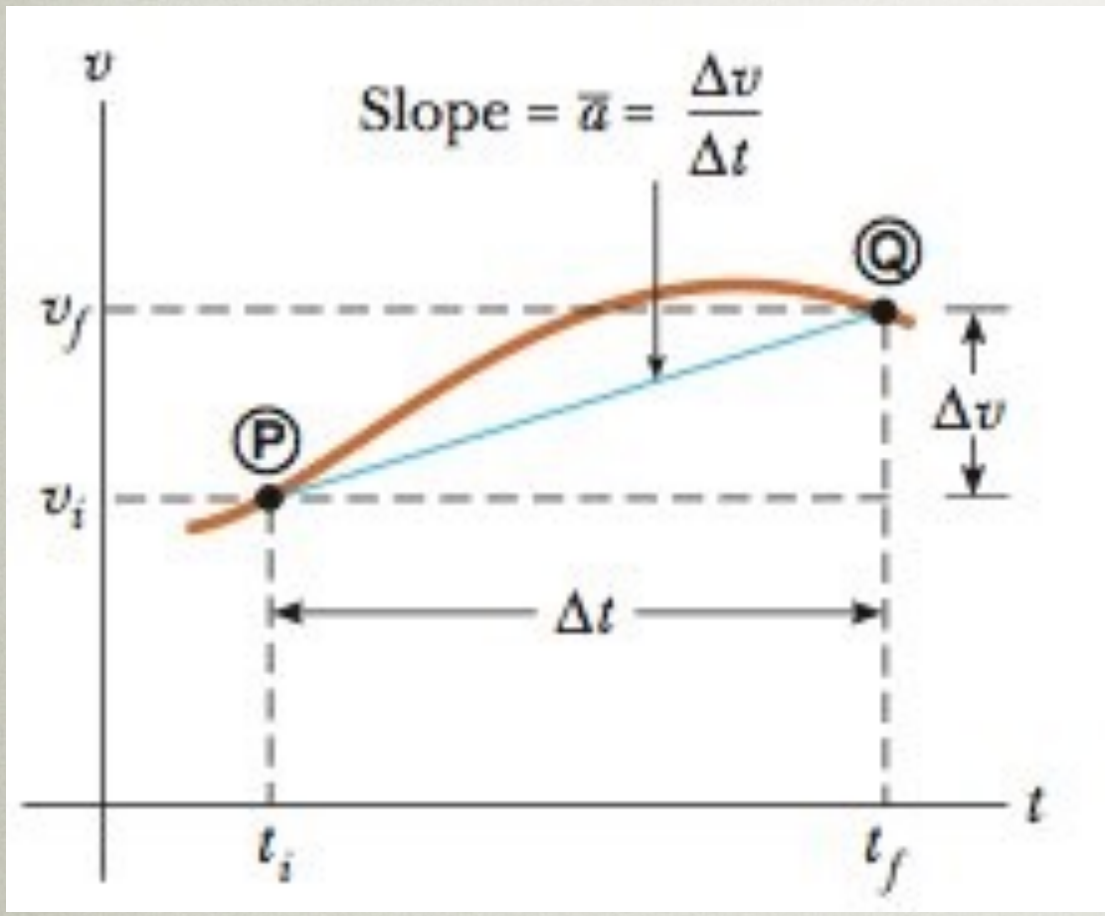
$$\vec{a} = \lim_{\Delta t \to 0} \frac{\vec{v}(t + \Delta t) - \vec{v}(t)}{\Delta t} = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t}$$

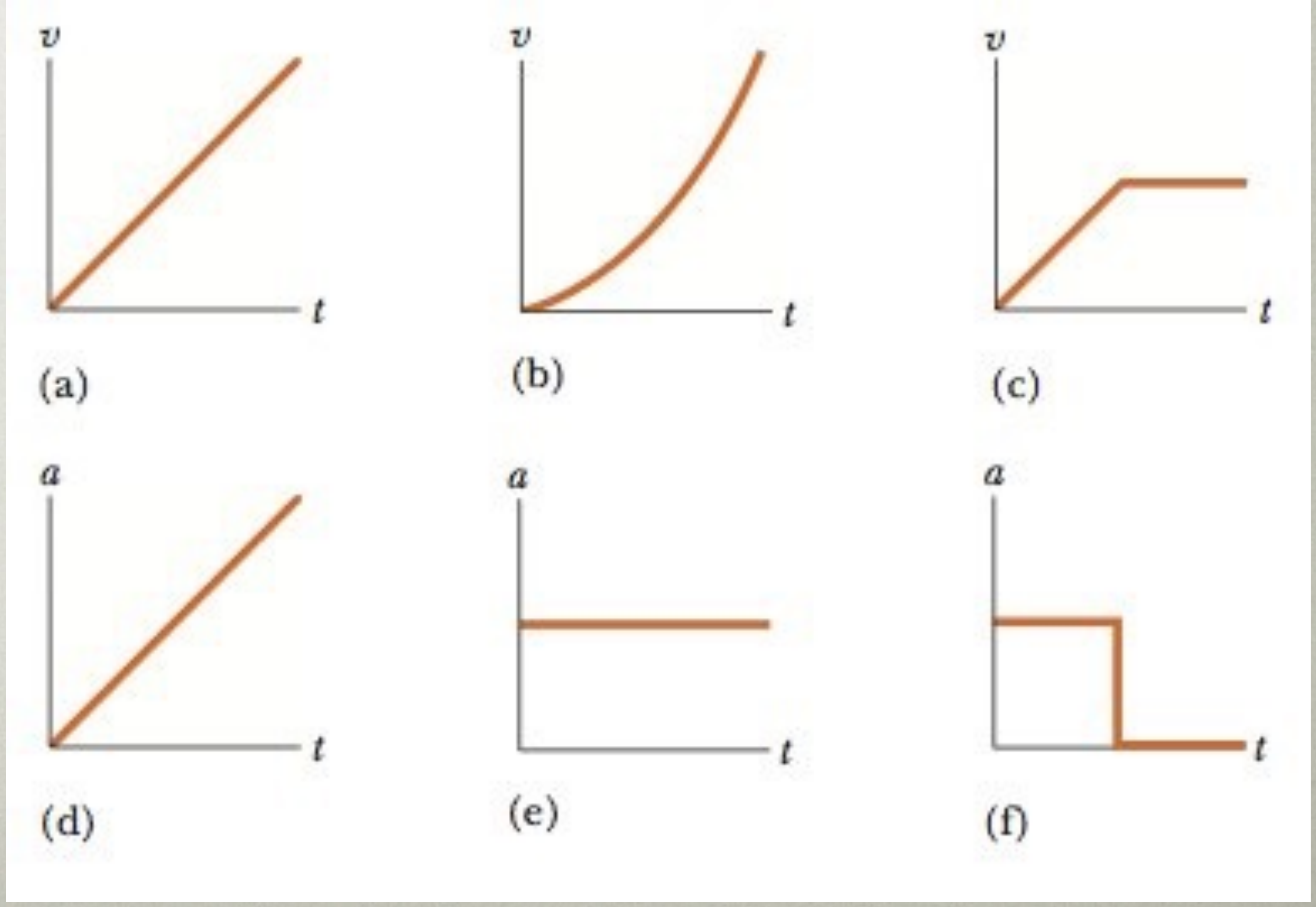


prosječna brzina $\vec{v} = \frac{\Delta \vec{s}}{\Delta t}$

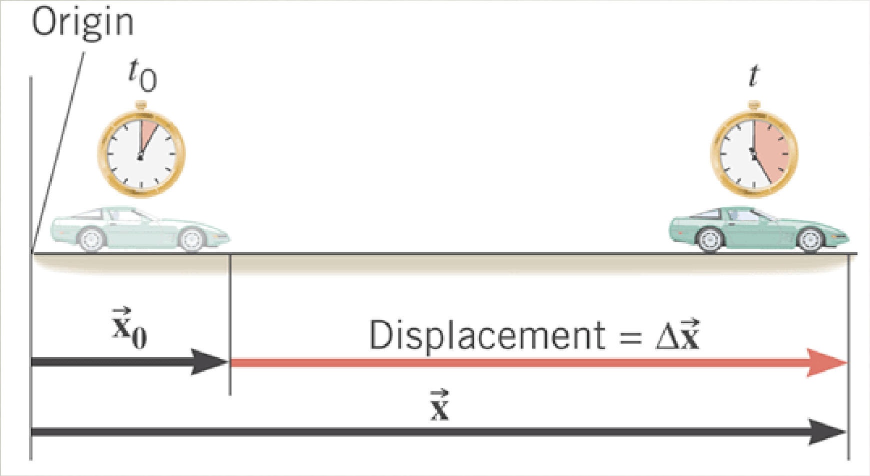
trenutna brzina $\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{s}}{\Delta t}$

AKCELERACIJA (UBRZANJE)





JEDNOLIKO I JEDNOLIKO UBRZANO GIBANJE



prosječna brzina $$\bar{\vec{v}} = \frac{\vec{x}-\vec{x}_0}{t-t_0} = \frac{\Delta\vec{x}}{\Delta t}$$

prosječno ubrzanje $$\bar{\vec{a}} = \frac{\vec{v}-\vec{v}_0}{t-t_0} = \frac{\Delta\vec{v}}{\Delta t}$$

pomak $\Delta x = x - x_0$

Jednoliko gibanje:

$$a = 0$$

$$v = v_0$$

$$x = x_0 + v_0 t$$

Jednoliko ubrzano gibanje:

$$a = \text{const.}$$

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

ako eliminiramo vrijeme:

$$x - x_0 = \tfrac{1}{2}(v_0 + v)t = \frac{v^2 - v_0^2}{2a}$$

varijable: x, x_0, v, v_0, a, t, $t_0 = 0$

Jednoliko gibanje

$$\vec{v} = \frac{d\vec{r}}{dt} \quad \Rightarrow \quad d\vec{r} = \vec{v}dt$$

$$\int d\vec{r} = \int \vec{v}\, dt$$ $v = \text{const},\ v = v_0$

$$\vec{r} + C_1 = \vec{v}_0 \cdot t + C_2$$

$$\vec{r} = \vec{v}_0 \cdot t + C$$ $t = 0,\ r = r_0$

$$\vec{r} = \vec{v}_0 \cdot t + \vec{r}_0$$

Jednoliko ubrzano gibanje

$$\vec{a} = \frac{d\vec{v}}{dt} \quad \Rightarrow \quad d\vec{v} = \vec{a}dt$$

$$\int d\vec{v} = \int \vec{a}\, dt$$ $a = \text{const},\ a = a_0$

$$\vec{v} + C_1 = \vec{a}_0 \cdot t + C_2$$

$$\vec{v} = \vec{a}_0 \cdot t + C$$ $t = 0,\ v = v_0$

$$\vec{v} = \vec{a}_0 \cdot t + \vec{v}_0$$

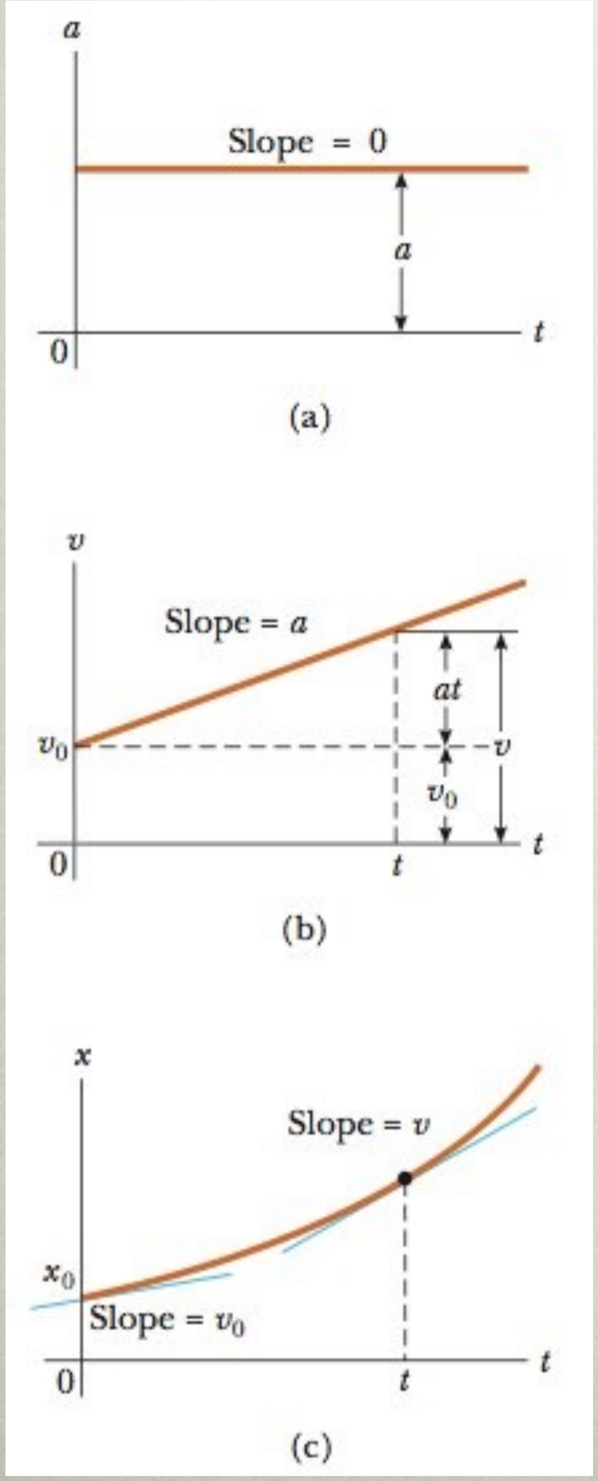
$$\int d\vec{r} = \int \vec{v}\, dt = \int \left(\vec{a}_0 \cdot t + \vec{v}_0\right)$$

$$\vec{r} + C_1 = \tfrac{1}{2}\vec{a}_0 t^2 + \vec{v}_0 \cdot t + C_2$$

$$\vec{r} = \tfrac{1}{2}\vec{a}_0 t^2 + \vec{v}_0 \cdot t + C$$ $t = 0,\ r = r_0$

$$\vec{r} = \tfrac{1}{2}\vec{a}_0 t^2 + \vec{v}_0 \cdot t + \vec{r}_0$$

JEDNOLIKO UBRZANO GIBANJE



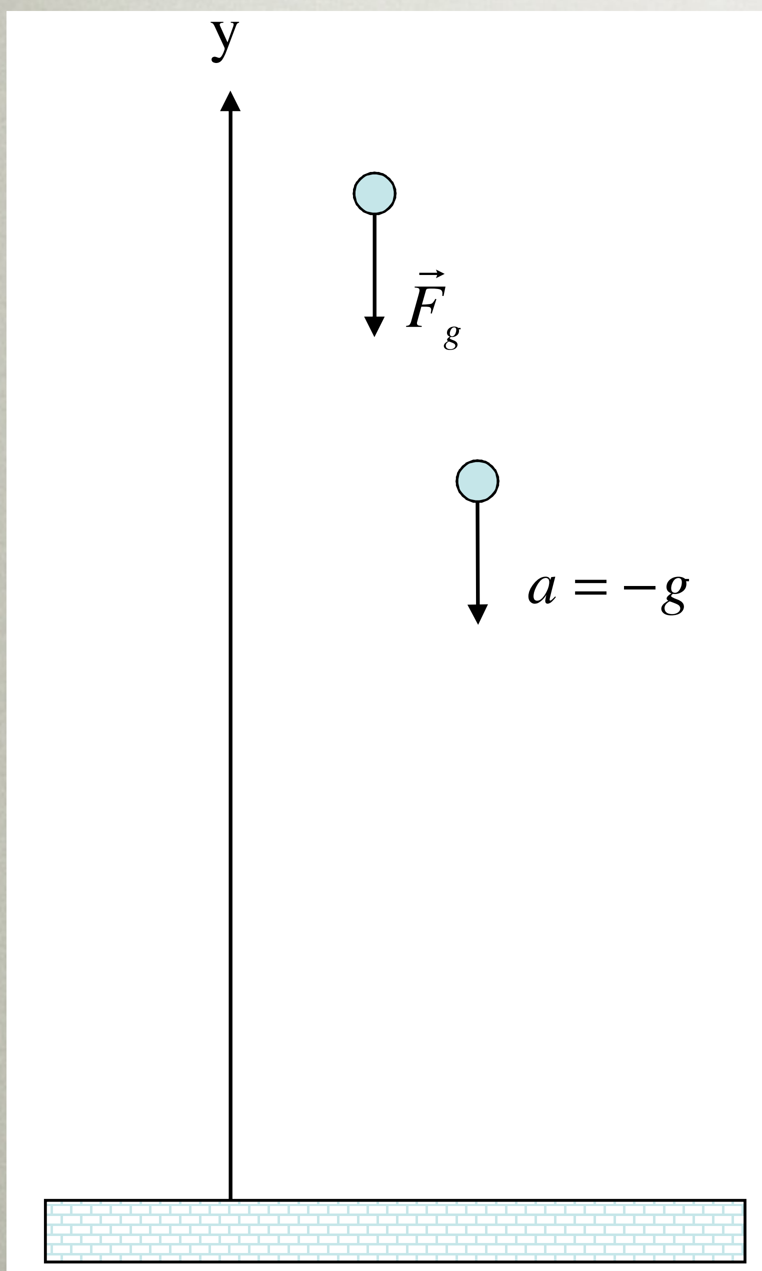
$$v = v_0 + at \qquad \text{(for constant } a\text{)}$$

$$\Delta x = \tfrac{1}{2}(v_0 + v)t$$

$$\Delta x = v_0 t + \tfrac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a\Delta x$$

SLOBODNI PAD



$$F_g = -mg$$

Gravitacijska sila u blizini površine Zemlje - sila teže

$$v = v_0 - gt$$

$$y = y_0 + v_0 t - \tfrac{1}{2} g t^2$$

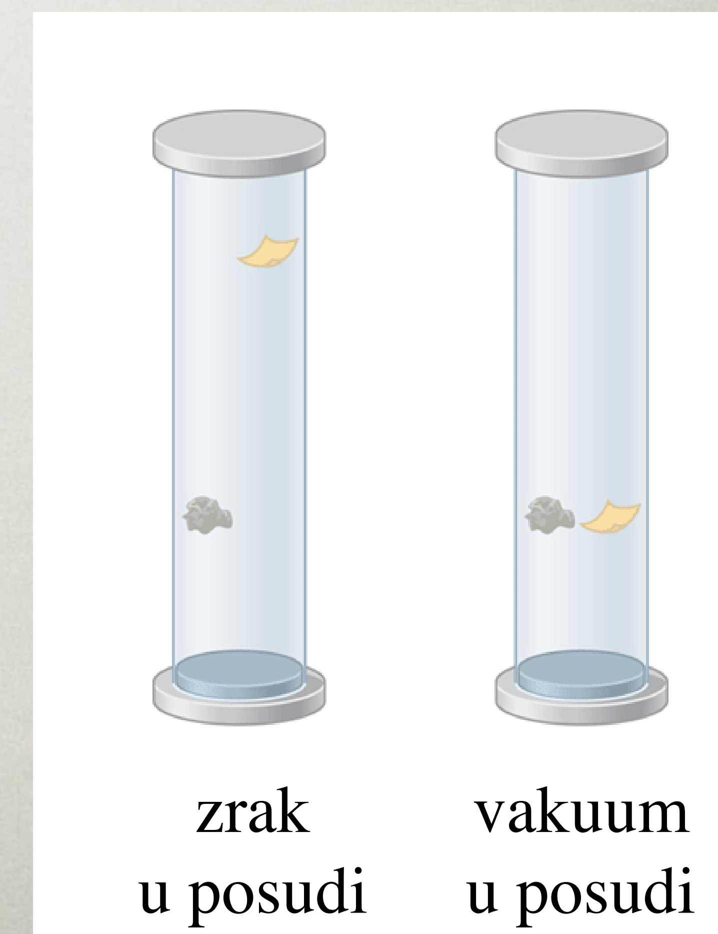
$$g = 9.81\,\mathrm{m/s^2}$$

Jednoliko ubrzano gibanje:

$$a = \text{const.}$$

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$



JEDNOLIKO UBRZANO GIBANJE

- početna brzina $v_0 = 0$ m/s

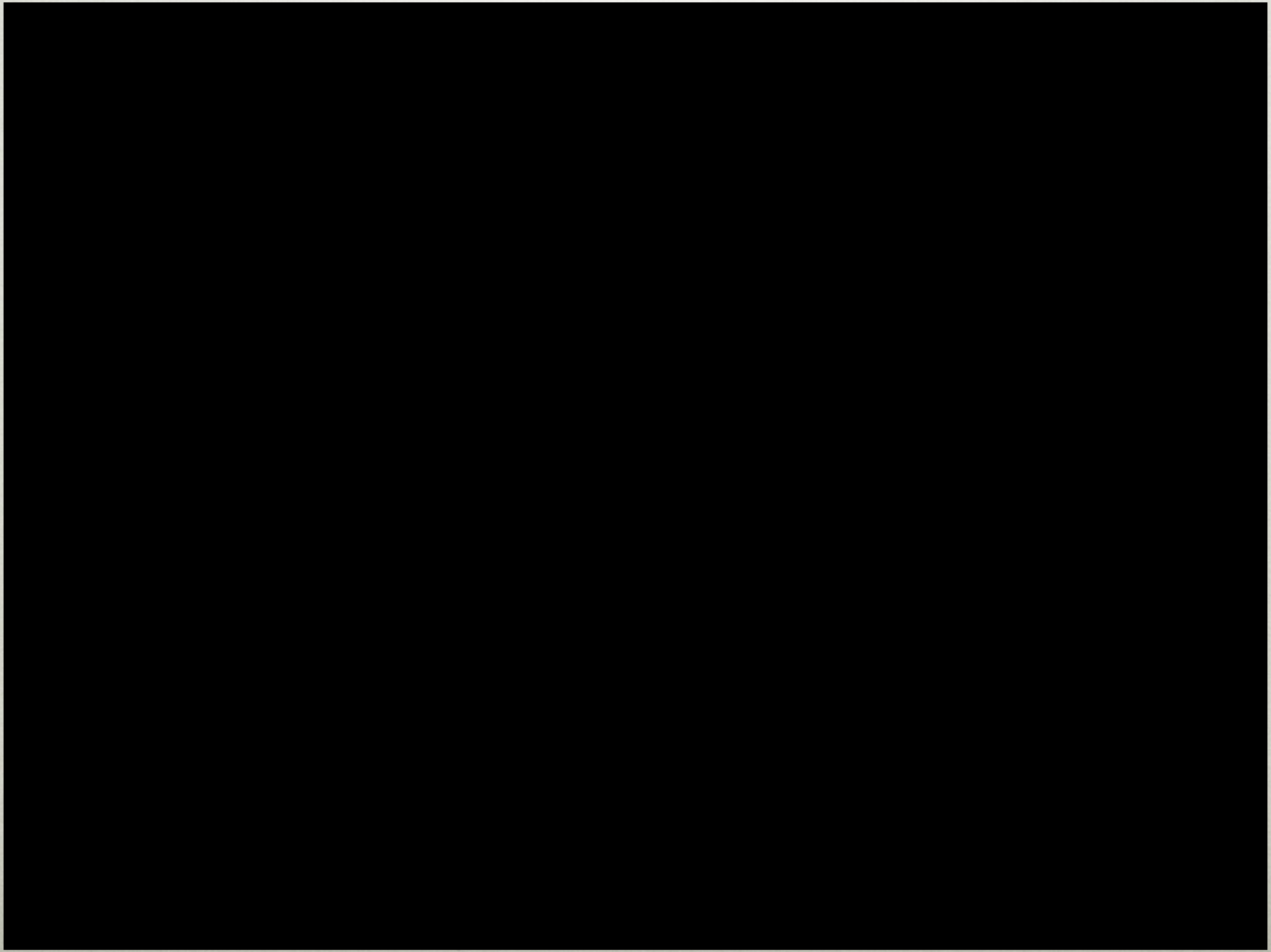
$$s = \frac{1}{2}at^2 \approx 5 \cdot t^2$$

$$y = y_0 + v_0 t - \tfrac{1}{2} g t^2$$
$$v = v_0 - gt$$
$$g = 9.81 \, \mathrm{m/s^2}$$

t (s)	s (m)
1	5
$t_1 = 2$	$s_1 = 2^2 \cdot 5 = 20$
$t_2 = 3$	$s_2 = 3^2 \cdot 5 = 45$
$t_3 = 4$	$s_3 = 4^2 \cdot 5 = 80$

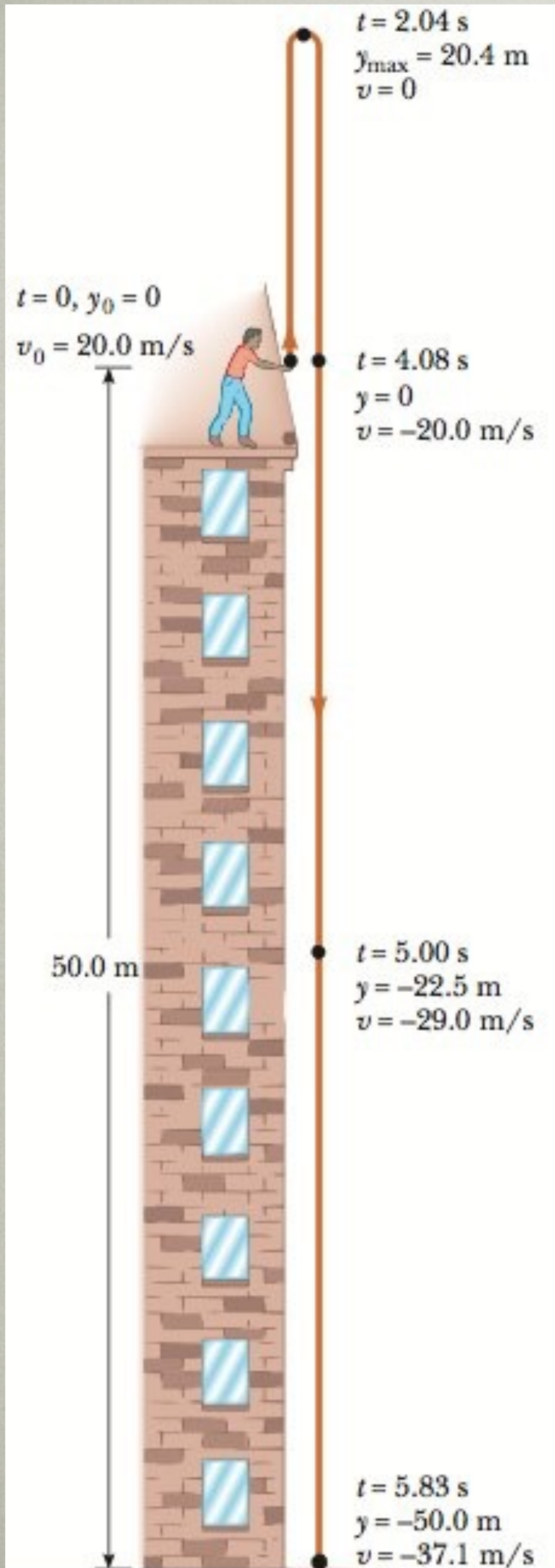
SLOBODNI PAD

SLOBODNI PAD



SLOBODNI PAD

- gustoća plinova na površini mjeseca je 10^{14} puta manja nego na površini Zemlje
- Na, K
- Zemlja: 10^{19} atoma/cm^3, mjesec: $<10^6$ atoma/cm^3



SLOBODNI PAD

$$v = at + v_0$$

$$\Delta y = y - y_0 = v_0 t + \tfrac{1}{2} a t^2$$

$$v = (-9.80 \text{ m/s}^2)t + 20.0 \text{ m/s}$$

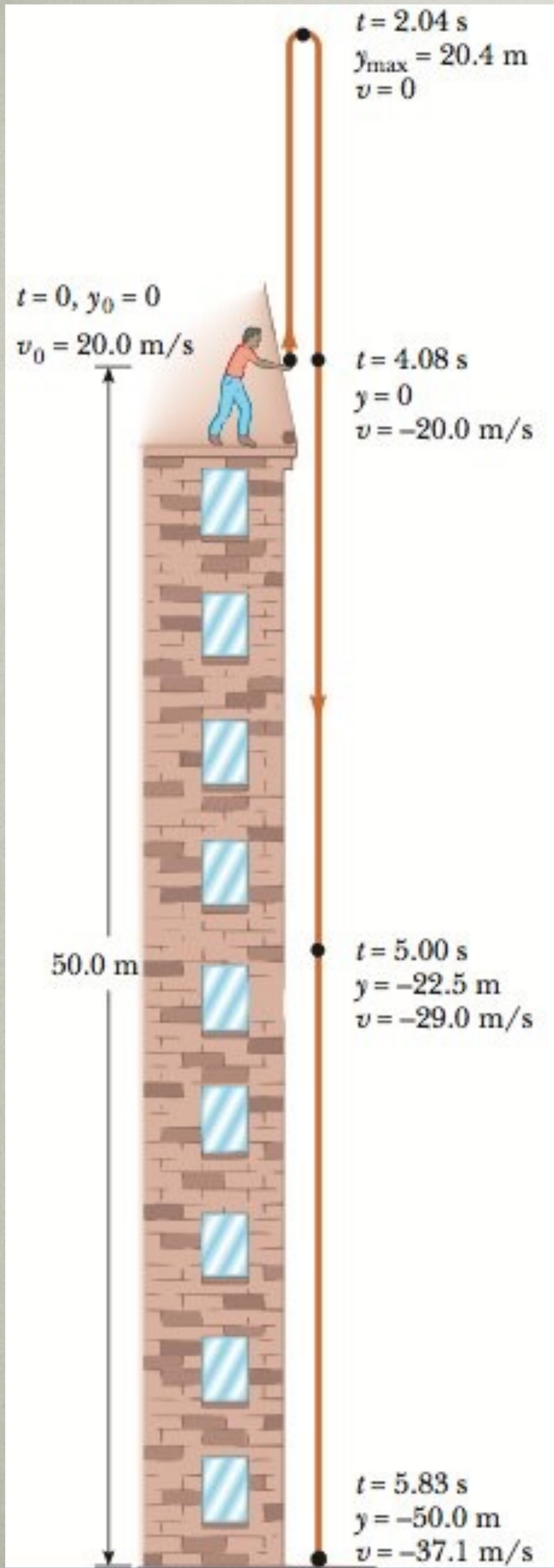
$$y = (20.0 \text{ m/s})t - (4.90 \text{ m/s}^2)t^2$$

maksimalna visina i u kojem trenutku?

$$0 = (-9.80 \text{ m/s}^2)t + 20.0 \text{ m/s}$$

$$t = \frac{-20.0 \text{ m/s}}{-9.80 \text{ m/s}^2} = \boxed{2.04 \text{ s}}$$

$$y_{max} = (20.0 \text{ m/s})(2.04) - (4.90 \text{ m/s}^2)(2.04)^2 = \boxed{20.4 \text{ m}}$$



SLOBODNI PAD

kada će kamen opet biti u početnom položaju i koja će tada biti njegova brzina?

$$0 = (20.0 \text{ m/s})t - (4.90 \text{ m/s}^2)t^2$$
$$= t(20.0 \text{ m/s} - 4.90 \text{ m/s}^2\, t)$$
$$t = 4.08 \text{ s}$$

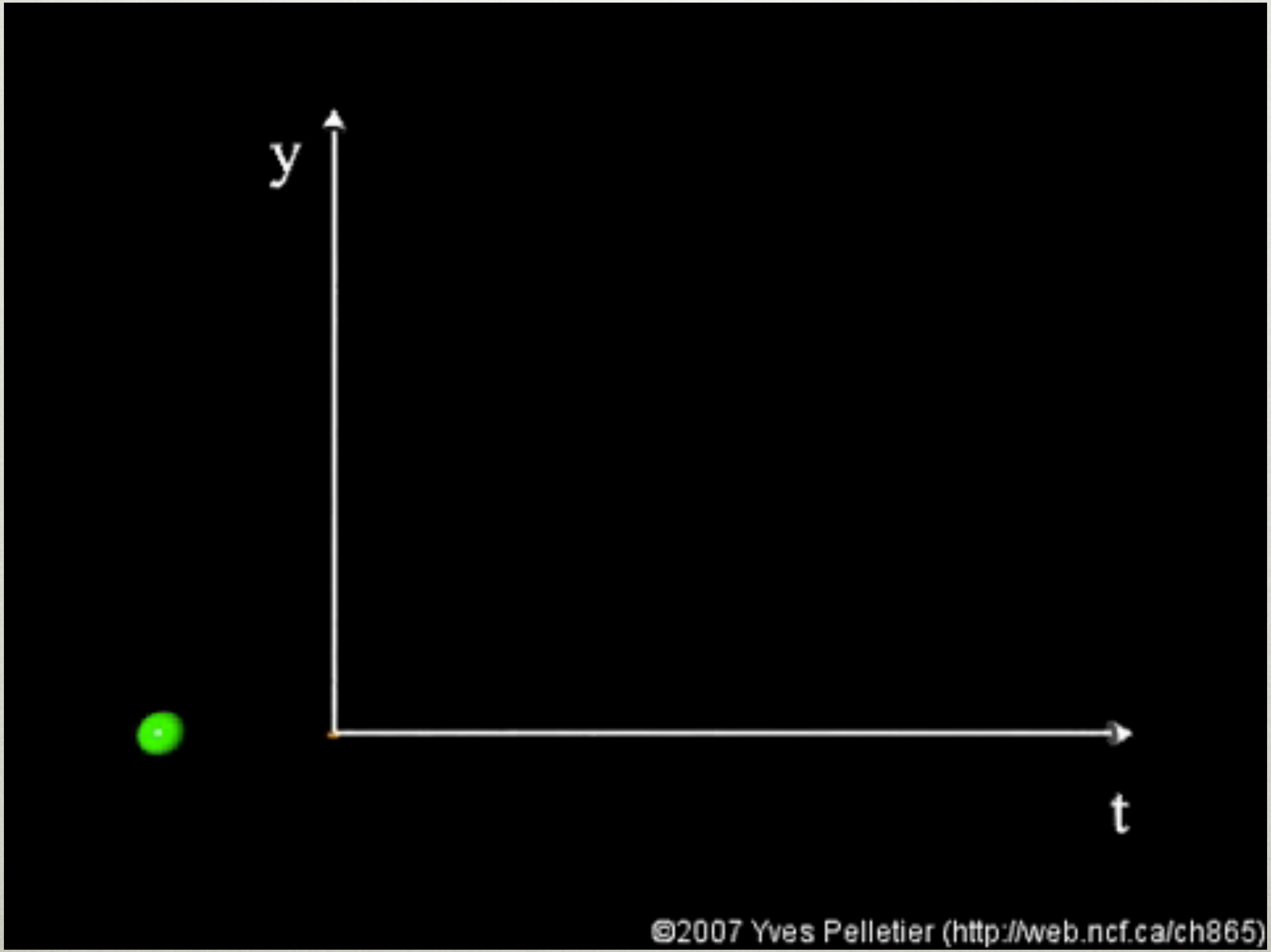
$$v = 20.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(4.08 \text{ s}) = -20.0 \text{ m/s}$$

kada će kamen pasti na zemlju?

$$-50.0 \text{ m} = (20.0 \text{ m/s})t - (4.90 \text{ m/s}^2)t^2$$

$$t = 5.83 \text{ s}$$

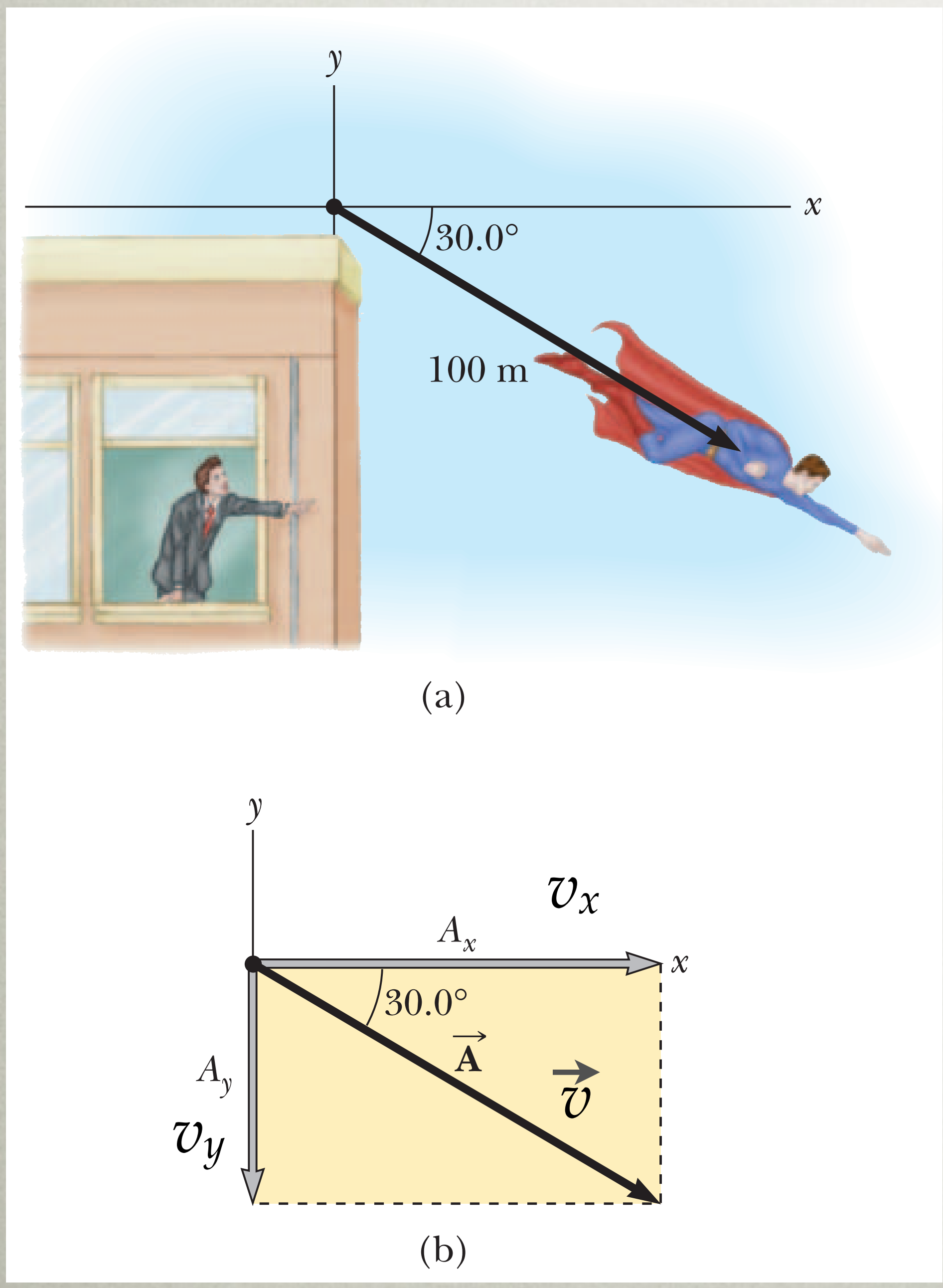
JEDNOLIKO UBRZANO GIBANJE



KONCEPTUALNA PITANJA:

- Dijete baca kamen u zrak s početnom brzinom v_0. Drugo dijete u istom trenutku ispušta loptu. Usporedite ubrzanja ova dva tijela dok se nalaze u zraku.
- Lopta je bačena vertikalno u vis. (a) Koliki su njena brzina i ubrzanje u trenutku kada doseže maksimalnu visinu? (b) Koliko je ubrzanje lopte u trenutku baš prije nego što padne na tlo?

SLOŽENO GIBANJE



Početnu brzinu rastavljamo na komponente

Početni uvjeti:

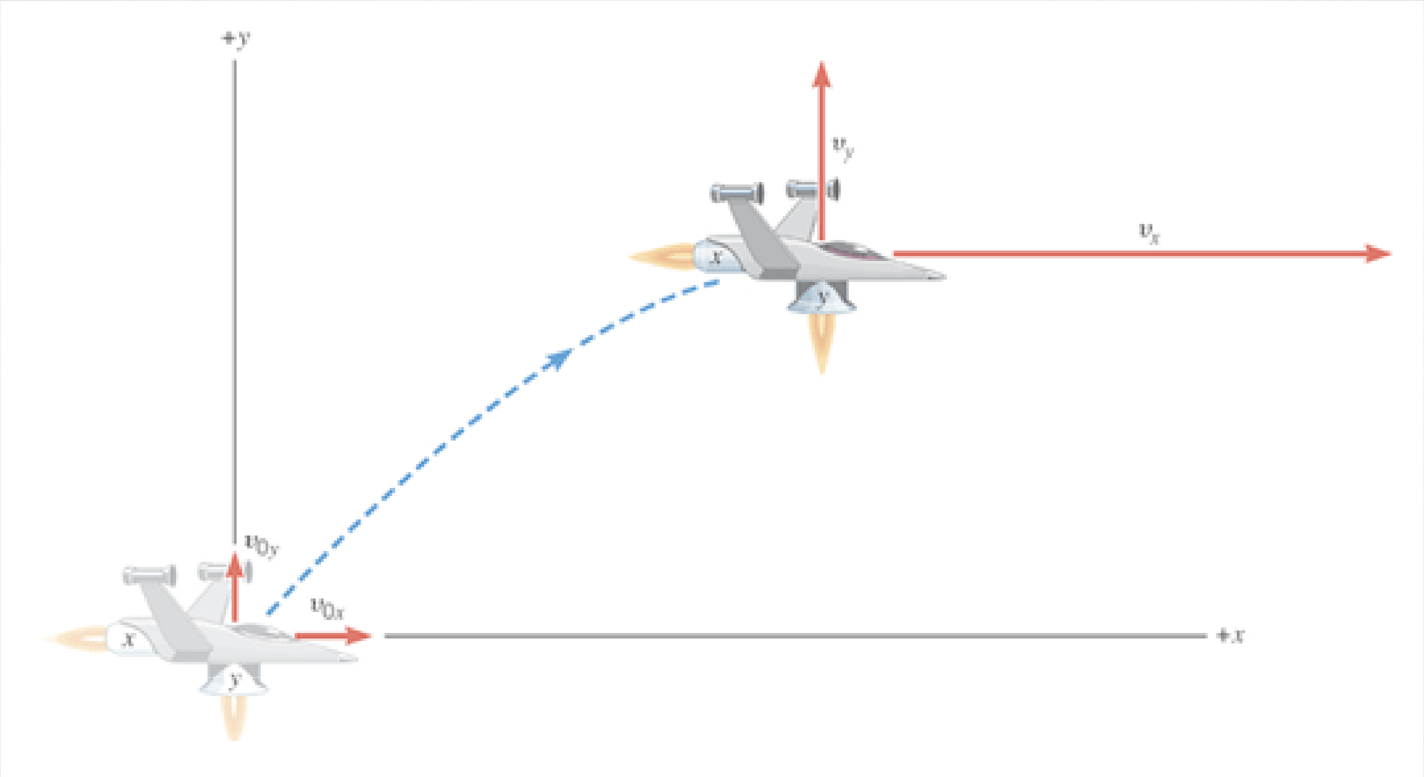
x_0, v_{0x}, a_x

y_0, v_{0y}, a_y

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(v_{0x} + a_x t\right)^2 + \left(v_{0y} + a_y t\right)^2}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{v_{0y} + a_y t}{v_{0x} + a_x t}\right)$$

JEDNADŽBE KINEMATIKE U 2D



x - komponenta

$$x = x_0 + v_{ox}t + \tfrac{1}{2}a_x t^2$$

$$v = v_{0x} + a_x t$$

$$a_x = \text{konst.}$$

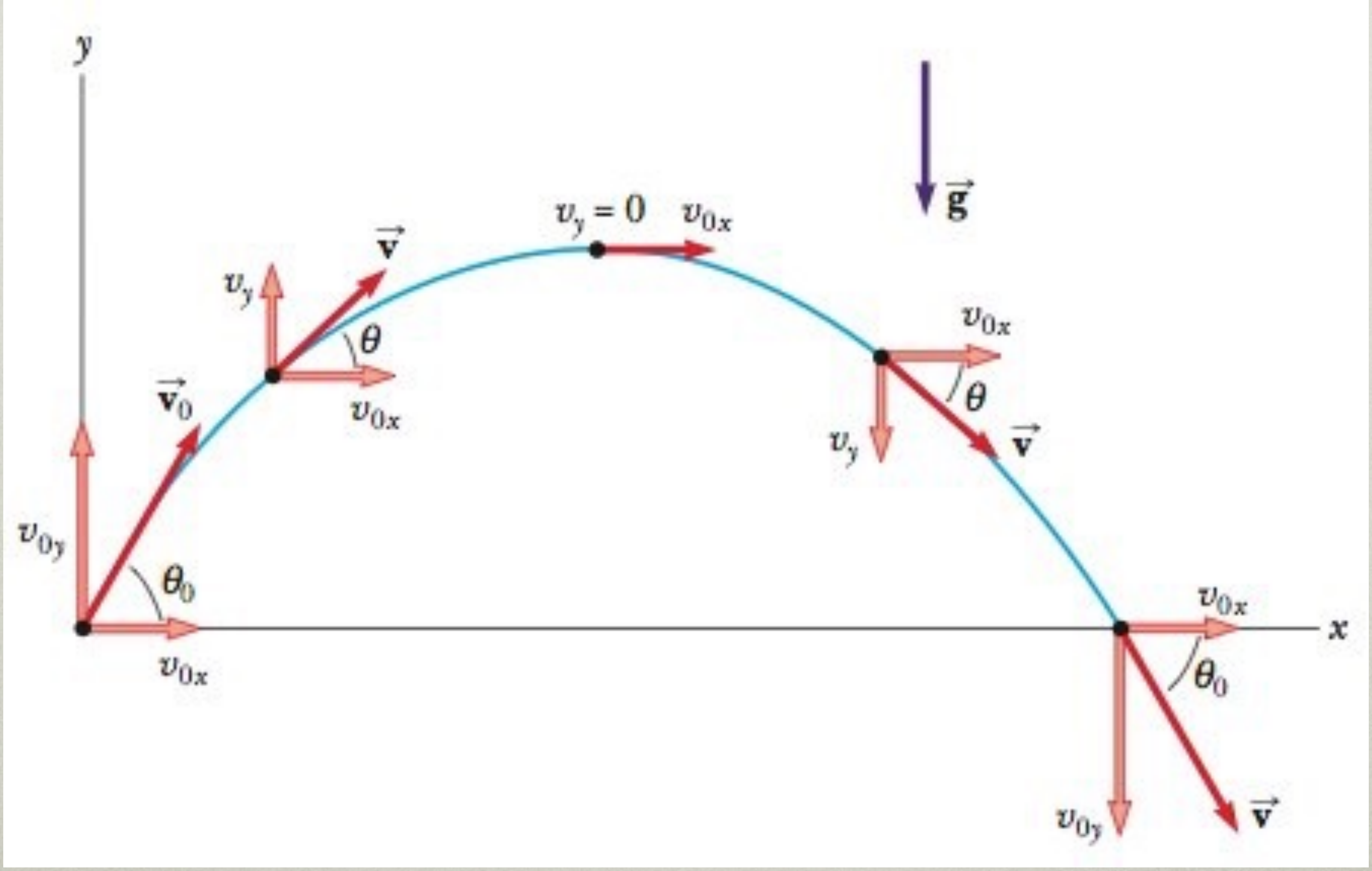
y - komponenta

$$y = y_0 + v_{oy}t + \tfrac{1}{2}a_y t^2$$

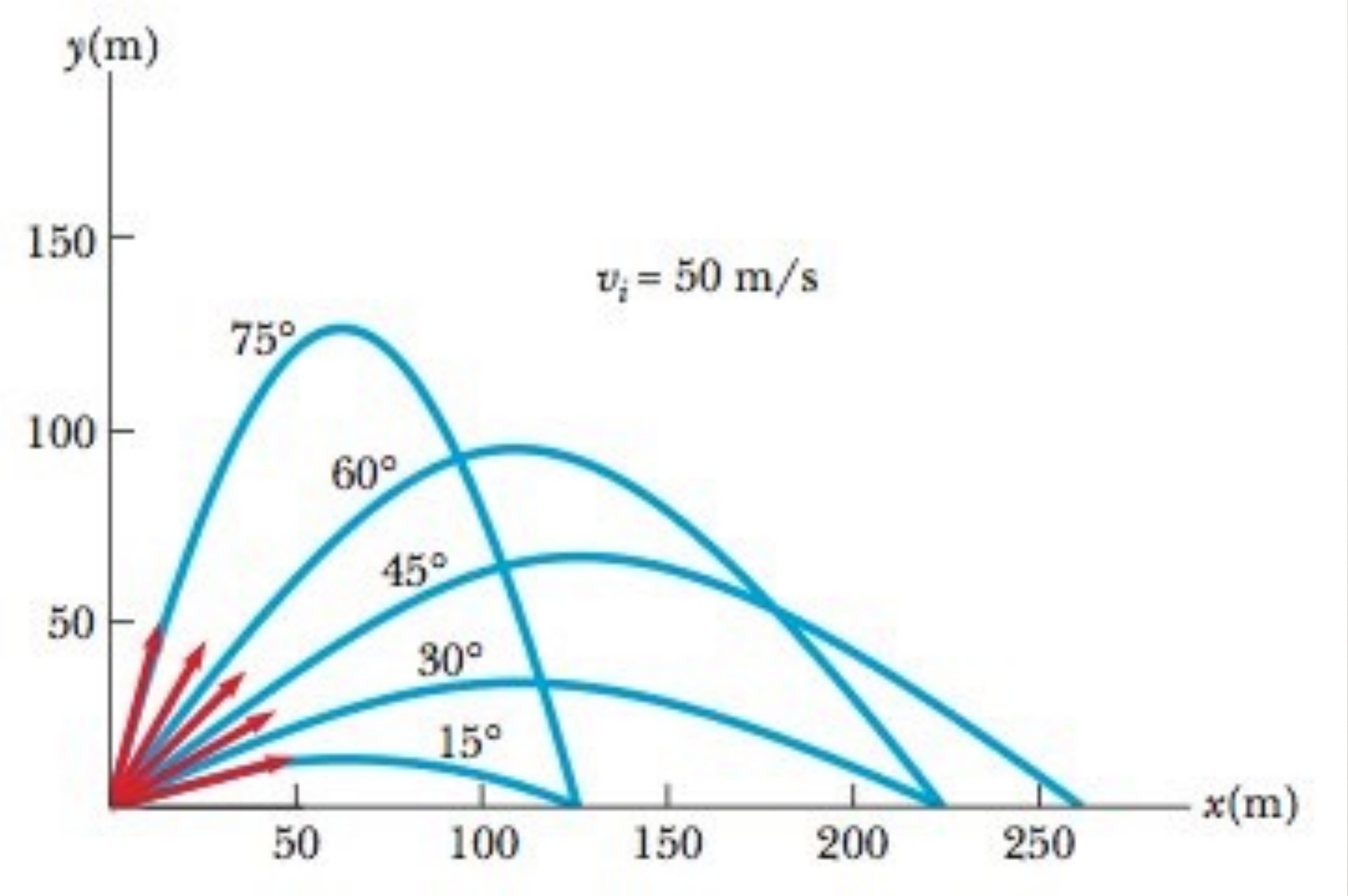
$$v = v_{0y} + a_y t$$

$$a_y = \text{konst.}$$

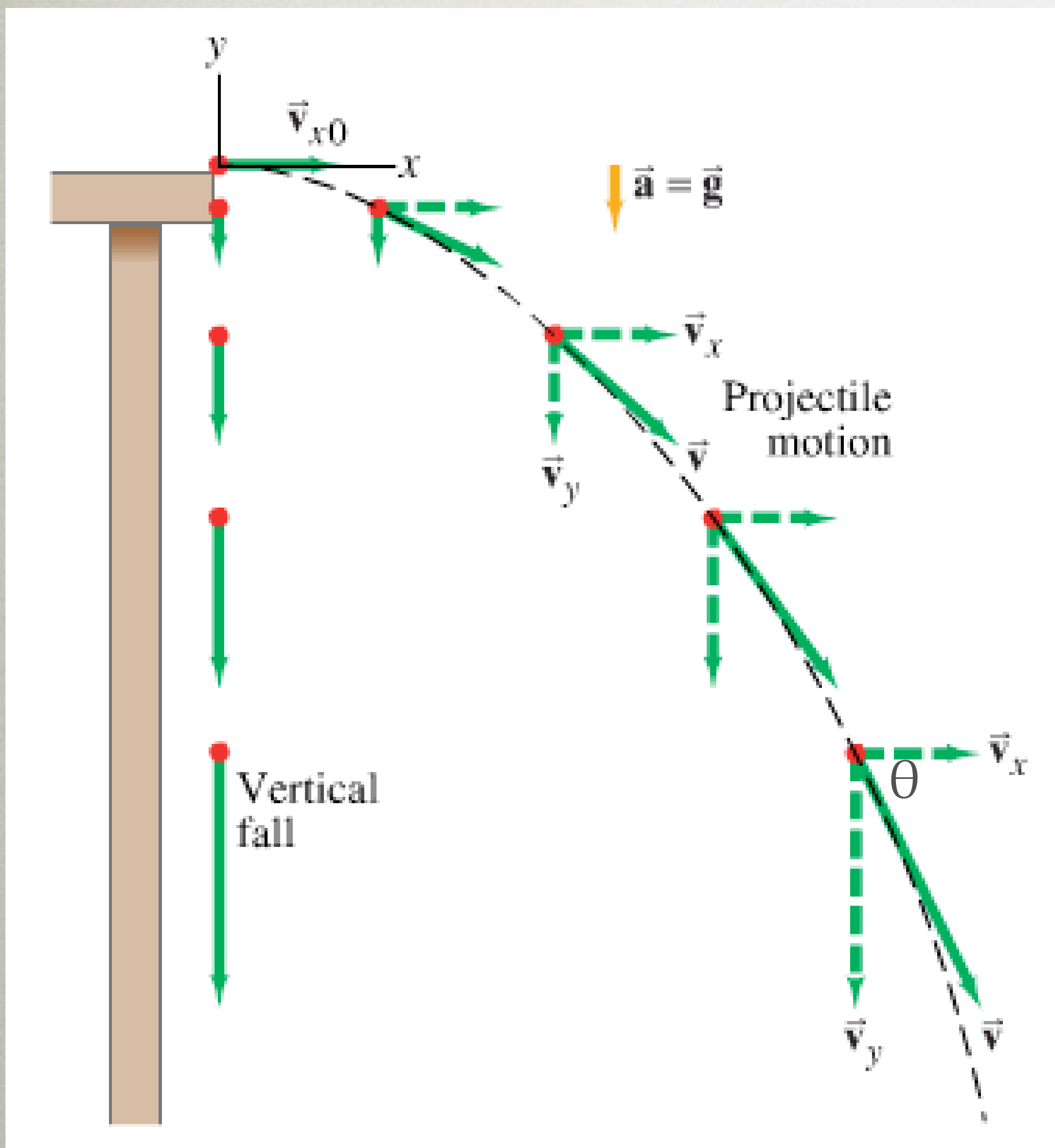
GIBANJE U DVIJE DIMENZIJE-HITAC



GIBANJE U DVIJE DIMENZIJE-HITAC



Specijalan slučaj: hici



x - komponenta

$$x = x_0 + v_{ox}t$$
$$v_x = v_{0x}$$
$$a_x = 0$$

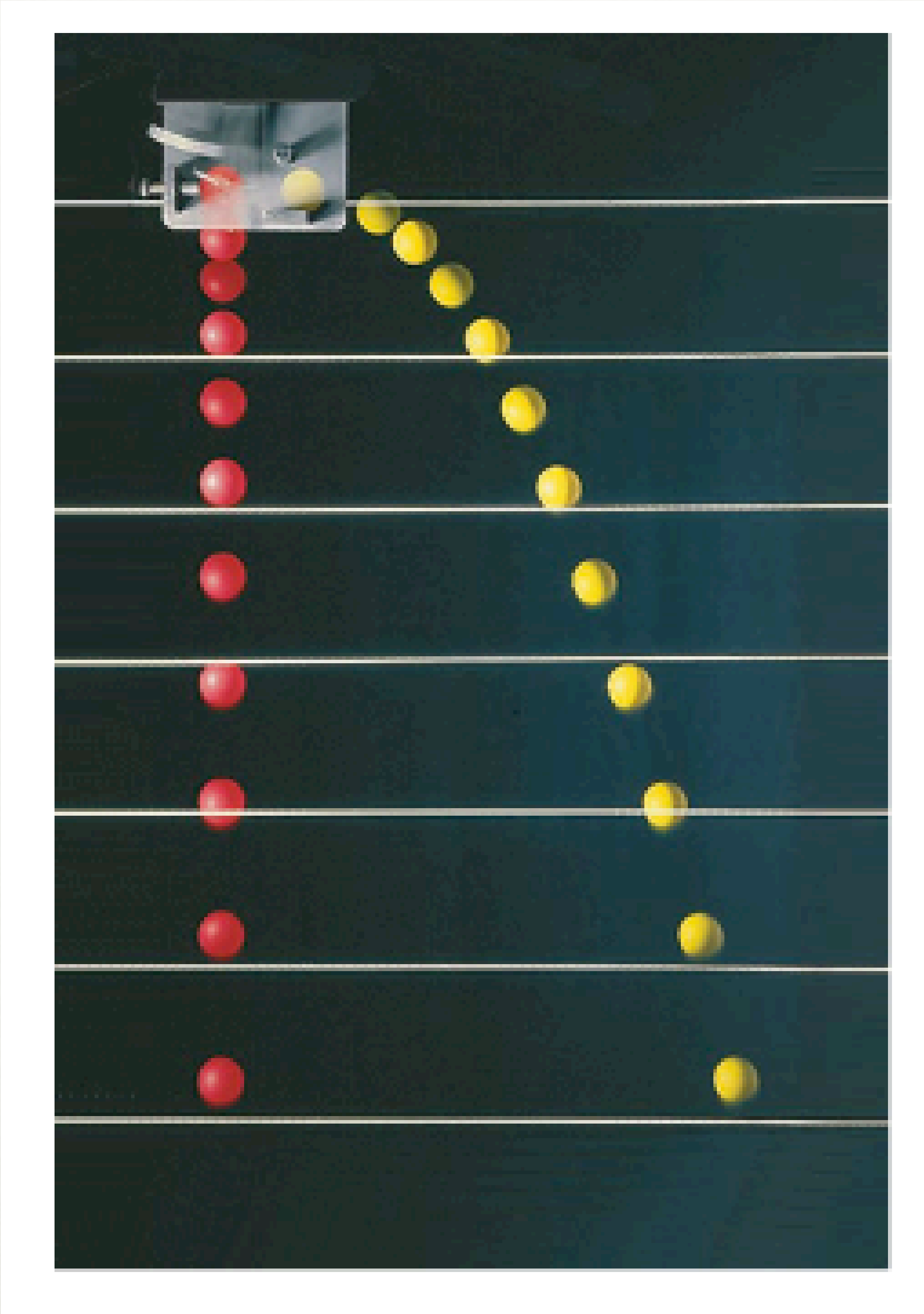
y - komponenta

$$y = y_0 + v_{oy}t - \tfrac{1}{2}gt^2$$
$$v_y = v_{0y} - gt$$
$$a_y = -g$$

$$v = \sqrt{v_x^2 + v_y^2}$$
$$v_x = v_{0x} \qquad v_y^2 = v_{0y}^2 + 2a_y y$$

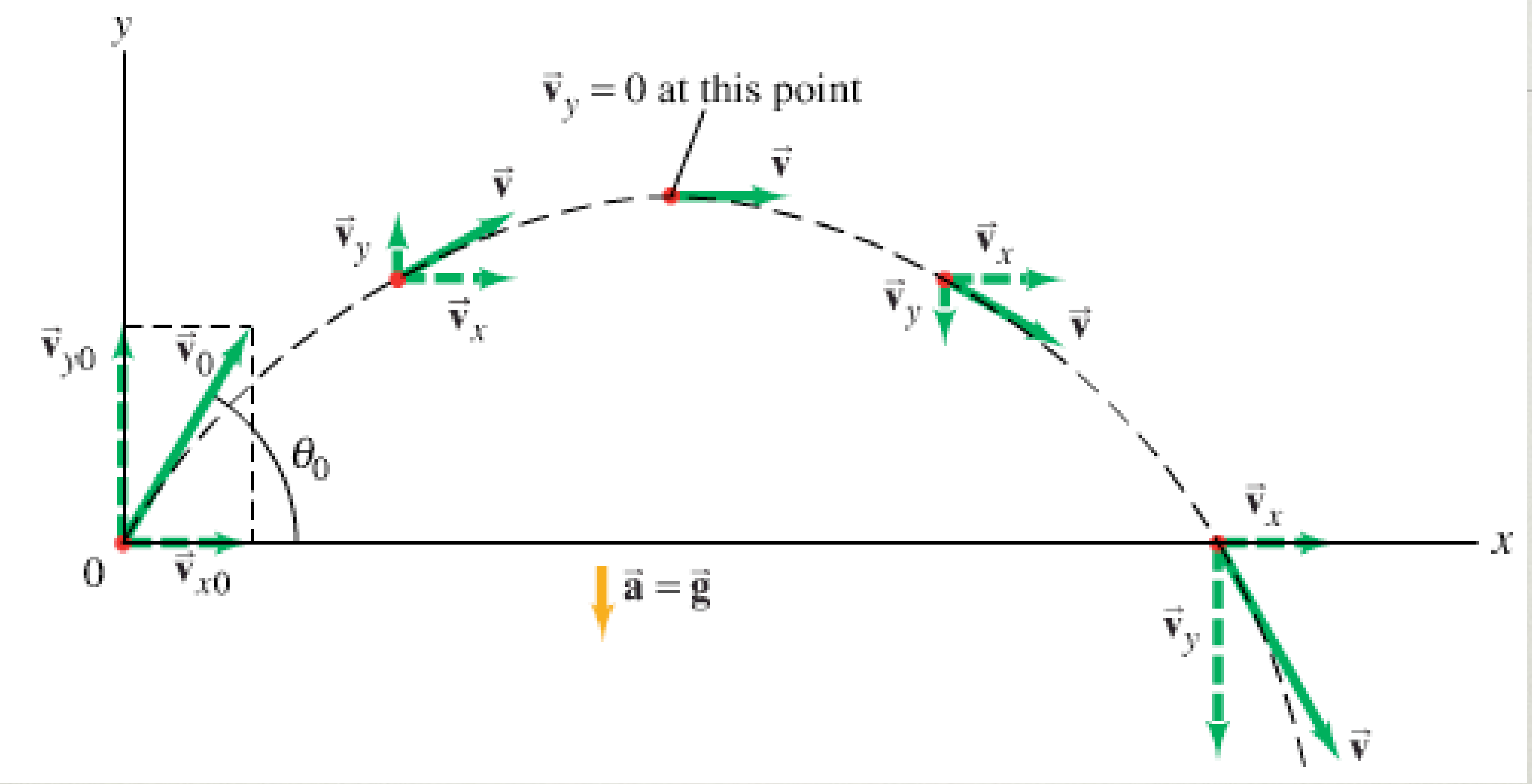
$$\theta = \cos^{-1}\left(\frac{v_x}{\sqrt{v_x^2 + v_y^2}}\right)$$
$$v_x = v_{0x} \qquad v_y^2 = v_{0y}^2 + 2a_y y$$

Konceptualno pitanje



Da li ova dva tijela u istom trenutku padaju na tlo?

KOSI HITAC



x - komponenta

$$x = x_0 + v_{xo}t$$

$$v = v_{x0}$$

$$a_x = 0$$

y - komponenta

$$y = y_0 + v_{y0}t - \tfrac{1}{2}gt^2$$

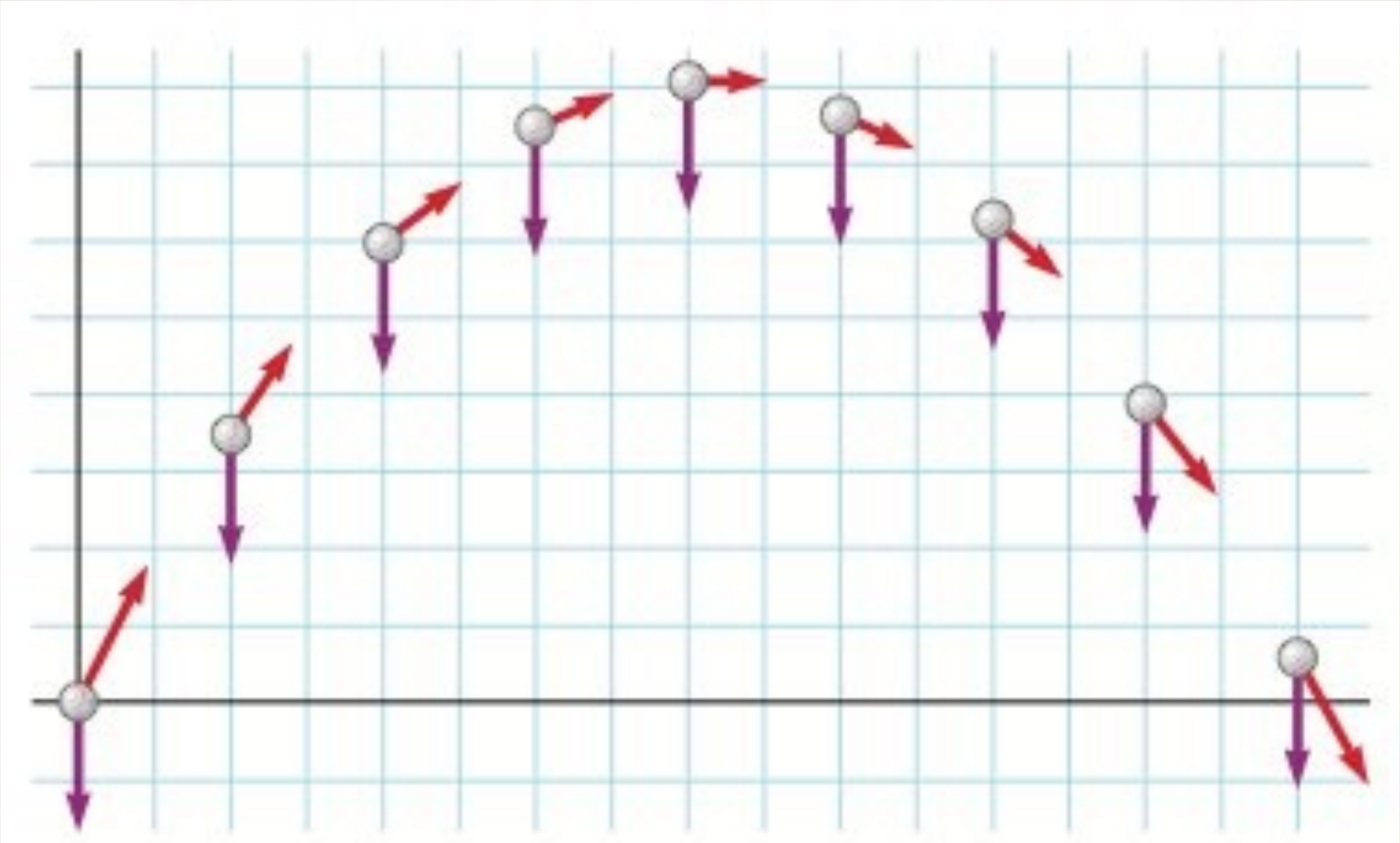
$$v_y = v_{y0} - gt$$

$$v_y^2 = v_{y0}^2 - 2g(y - y_0)$$

$$a_y = -g = \text{konst.}$$

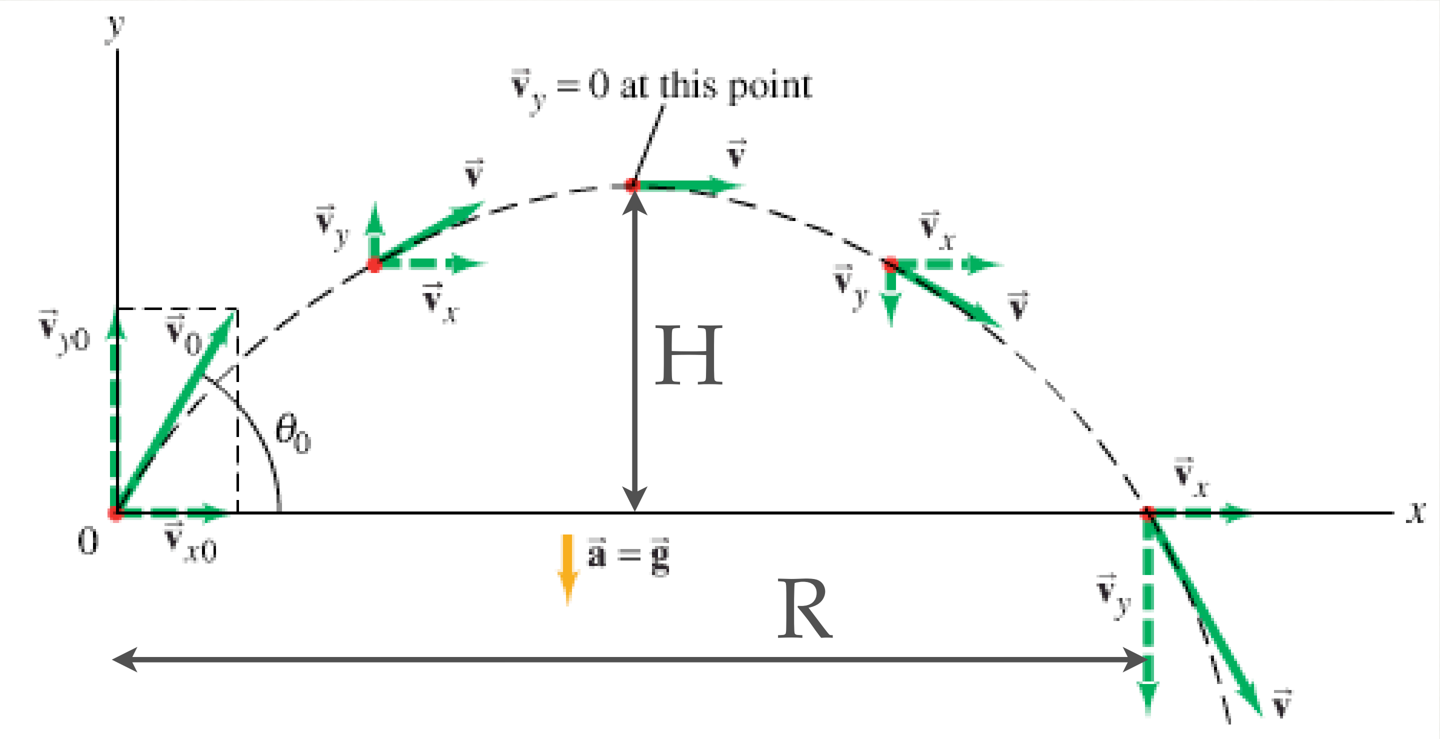
$$y = \tan\theta_0 \cdot x - \frac{g}{2v_0^2\cos^2\theta_0} \cdot x^2 = C_1 \cdot x + C_2 \cdot x^2$$

KOSI HITAC



dijagram gibanja za kosi hitac

KOSI HITAC



Koja je maksimalna visina H koju može dostići tijelo izbačeno početnom brzinom v_0 (poznati kut i iznos brzine)?

- $v_y = 0 \longrightarrow$ odredimo $t_H \longrightarrow$ izračunamo $H = y(t_H)$

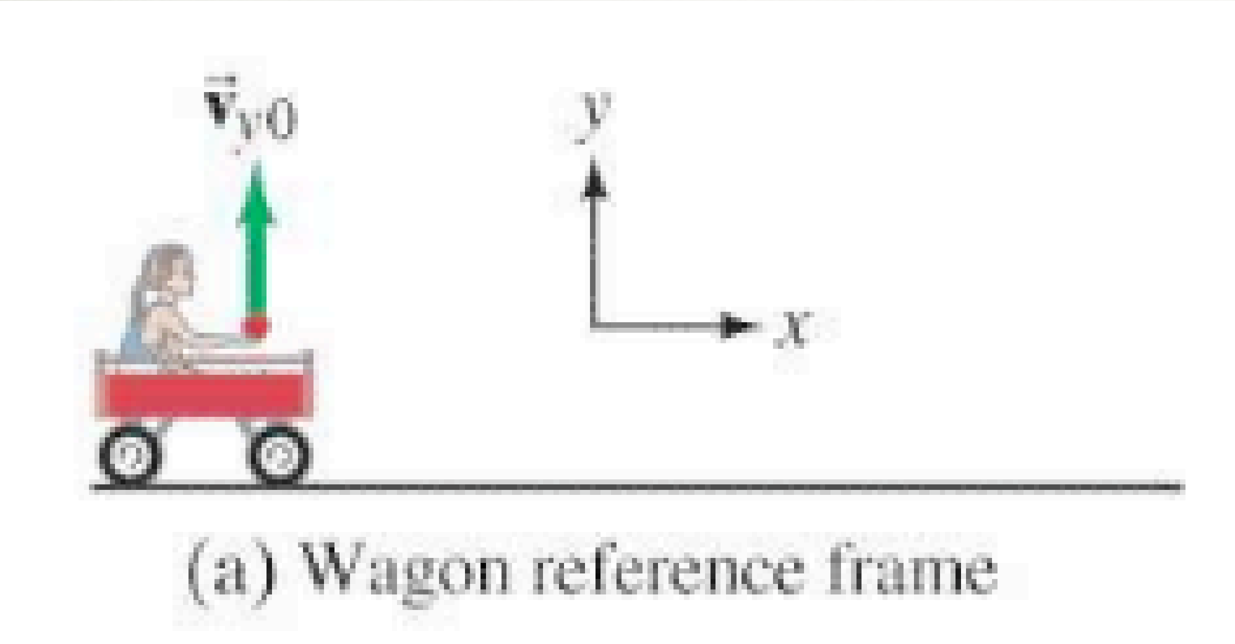
Koliki je domet tijela, R?

$x = 0 \longrightarrow$ odredimo $t_R \longrightarrow$ izračunamo $R = x(t_R)$

Pokažite da vrijedi $t_R = 2\ t_H$

KOSI HITAC

Konceptualna pitanja:

1. 

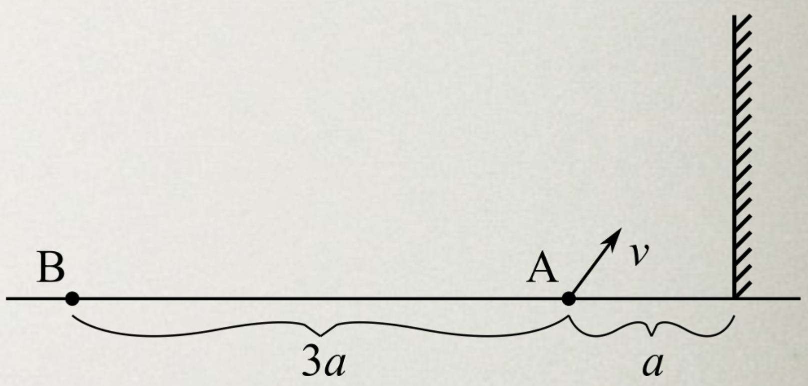
Kolica se gibaju brzinom v_x u smjeru osi x. Ukoliko je lopta izbačena vertikalno u vis brzinom v_{yo} gdje će, u odnosu na kolica, pasti?

2. Dvije kugle kotrljaju se po stolu različitim brzinama. Dolaze do ruba stola u istom trenutku. Koja kugla će prije pasti na tlo, sporija ili brža?

Zadatak

Malo tijelo izbačeno je iz točke *A* brzinom *v* prema vertikalnom zidu od kojeg se savršeno elastično odbija. Udaljenost točke *A* od zida je a = 2 m. Izračunajte:

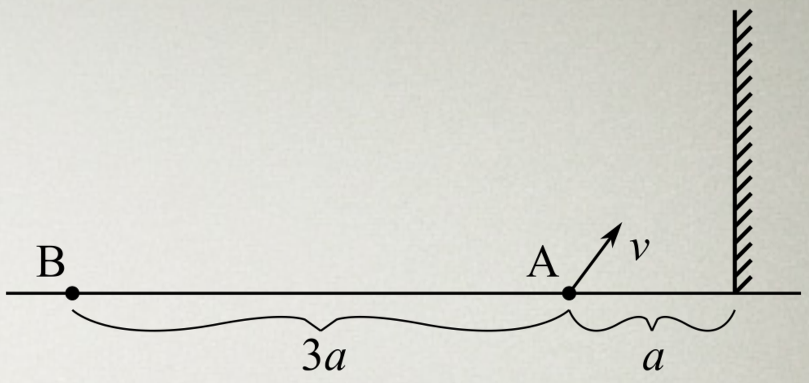
a) minimalnu brzinu *v* takvu da tijelo padne na tlo u točki *B*,
b) visinu na kojoj malo tijelo udari o zid i
c) maksimalnu visinu koju malo tijelo postigne za vrijeme leta te horizontalnu udaljenost od zida u istom trenutku.



Za određenu brzinu *v* tijelo postiže maksimalan domet, ako je ispaljeno pod kutom 45° **(1 bod)**. Nakon odbijanja od zida horizontalna komponenta brzine promijeni smjer dok je vertikalna komponenta brzine nepromijenjena **(1 bod)**. Gibanje malog tijela ekvivalentno je kosom hicu sa smjerom početne brzine 45° u odnosu na horizontalu i dometom 5*a* **(1 bod)**. Možemo napisati sljedeće jednadžbe:

$v_{0x} = \frac{1}{\sqrt{2}} v_0$, $v_{0y} = \frac{1}{\sqrt{2}} v_0$ **(2 boda)**

$5a = v_{0x} T$ **(1 bod)**



$$v_{0y} = g\frac{T}{2} \;\Rightarrow\; T = \frac{2v_{0y}}{g}$$ **(2 boda)**

Uvrštavanjem prve i treće jednadžbe u drugu dobije se:

$$5a = \left(\frac{v_0}{\sqrt{2}}\right)^2 \frac{2}{g} \;\Rightarrow\; v_0^2 = 5ag \;\Rightarrow\; v_0 = \sqrt{5ag} = 10\ \text{m/s}$$ **(2 boda)**

Tijelo udari o zid u trenutku:

$$a = v_{0x}t \;\Rightarrow\; t = \frac{a}{v_{0x}}$$ **(1 bod)**

Te se tada nalazi na visini:

$$h = v_{0y}t - \frac{1}{2}gt^2 = v_{0y}\frac{a}{v_{0x}} - \frac{1}{2}g\left(\frac{a}{v_{0x}}\right)^2 = a - \frac{ga^2}{v_0^2} = 1.6\ \text{m}$$ **(2 boda)**

Maksimalna visina, koju postiže tijelo, jednaka je:

$$H = \frac{1}{2}g\left(\frac{T}{2}\right)^2 = \frac{1}{2}g\left(\frac{v_{0y}}{g}\right)^2 = \frac{v_0^2}{4g} = 2.5\ \text{m}$$ **(2 boda)**

Horizontalna udaljenost od zida u tom trenutku jednaka je $4a - 2.5a = 1.5a = 3$ m **(1 bod)**