

Note for two-fermion tunneling [2021.12.06]

Tomohiro Oishi (toishi@phy.hr)

1 Two-fermion tunneling

In Ref. [1], we investigated the quantum tunneling process of two fermions in one dimensional space. The Hamiltonian of this two-fermion (2F) system is given as follows.

$$\begin{aligned}\hat{H} &= h_1(x_1) + h_2(x_2) + v_{12}(|x_1 - x_2|), \\ h_i &= \left[-\frac{\hbar^2}{2m} \frac{d^2}{dx_i^2} + V_c(x_i) \right],\end{aligned}\tag{1}$$

where x_i is the coordinate of the i th fermion, and m is the one-fermion mass. Thus, h_i indicates the single-particle (s.p.) Hamiltonian, whereas v_{12} is the interaction between the two fermions. For $V_c(x)$, we employ a simple, square-well-plus-barrier potential. For the two-fermion interaction $v_{12}(|x_1 - x_2|)$, on the other hand, a square-well attraction is used: we employ the finite-range force between the two fermions. With this Hamiltonian \hat{H} , we simulated the quantum-tunneling process as a time-dependent evolution of the 2F state, $\Phi(t, x_1, x_2)$. That is,

$$|\Phi(t)\rangle \equiv \exp \left[-it \frac{\hat{H}}{\hbar} \right] |\Phi(0)\rangle.\tag{2}$$

In order to solve this time-dependent equation, we expand the initial state, $|\Phi(0)\rangle$, on the eigenstates of the total Hamiltonian. Then, one can follow the evolution in time as

$$|\Phi(t)\rangle = \sum_N C_N(t) |\Psi_N\rangle,\tag{3}$$

with $C_N(t) = \exp \left[-it \frac{E_N}{\hbar} \right] C_N(0)$. Then in Ref. [1], from this time-dependent simulation, we found that the 2F interaction v_{12} is essential to control the tunneling probability, which can be connected with e.g. the two-proton radioactivity [2]. Note that the time-dependent state as well as each eigenstate of \hat{H} has the quantum entanglement with the total spin $S_{12} = 0$.

2 Current interest

I am now planning to expand the previous model to take the density dependence of the 2F interaction into account. That is,

$$v_{12} = v_{12}(\rho(t, x_1, x_2)),\tag{4}$$

where $\rho(t, x_1, x_2)$ is the time-dependent density of the 2F state: $\rho(t, x_1, x_2) \equiv |\Phi(t, x_1, x_2)|^2$. In this case, however, the numerical computation turns to become very difficult, since the time-development operator, $\exp \left[-it \frac{\hat{H}}{\hbar} \right]$, is also time-dependent via the state self-consistently. In such a case, we must solve the equation of motion at each step of time, where a huge amount of computational resource is necessary as long as we are using the classical digital computers.

In this situation, now I am studying whether the employment of quantum computers can hasten this computation or not. Especially I search for the quantum-computing algorithm suitable to (i) the solution of 2F Schroedinger equation with entanglement, and (ii) the time-development process via the time-dependent Hamiltonian. I will be happy if you could inform the corresponding information and/or literature.

References

- [1] Tomohiro Oishi, Lorenzo Fortunato, and Andrea Vitturi, J. Phys. G: Nucl. Part. Phys 45(10), 105101 (2018), with supplemental note in arXiv: arXiv:1810.05521 (2018).
- [2] Tomohiro Oishi, Markus Kortelainen, and Alessandro Pastore, Phys. Rev. C 96, 044327 (2017), and references therein.