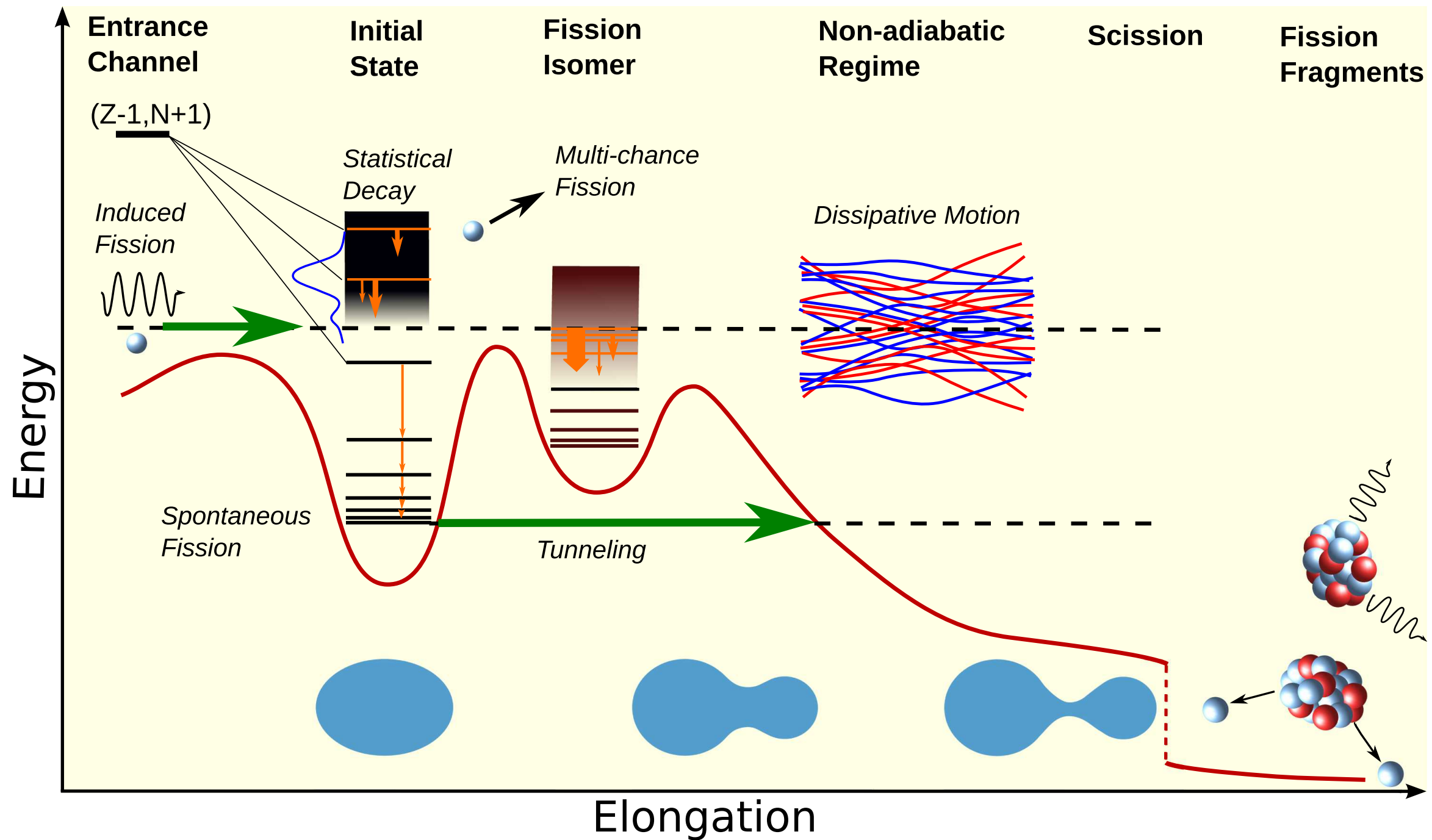


# Microscopic Analysis of Induced Nuclear Fission Dynamics



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University of Zagreb





Two basic microscopic approaches to the description of induced fission dynamics:

### The time-dependent generator coordinate method (TDGCM)

$$|\Psi(t)\rangle = \int_{\mathbf{q} \in E} d\mathbf{q} |\phi(\mathbf{q})\rangle f(\mathbf{q}, t). \quad \Rightarrow \text{represents the nuclear wave function by a superposition of generator states that are functions of collective coordinates.}$$

$\Rightarrow$  a fully quantum mechanical approach but only takes into account collective degrees of freedom in the adiabatic approximation.

$\Rightarrow$  no dissipation mechanism.

### TDGCM in the Gaussian overlap approximation (TDGCM+GOA)

#### Example

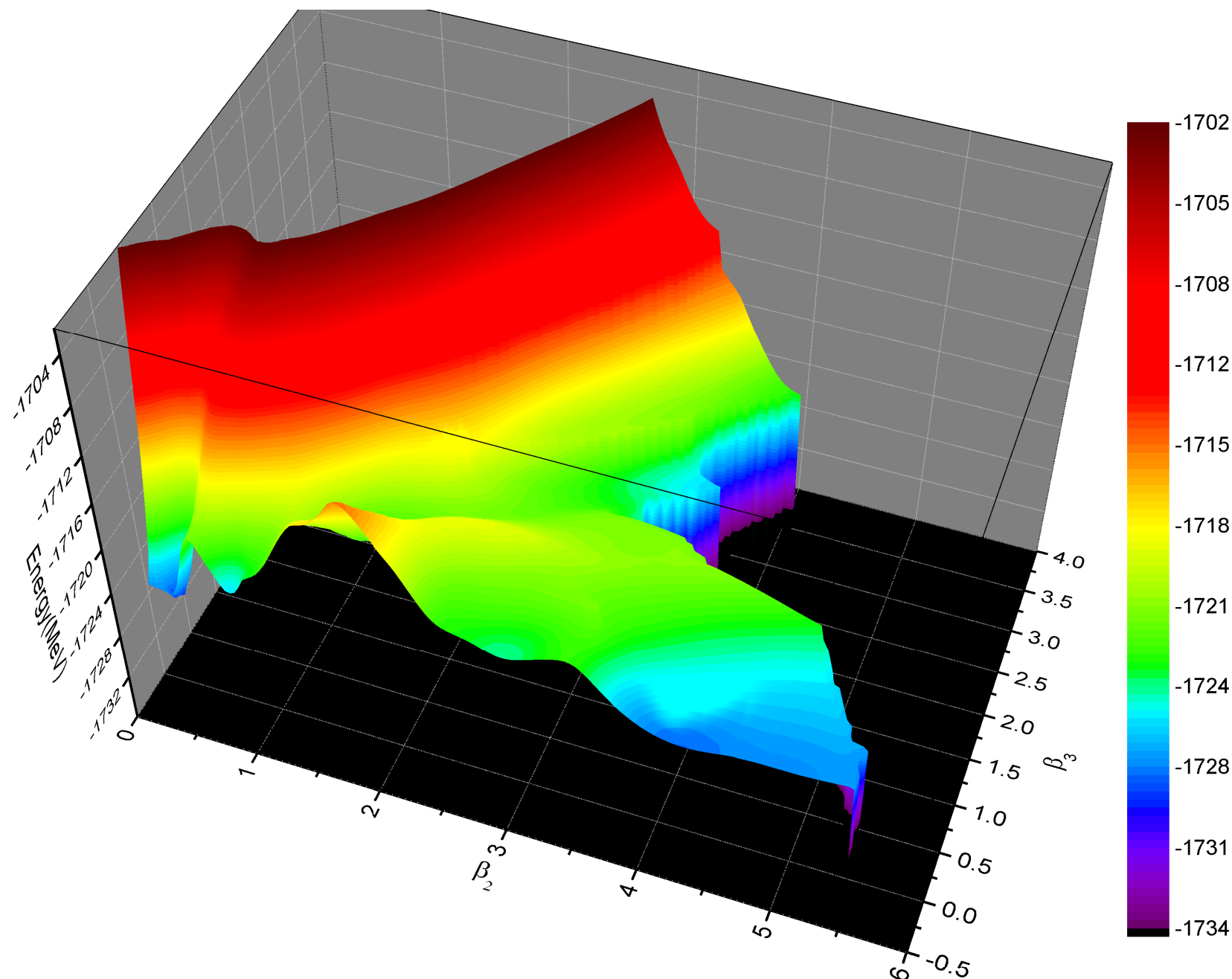
Time-dependent Schroedinger-like equation for fission dynamics (axial quadrupole and octupole deformation parameters as collective degrees of freedom):

$$i\hbar \frac{\partial}{\partial t} g(\beta_2, \beta_3, t) = \left[ -\frac{\hbar^2}{2} \sum_{kl} \frac{\partial}{\partial \beta_k} B_{kl}(\beta_2, \beta_3) \frac{\partial}{\partial \beta_l} + V(\beta_2, \beta_3) \right] g(\beta_2, \beta_3, t)$$

RMF+BCS quadrupole and octupole constrained deformation energy surface of  $^{226}\text{Th}$  in the  $\beta_2 - \beta_3$  plane.

TAO, ZHAO, LI, NIKŠIĆ, AND VRETENAR

PHYSICAL REVIEW C **96**, 024319 (2017)

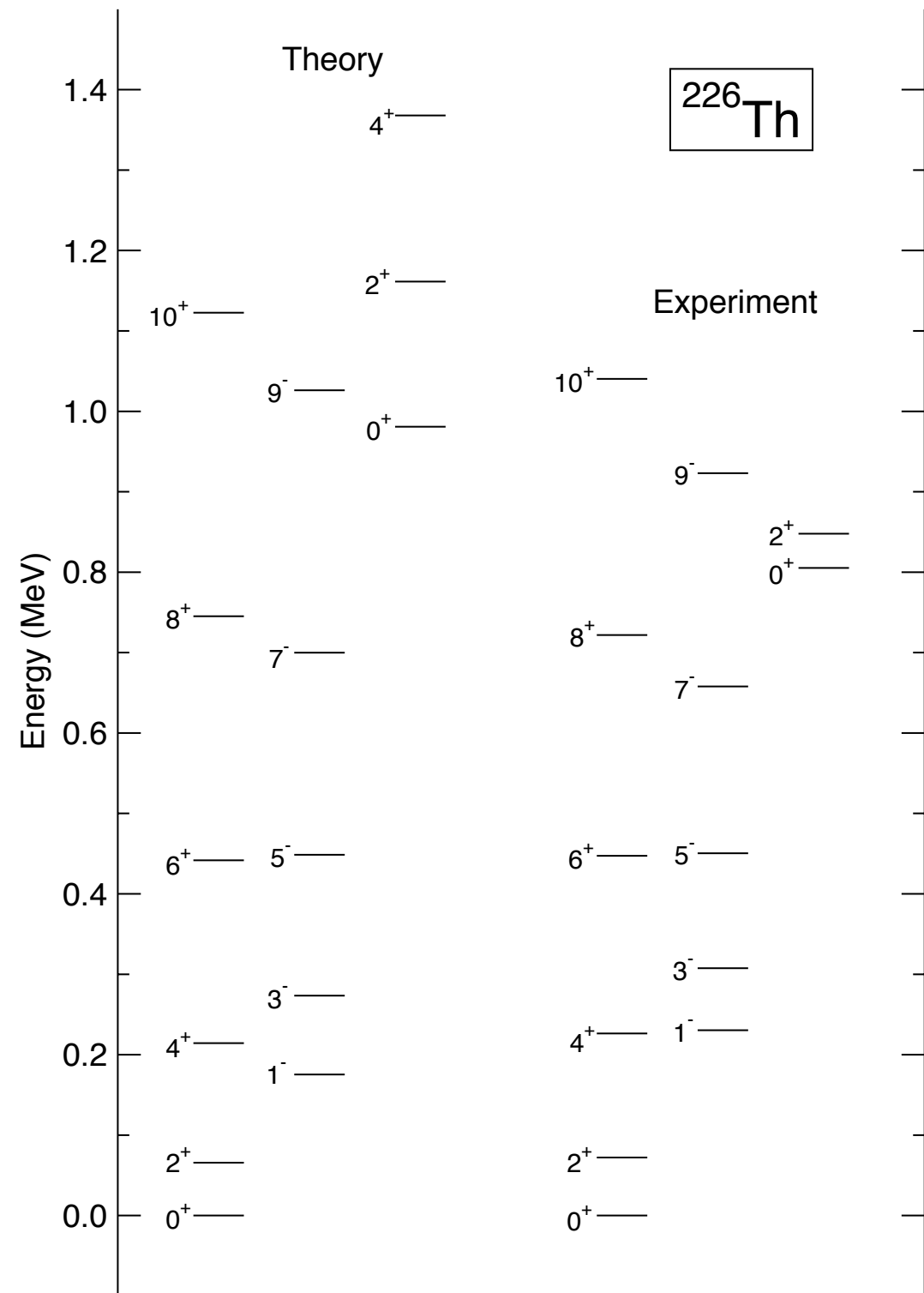
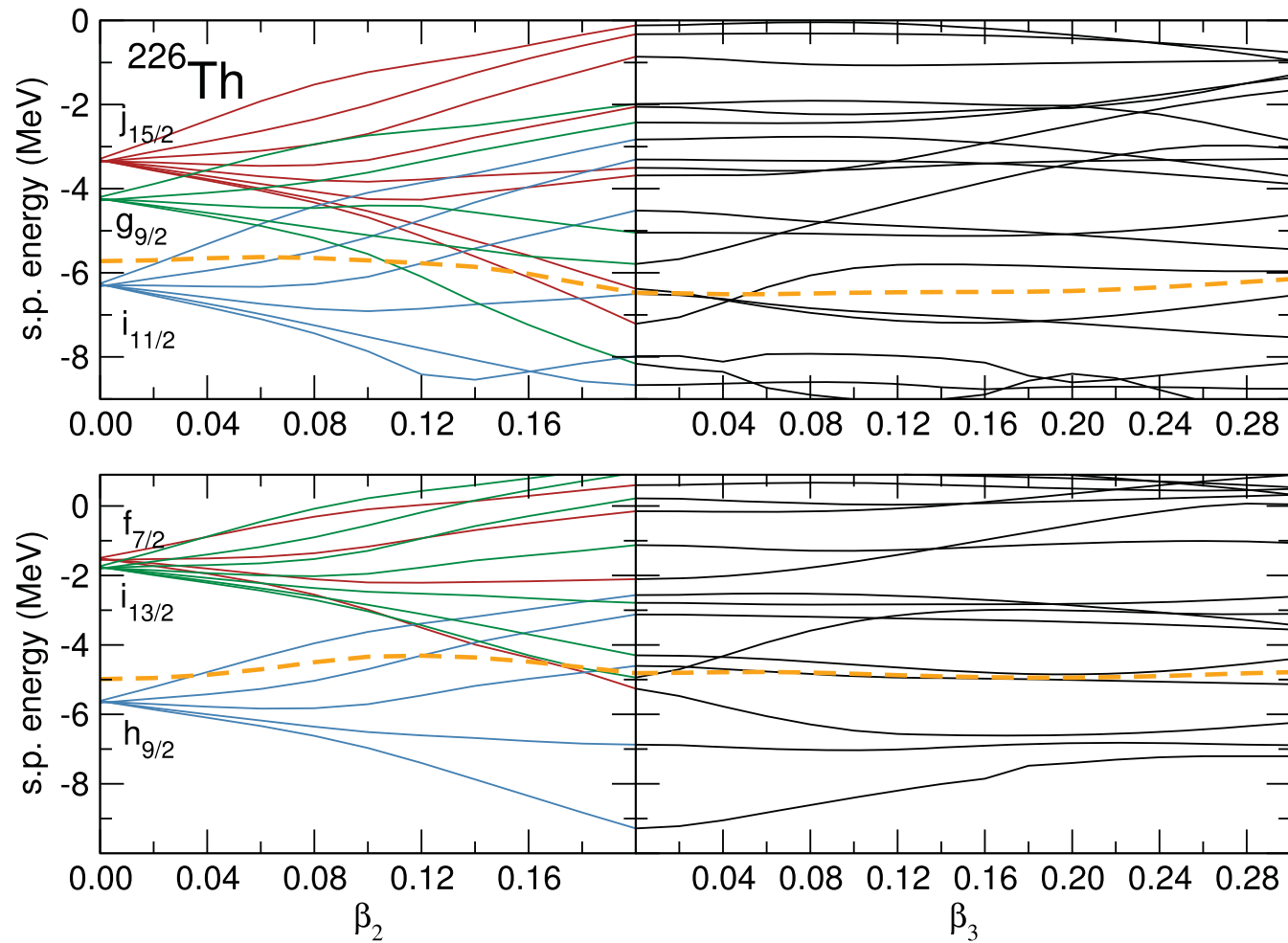
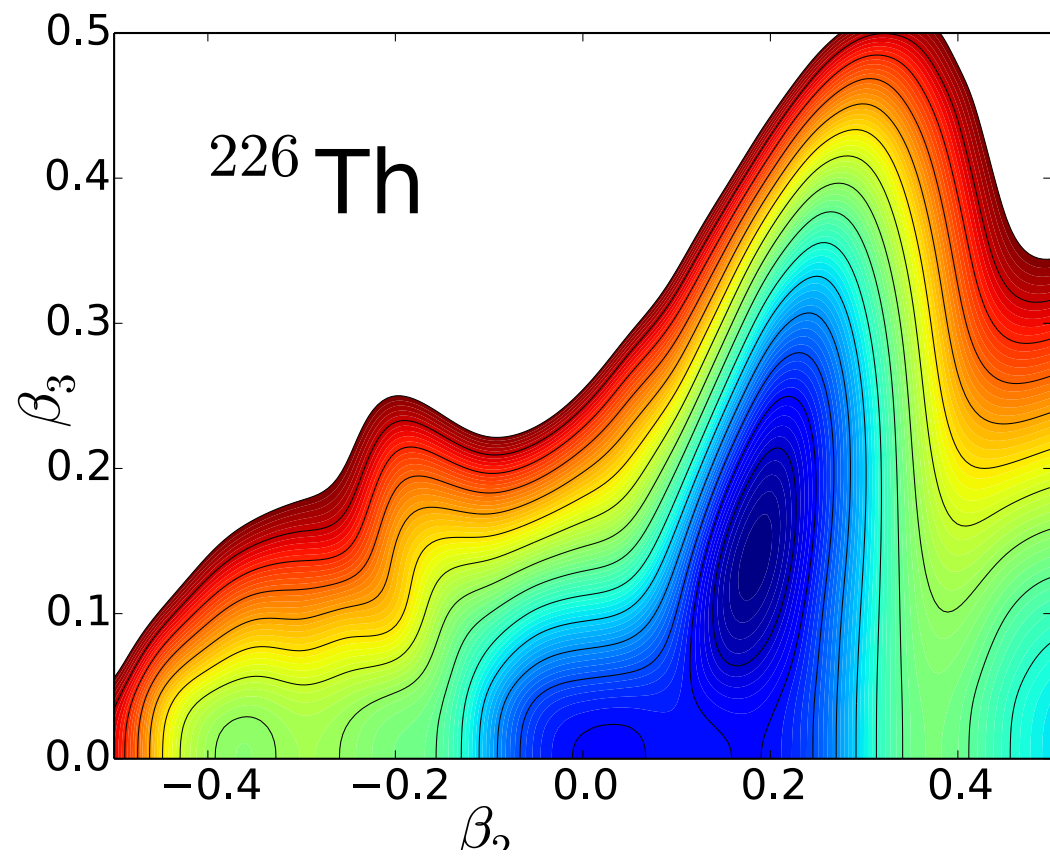


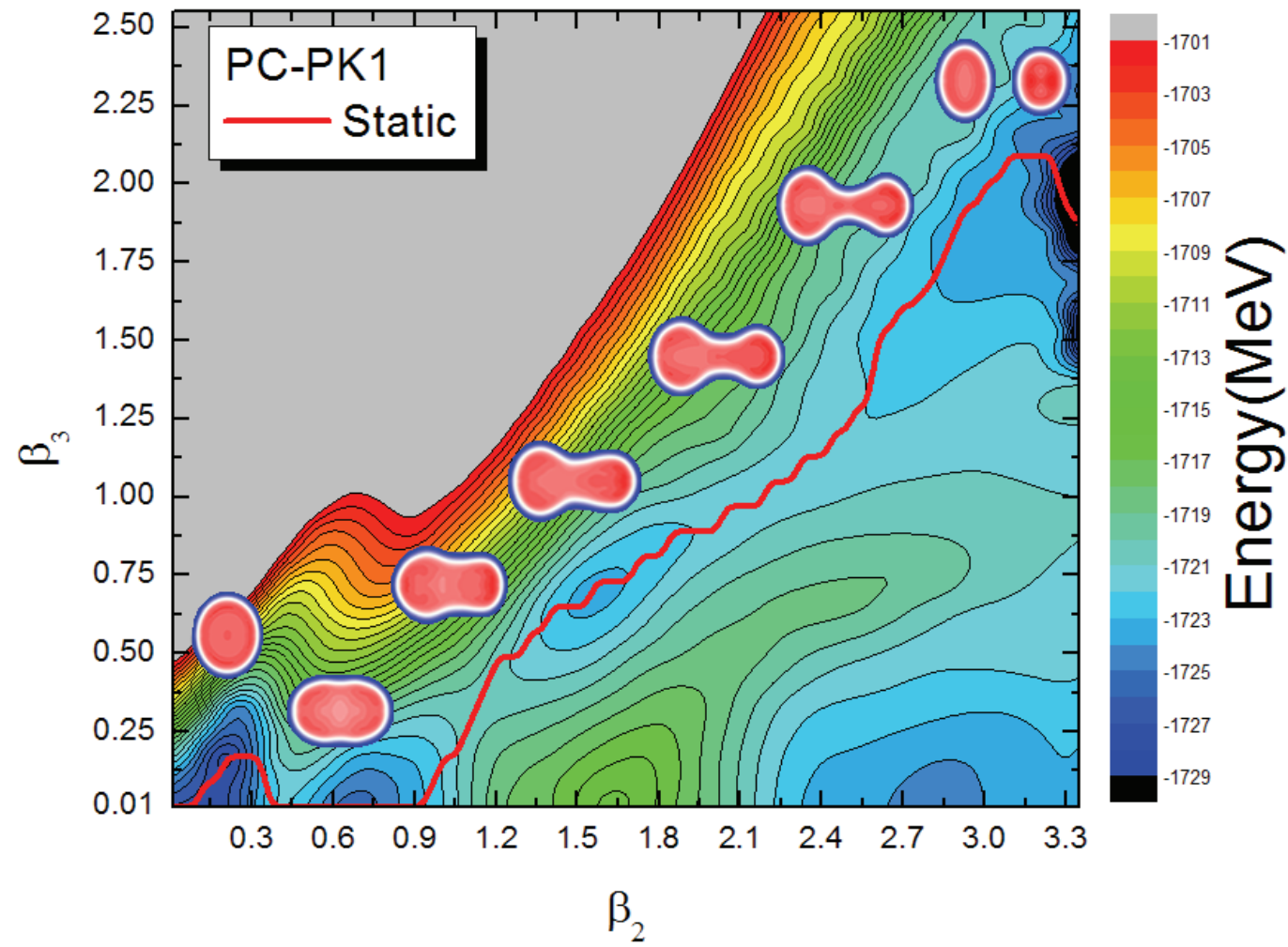
→ includes **static correlations**:  
deformations & pairing

→ does not include **dynamic (collective) correlations** that  
arise from symmetry restoration  
and quantum fluctuations  
around mean-field minima

PC-PK I plus  $\delta$ -force pairing



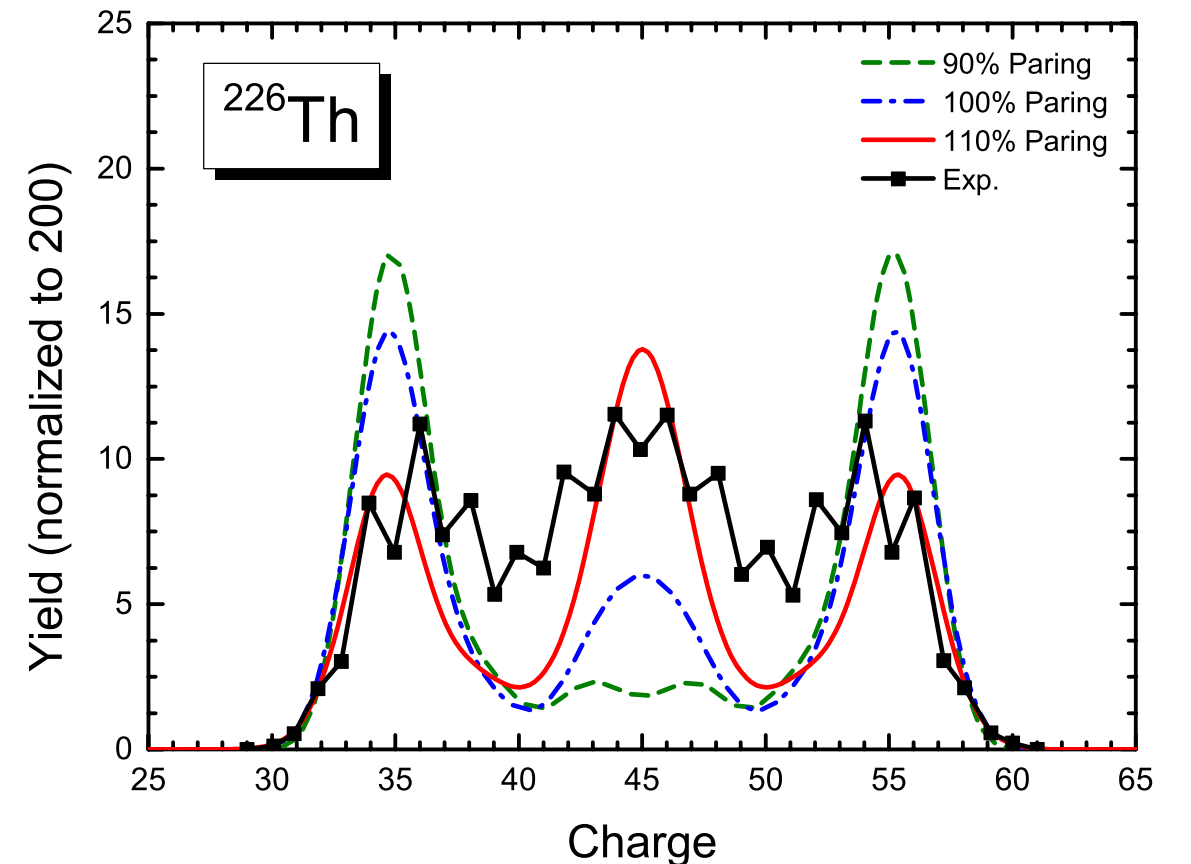




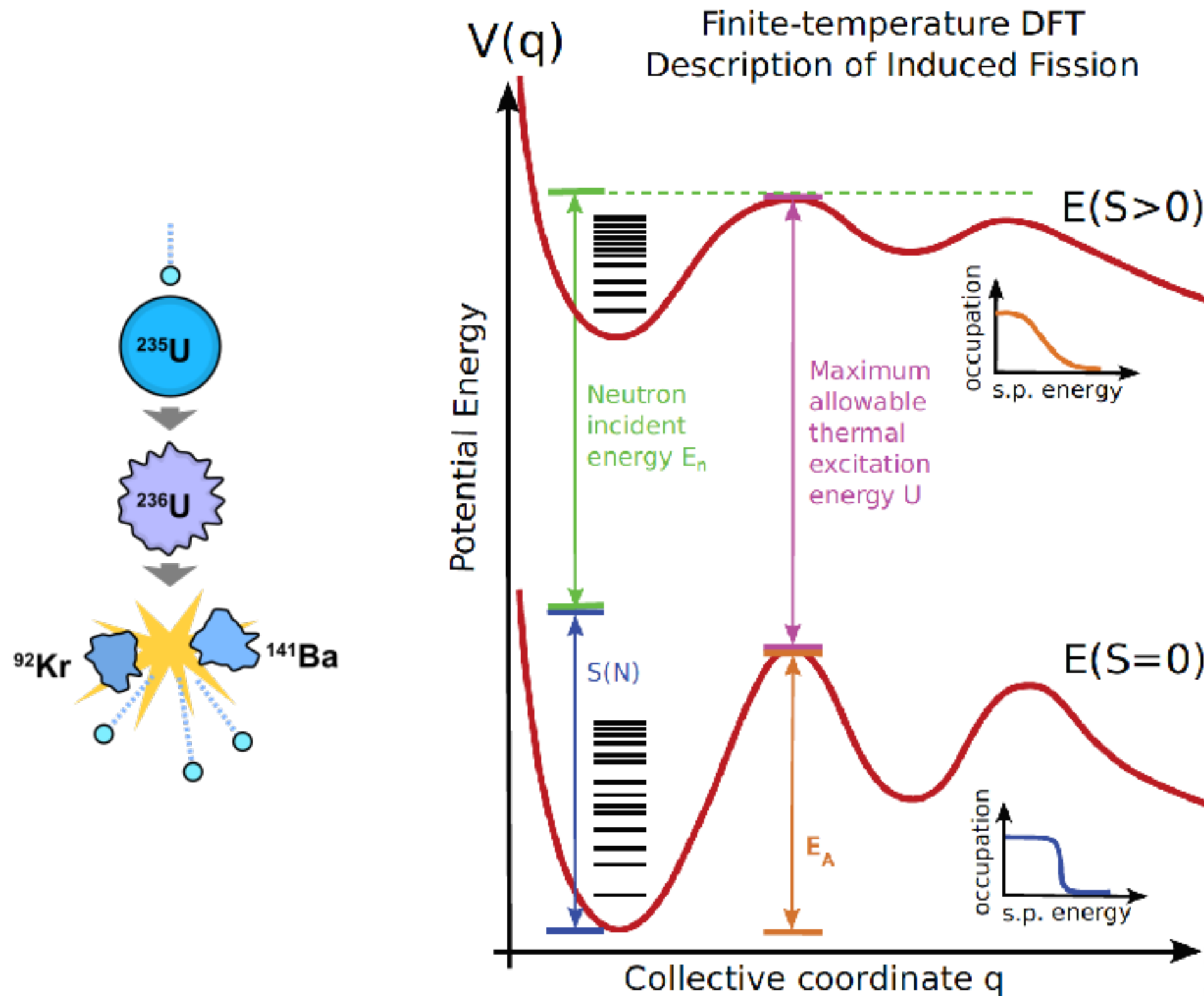
A triple-humped fission barrier is predicted along the static fission path, and the calculated heights are **7.10, 8.58, and 7.32 MeV** from the inner to the outer barrier.

The height of the fission barriers (in MeV) with respect to the corresponding ground-state minima:

	$B_I$	$B_{II}^{\text{asy}}$	$B_{III}^{\text{asy}}$	$B_{II}^{\text{sym}}$	$B_{III}^{\text{sym}}$
90% pairing	8.23	9.47	7.74	15.64	6.38
100% pairing	7.10	8.58	7.32	14.21	5.72
110% pairing	5.92	7.78	7.09	12.72	5.17



# Induced Fission - Finite Temperature Effects



# Finite temperature effects:

$$i\hbar \frac{\partial g(\mathbf{q}, t)}{\partial t} = \hat{H}_{\text{coll}}(\mathbf{q}) g(\mathbf{q}, t)$$

$$\hat{H}_{\text{coll}}(\mathbf{q}) = -\frac{\hbar^2}{2} \sum_{ij} \frac{\partial}{\partial q_i} B_{ij}(\mathbf{q}) \frac{\partial}{\partial q_j} + V(\mathbf{q})$$

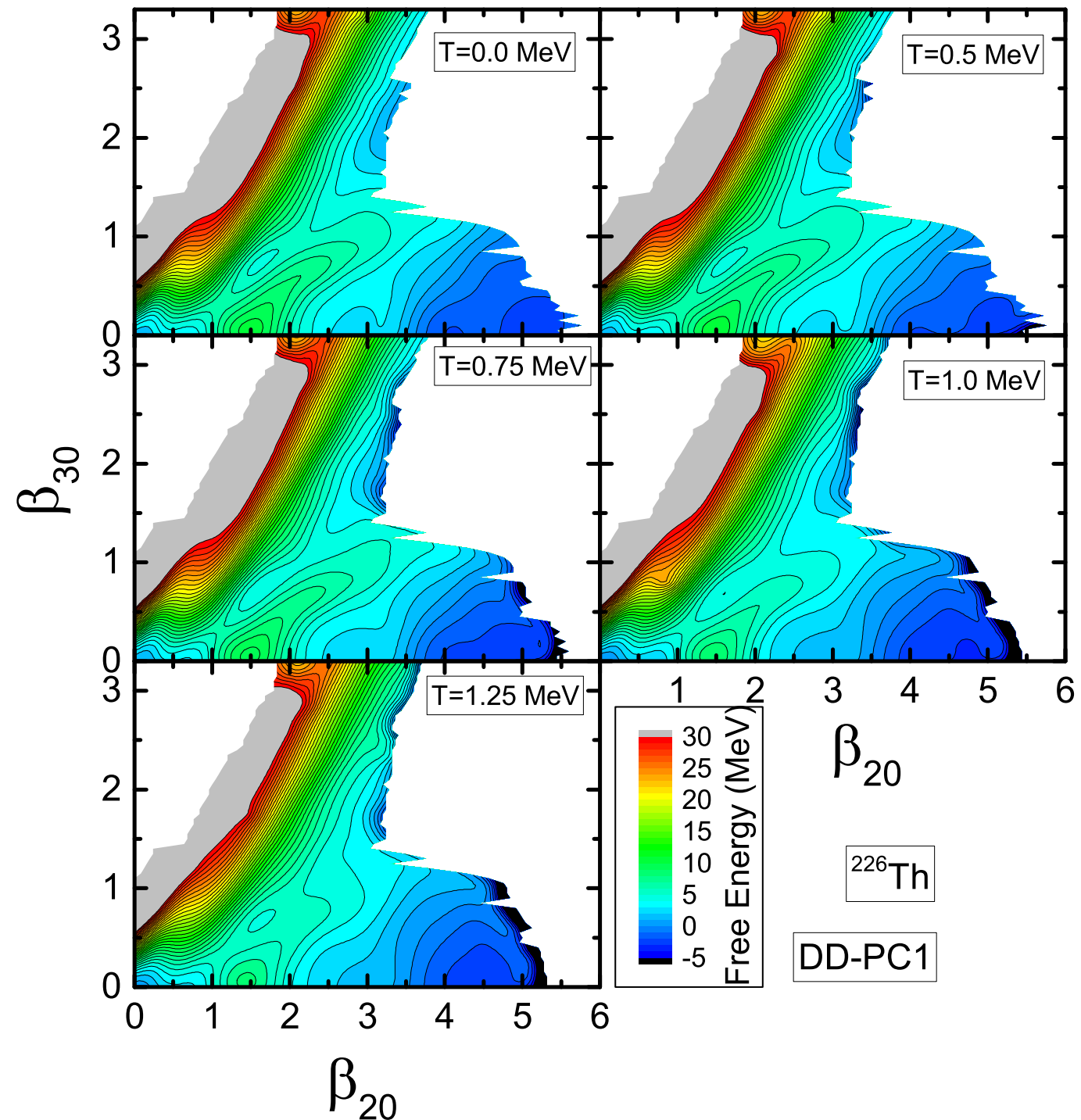
Helmholtz free energy:  $F = E(T) - TS$

... entropy of the compound nuclear system:

$$S = -k_B \sum_k [f_k \ln f_k + (1 - f_k) \ln(1 - f_k)]$$

... thermal occupation probabilities:

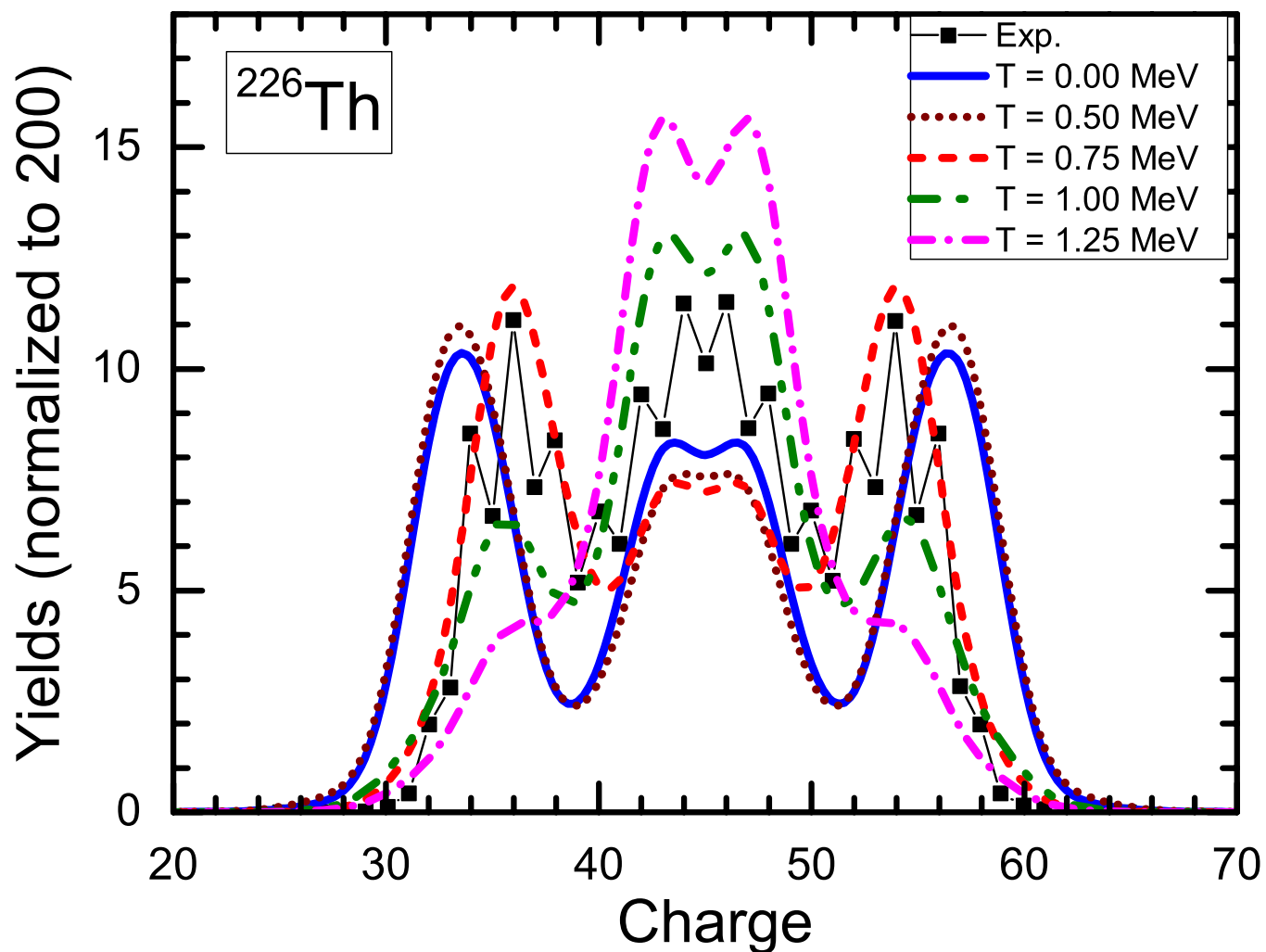
$$f_k = \frac{1}{1 + e^{\beta E_k}}$$



# Dynamics of induced fission

Zhao, Nikšić, Vretenar, Zhou  
Phys. Rev. C **99**, 014618 (2019).

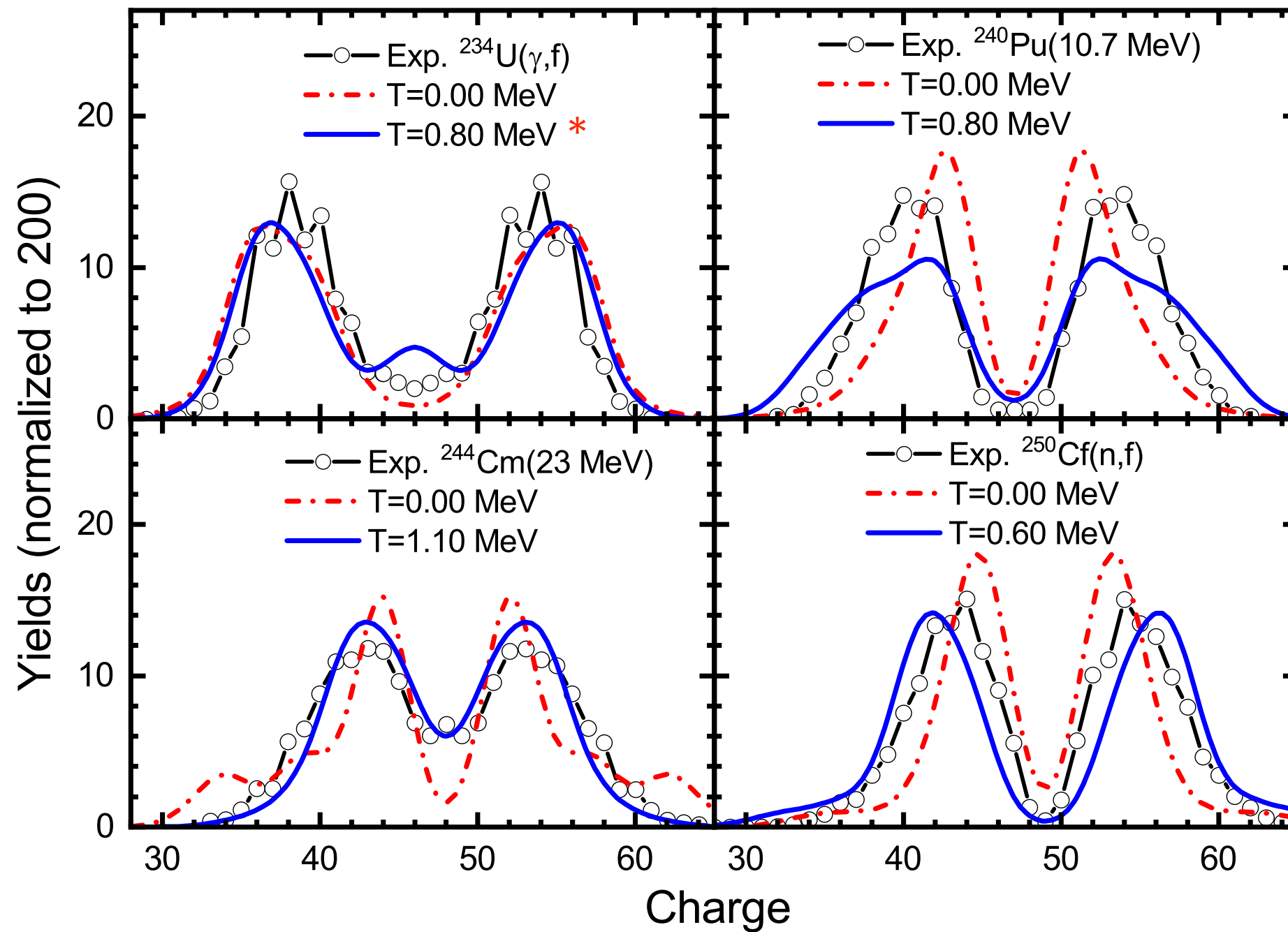
## Charge yields:



Experimental results  $\Rightarrow$  photoinduced fission with photon energies in the interval 8 – 14 MeV, and a peak value  $E_\gamma = 11$  MeV.

$T = 0.5, \mathbf{0.75}, \mathbf{1.0}$ , and 1.25 MeV  $\Rightarrow$  corresponding internal excitation energies  $E^*$  are: 2.58, **8.71**, **16.56**, and 27.12 MeV, respectively.





\*The temperature is adjusted so that the intrinsic excitation energy corresponds to the experimental exc. energy.

# Induced fission: dynamical pairing degree of freedom

Zhao, Nikšić, Vretenar  
Phys. Rev. C **104**, 044612 (2021).

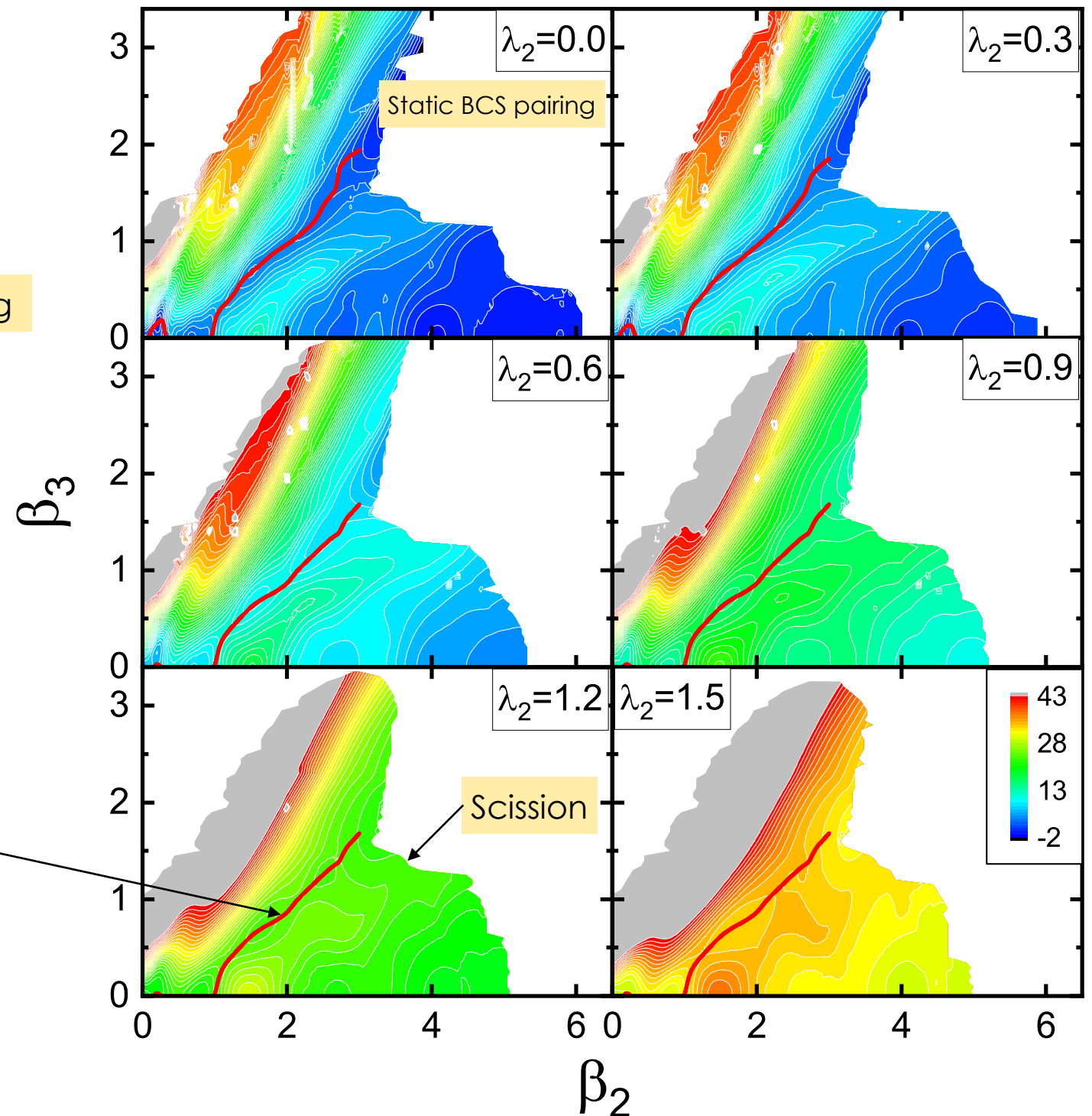
SCMF deformation energy surface  $\Rightarrow$  constraints on the mass multipole moments and the particle-number dispersion operator:  $\Delta\hat{N}^2 = \hat{N}^2 - \langle\hat{N}\rangle^2$ .

... the Routhian:

$$E' = E_{\text{RMF}} + \sum_{\lambda\mu} \frac{1}{2} C_{\lambda\mu} Q_{\lambda\mu} + \underbrace{\lambda_2 \Delta\hat{N}^2}_{\text{isoscalar dynamical pairing}}$$

2D projections of the deformation-energy manifold of  $^{228}\text{Th}$  on the quadrupole-octupole axially symmetric plane, for selected values of the pairing coordinate  $\lambda_2$ .

Static fission path of minimum energy

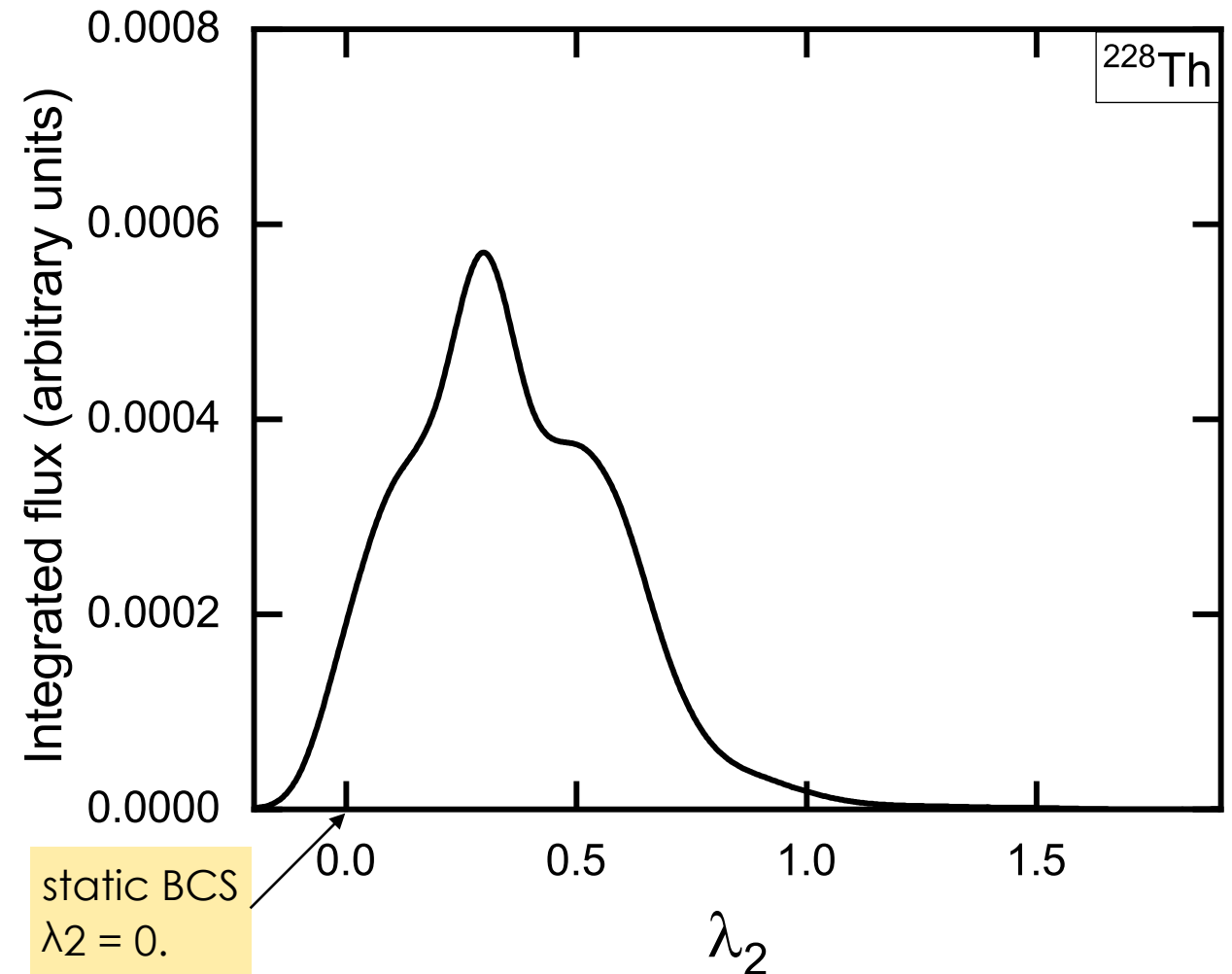
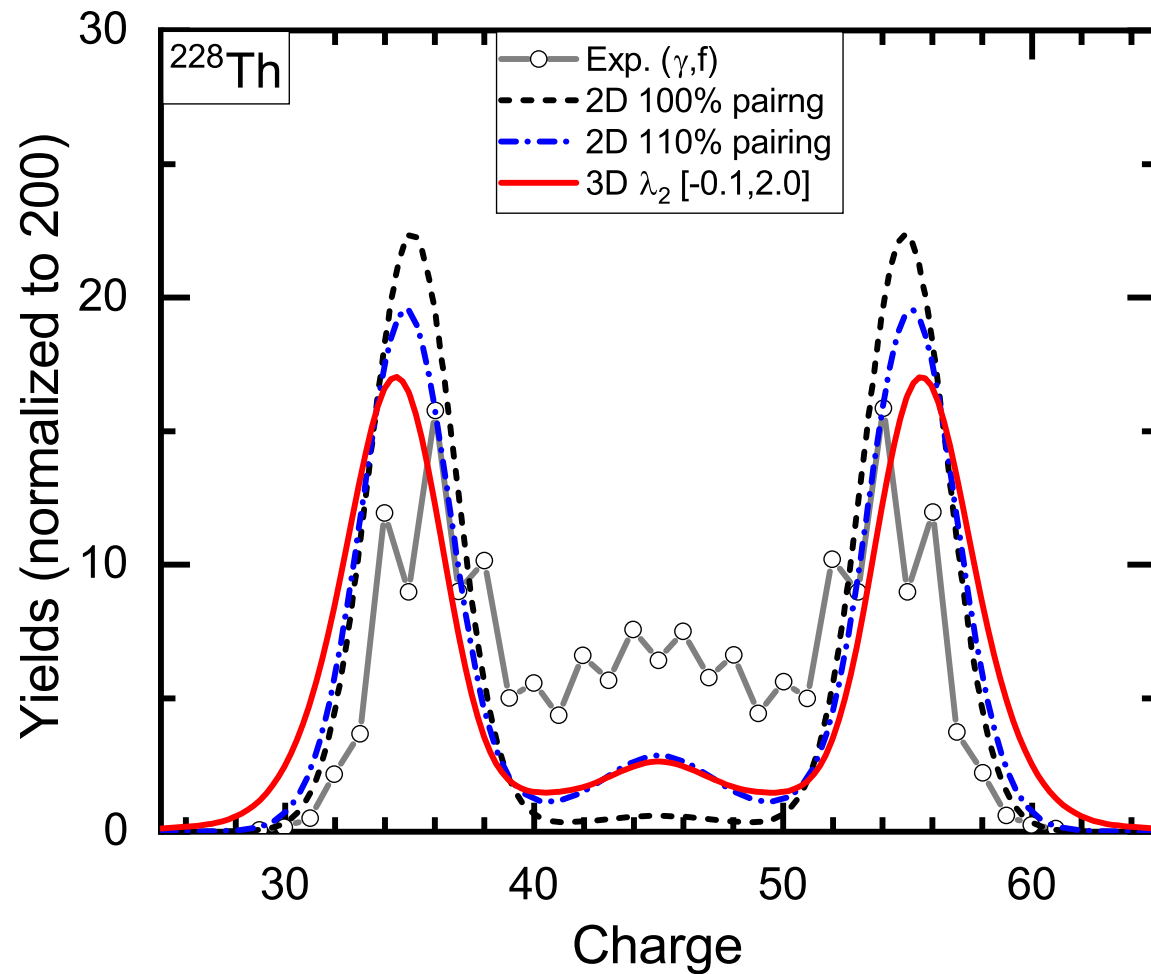




# 3D TDGCM+GOA calculation

$$\hat{H}_{\text{coll}}(\mathbf{q}) = -\frac{\hbar^2}{2} \sum_{ij} \frac{\partial}{\partial q_i} B_{ij}(\mathbf{q}) \frac{\partial}{\partial q_j} + V(\mathbf{q})$$

$$\mathbf{q} = \{\beta_2, \beta_3, \lambda_2\}$$



Charge yields calculated in the 3D collective space  
→ deformation  $\beta_2$ ,  $\beta_3$  and dynamical pairing  $\lambda_2$   
coordinates.

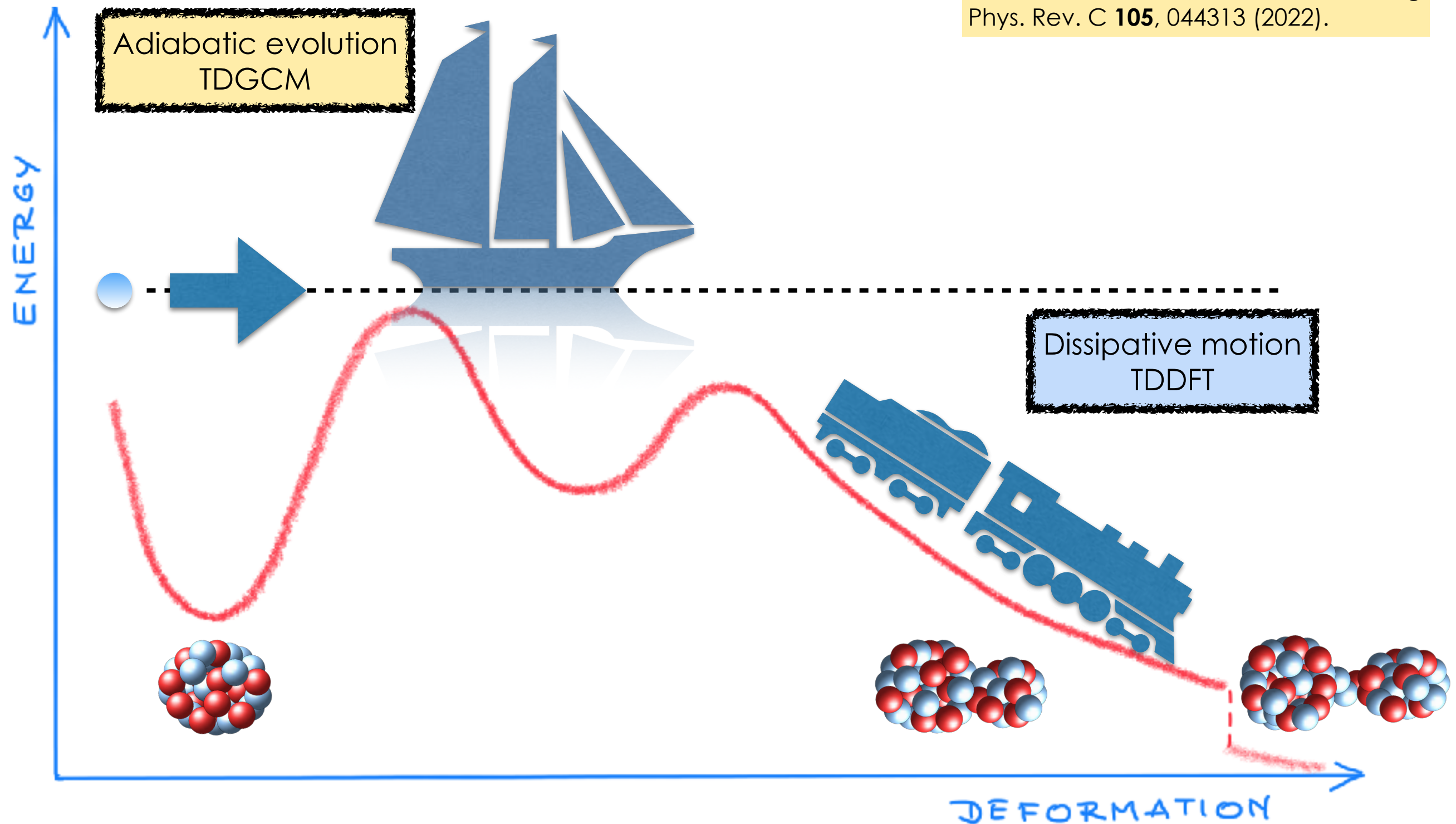
Effect of dynamical pairing on the flux of the probability current  
through the scission hyper-surface:

$$B(\lambda_2) \propto \sum_{\xi \in \mathcal{B}} \lim_{t \rightarrow \infty} F(\xi, \lambda_2, t).$$

→ time-integrated flux through the scission contour in the  $(\beta_2, \beta_3)$   
plane, for a given value of the pairing collective coordinate  $\lambda_2$ .

# Adiabatic evolution and dissipative dynamics

Ren, Zhao, Vretenar, Nikšić, Zhao, Meng  
Phys. Rev. C **105**, 044313 (2022).



## Time-dependent density functional theory (TDDFT)

$$i \frac{\partial}{\partial t} \psi_k(\mathbf{r}, t) = \left[ \hat{h}(\mathbf{r}, t) - \varepsilon_k(t) \right] \psi_k(\mathbf{r}, t),$$

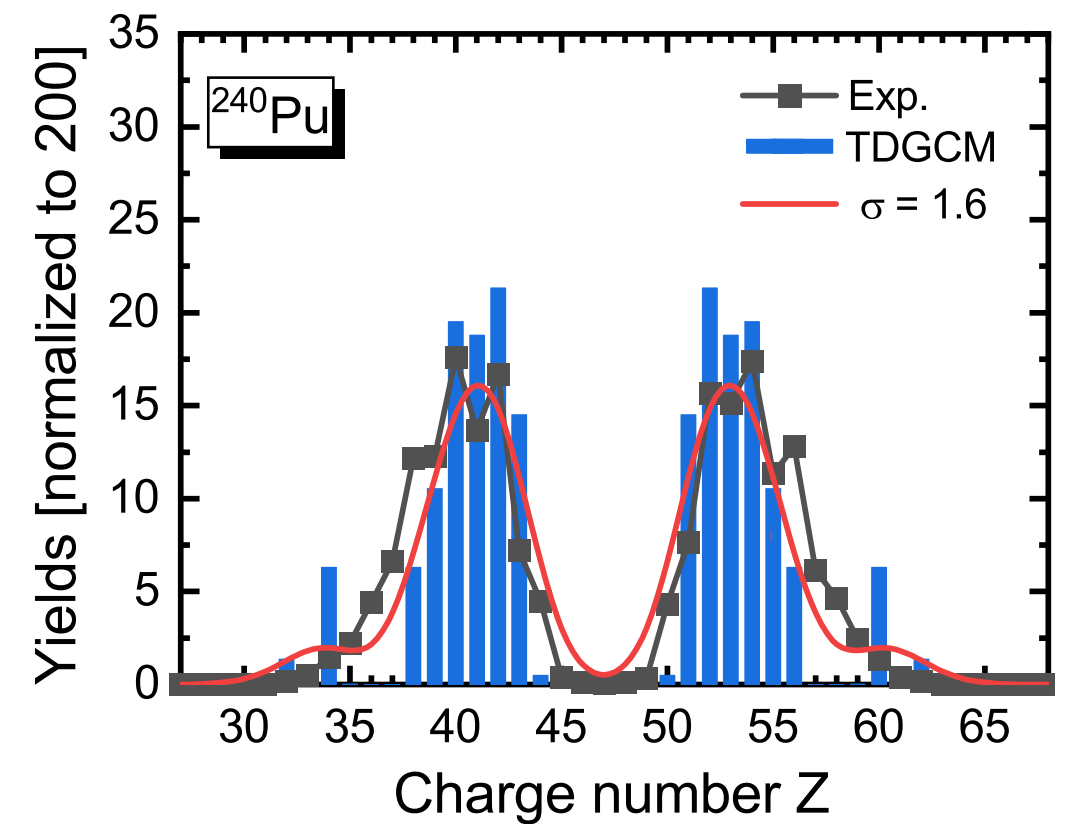
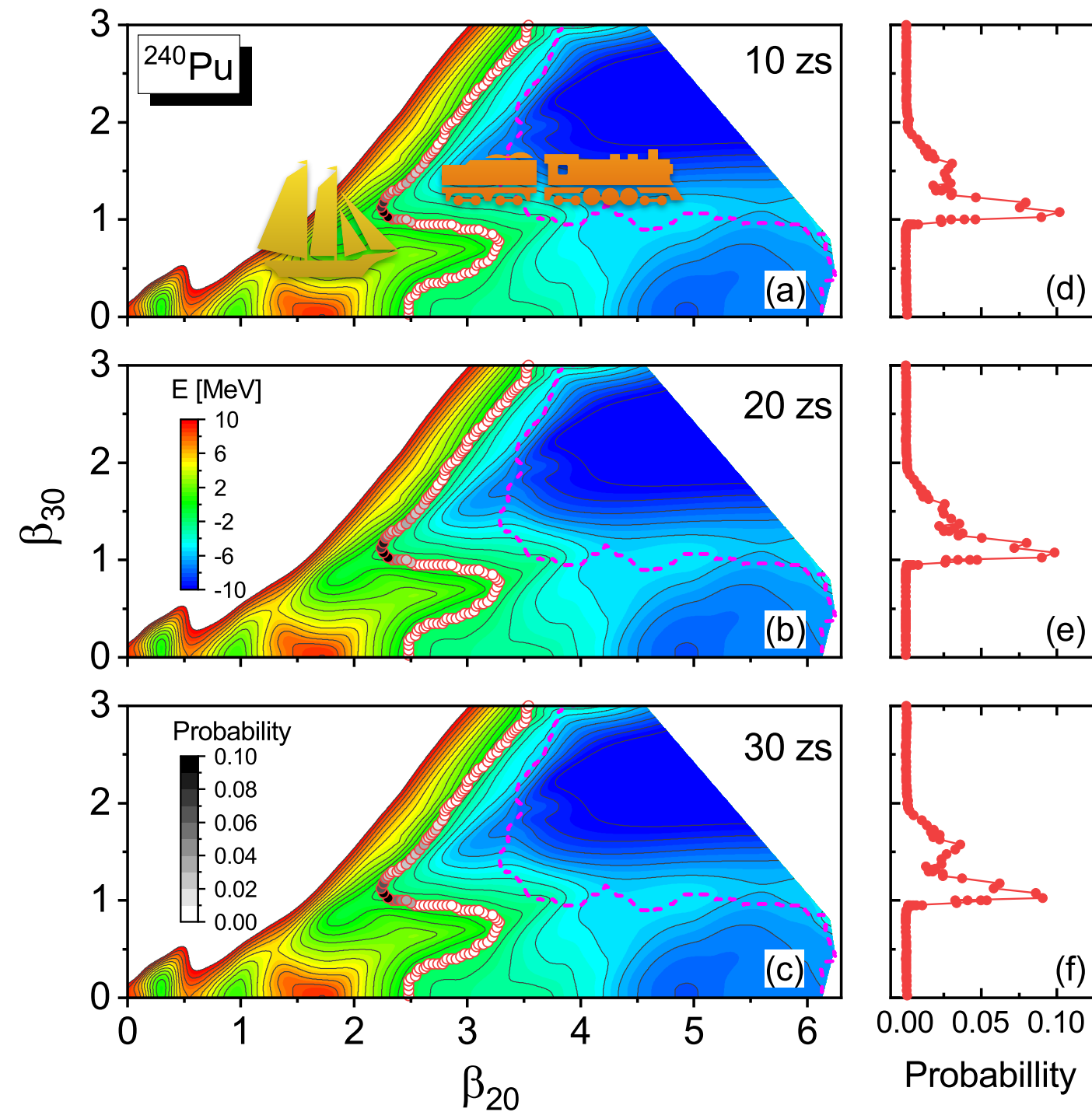
$$i \frac{d}{dt} n_k(t) = n_k(t) \Delta_k^*(t) - n_k^*(t) \Delta_k(t),$$

$$i \frac{d}{dt} \kappa_k(t) = [\varepsilon_k(t) + \varepsilon_{\bar{k}}(t)] \kappa_k(t) + \Delta_k(t) [2n_k(t) - 1].$$

⇒ classical evolution of independent nucleons in mean-field potentials, cannot be applied in classically forbidden regions of the collective space, nor does it take into account quantum fluctuations.

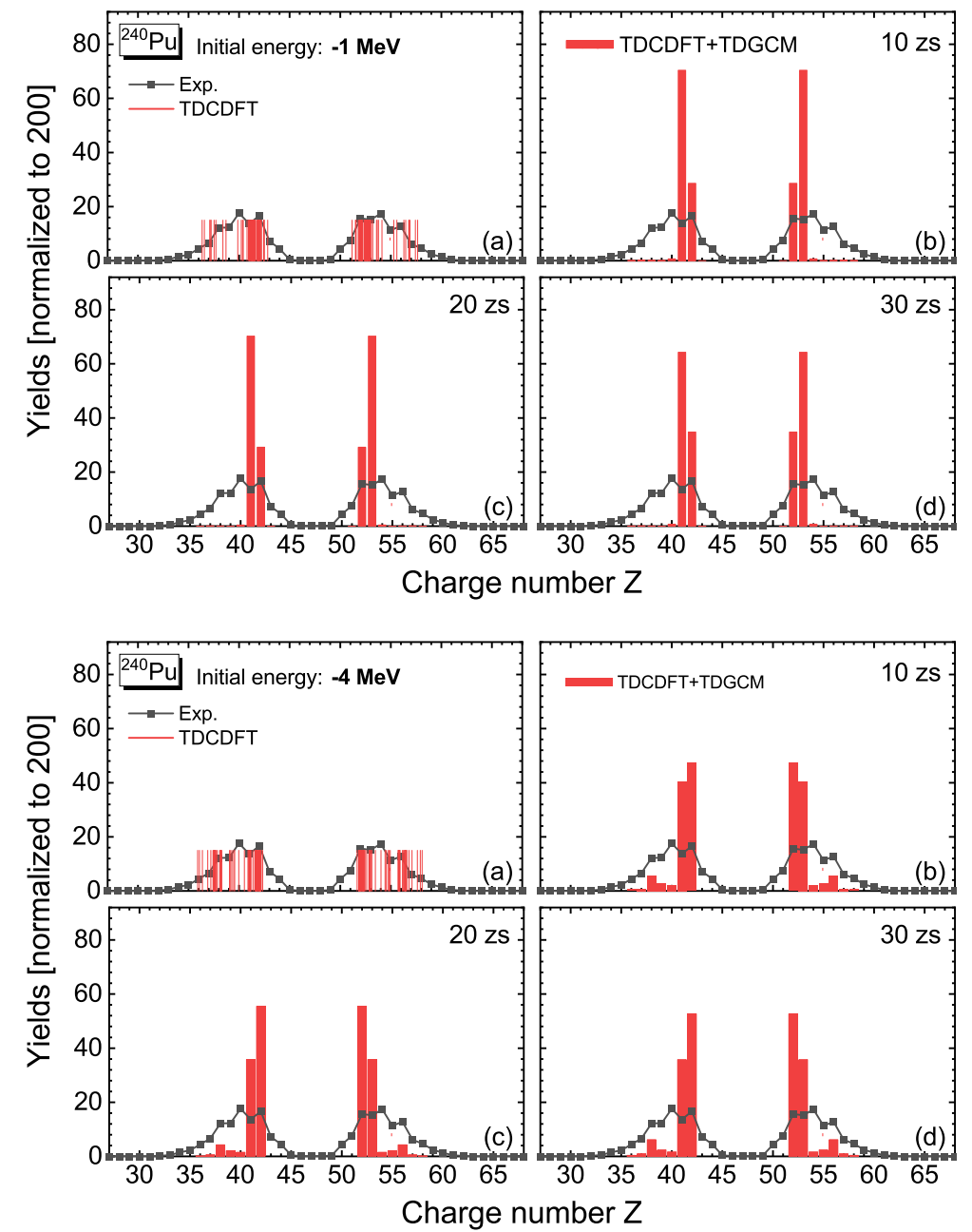
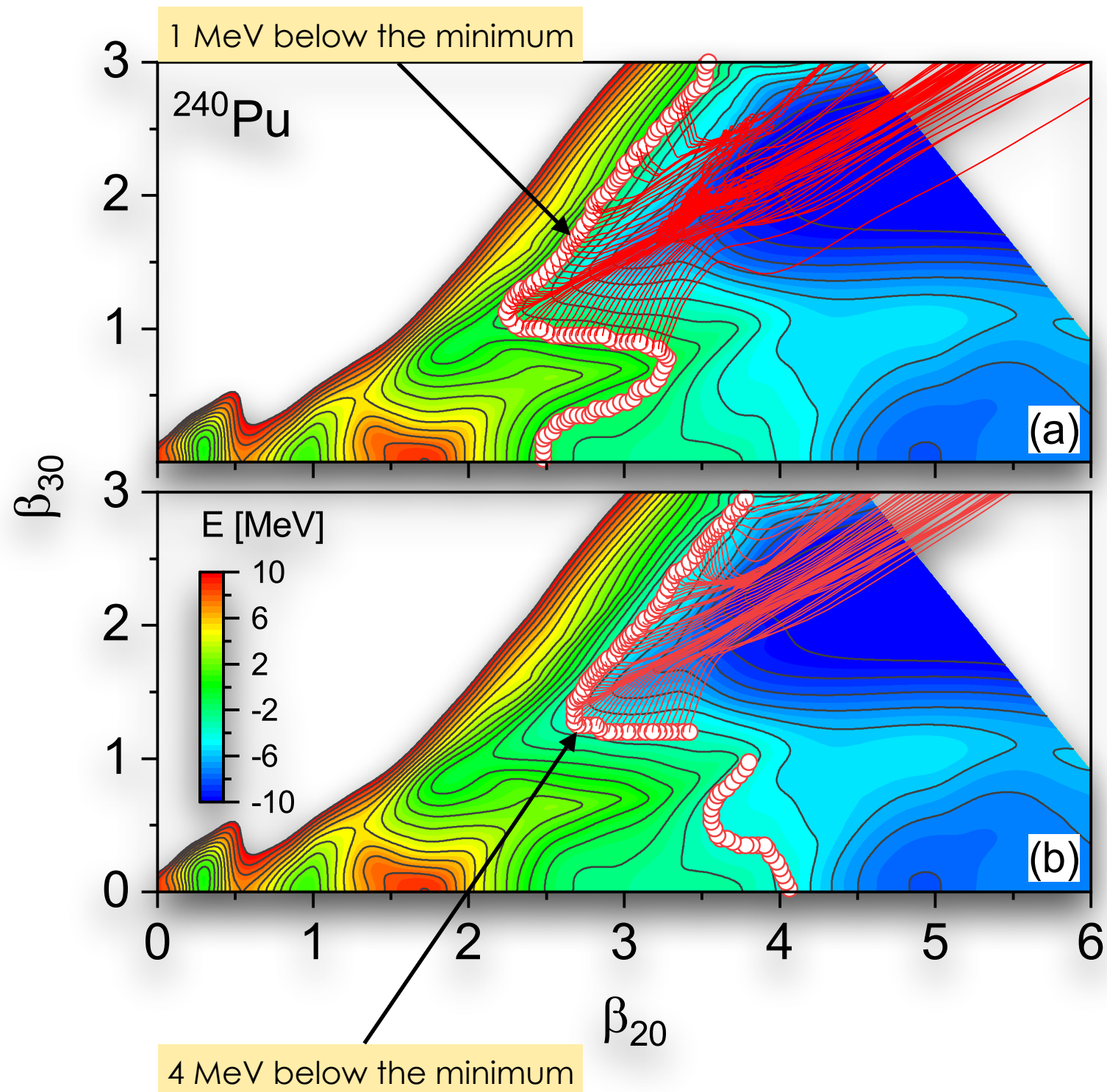
⇒ automatically includes the one-body dissipation mechanism, but can only simulate a single fission event by propagating the nucleons independently.

Negele et al. (1978)  $\Rightarrow$  use an adiabatic model for the time interval in which the fissioning nucleus evolves from the quasi-stationary initial state to the saddle point, and a non-adiabatic method for the saddle-to-scission and beyond-scission dynamics.



Ren, Zhao, Vretenar, Nikšić, Zhao, Meng  
Phys. Rev. C **105**, 044313 (2022).

# TDDFT fission trajectories





# Total kinetic energies (TKEs) of the fragments

TDGCM+GOA

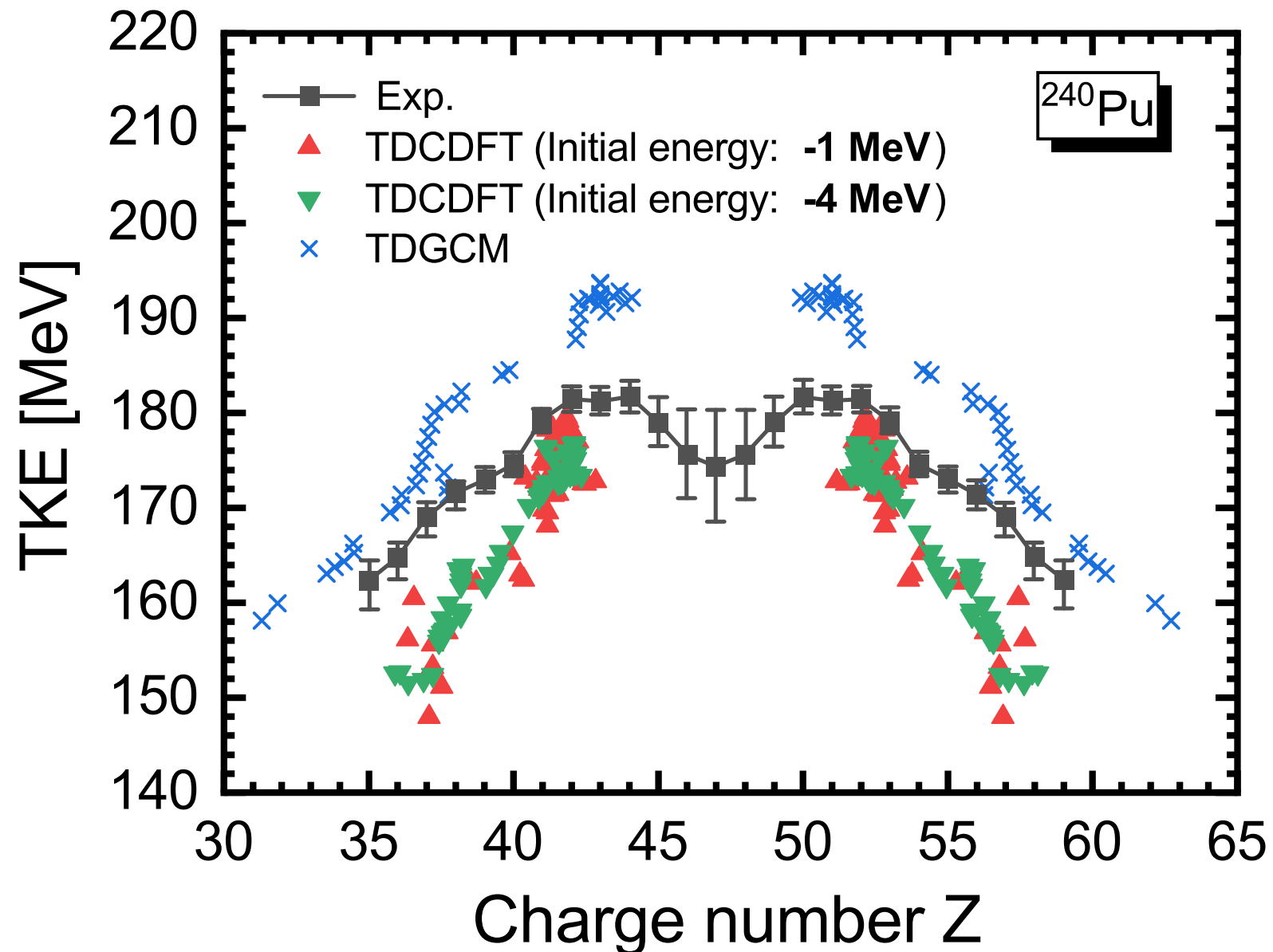
$$E_{\text{TKE}} = \frac{e^2 Z_H Z_L}{d_{\text{ch}}},$$

$d_{\text{ch}} \rightarrow$  distance between centers of charge at the point of scission.

TDDFT

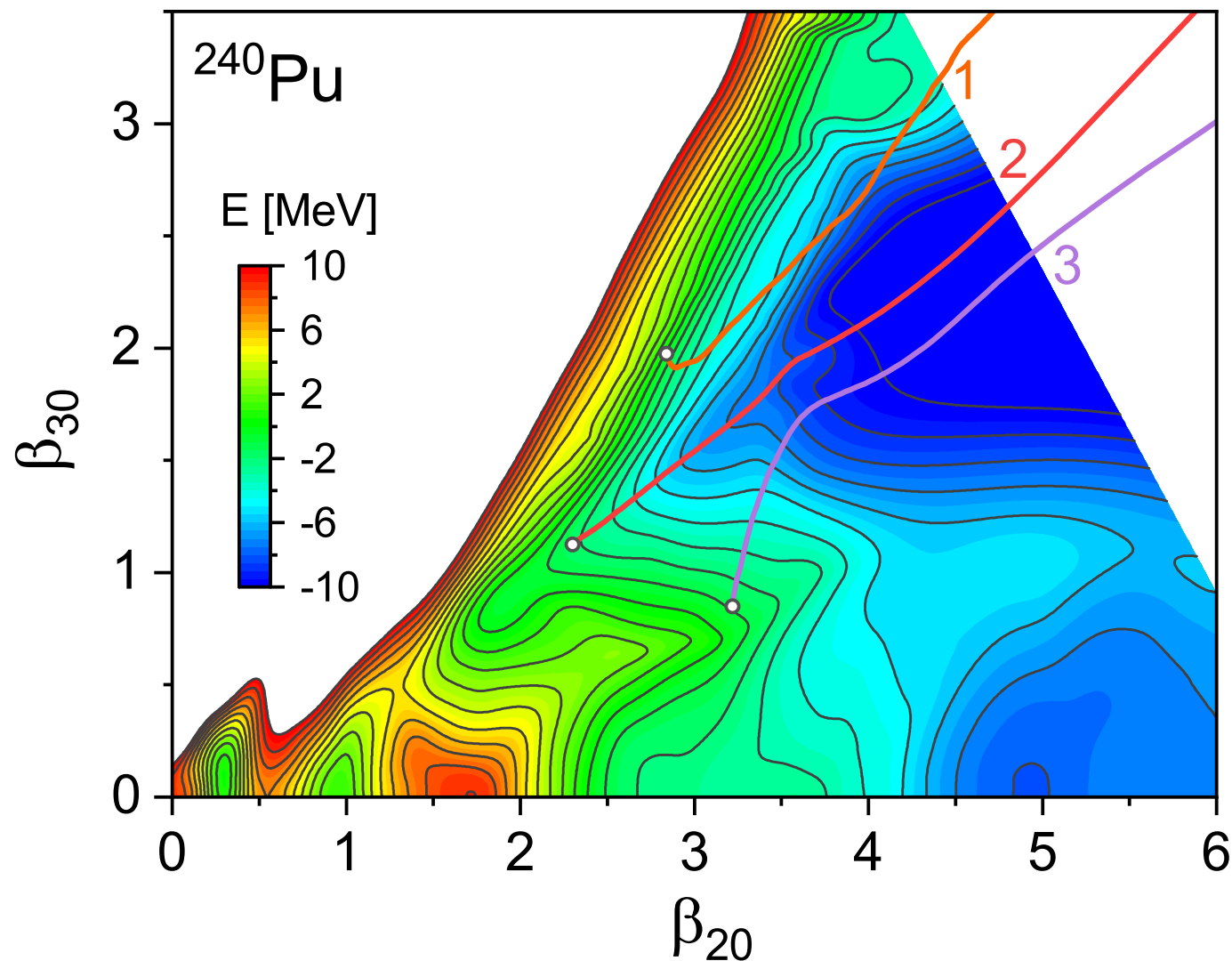
$$E_{\text{TKE}} = \frac{1}{2} m A_H v_H^2 + \frac{1}{2} m A_L v_L^2 + E_{\text{Coul}},$$

( $\approx 25$  fm, at which shape relaxation brings the fragments to their equilibrium shapes)

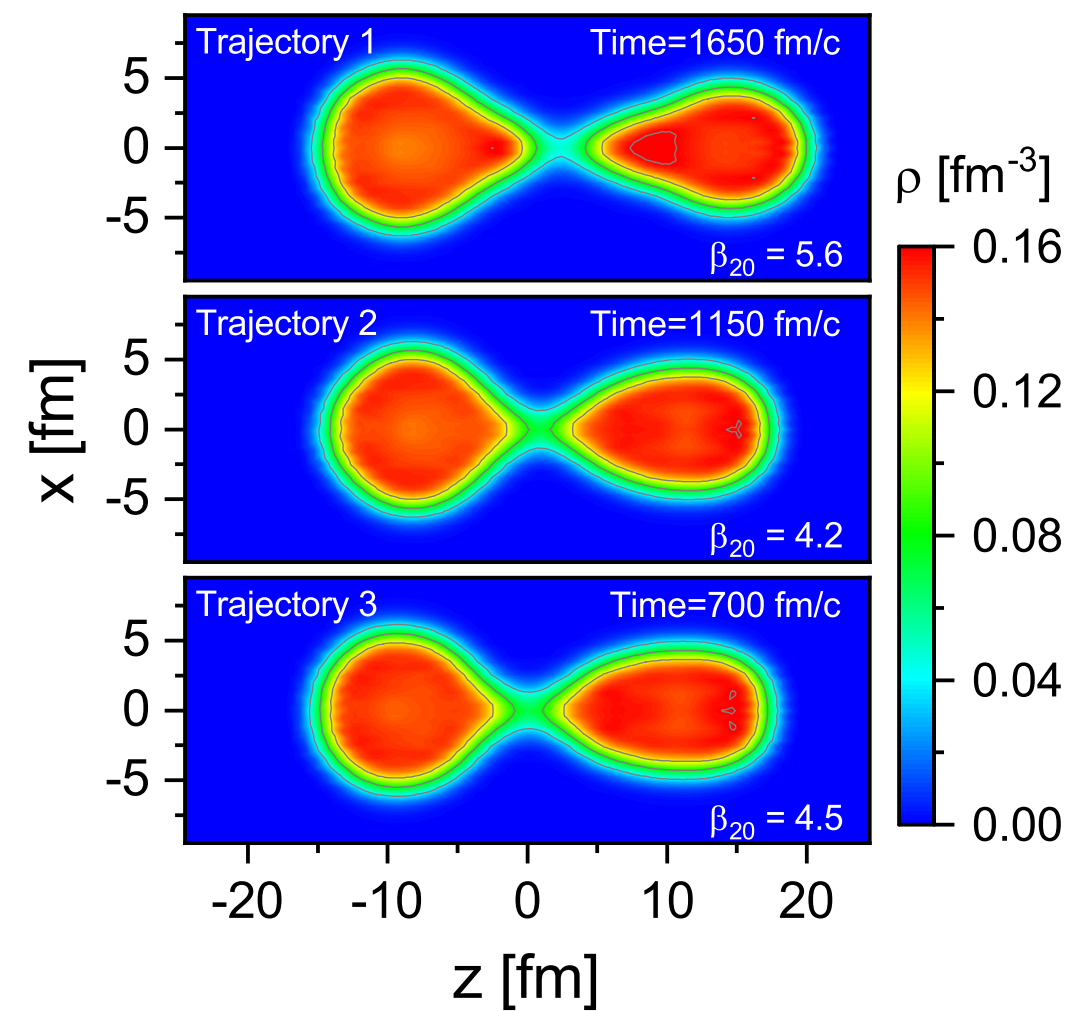


# Dynamical synthesis of $^4\text{He}$ in the scission phase of nuclear fission

TDDFT fission trajectories



Density profiles at times immediately prior to the scission event.





# Nucleon localization functions:

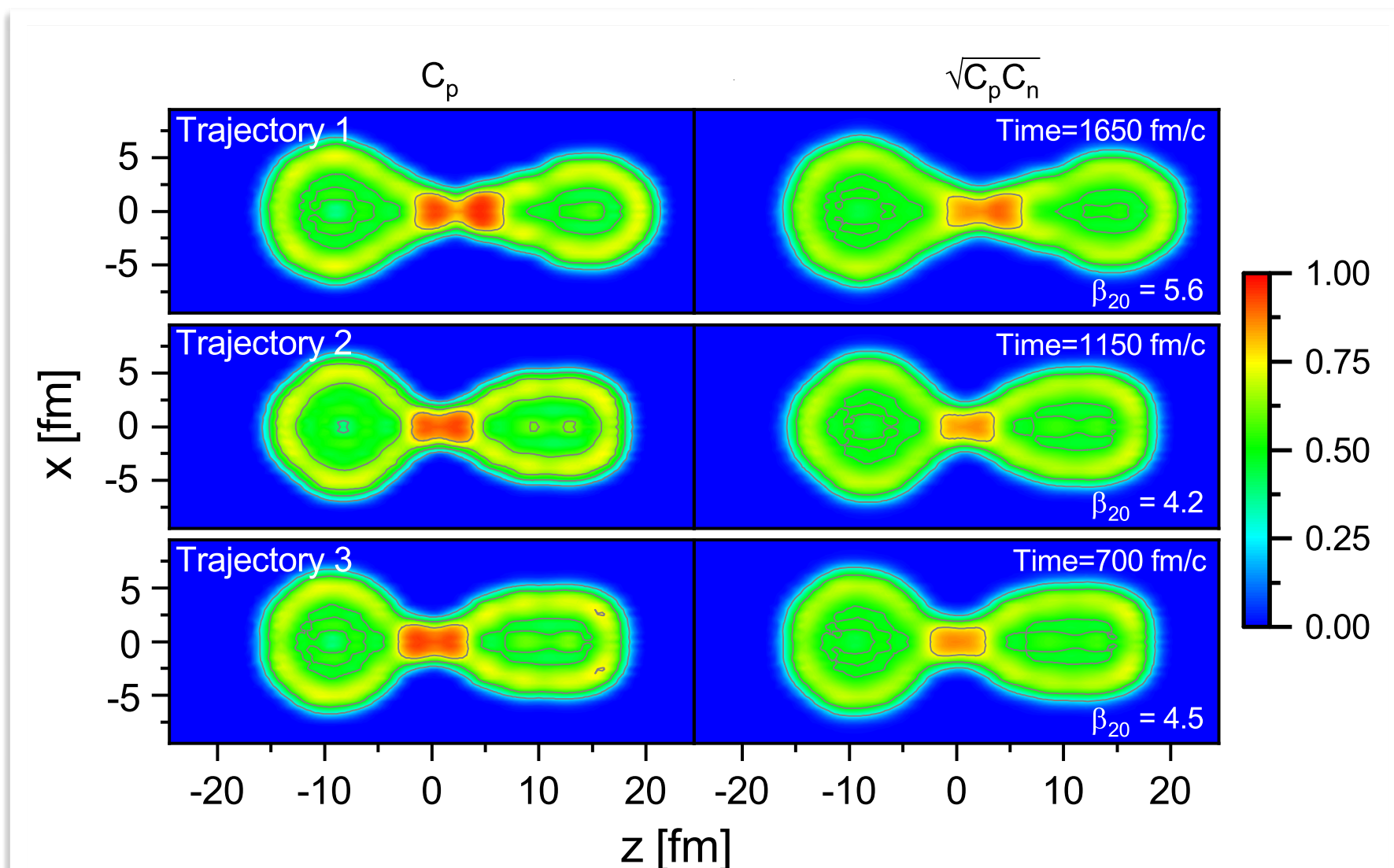
$\sigma$  ( $\uparrow$  or  $\downarrow$ )  
 $q$  ( $n$  or  $p$ )

$$C_{q\sigma}(\vec{r}) = \left[ 1 + \left( \frac{\overset{\text{kinetic energy density}}{\tau_{q\sigma} \rho_{q\sigma}} - \frac{1}{4} \frac{\overset{\text{density}}{|\vec{\nabla} \rho_{q\sigma}|^2}}{\rho_{q\sigma} \tau_{q\sigma}^{\text{TF}}} - \frac{\overset{\text{current density}}{j_{q\sigma}^2}}{\rho_{q\sigma} \tau_{q\sigma}^{\text{TF}}} \right)^2 \right]^{-1}$$

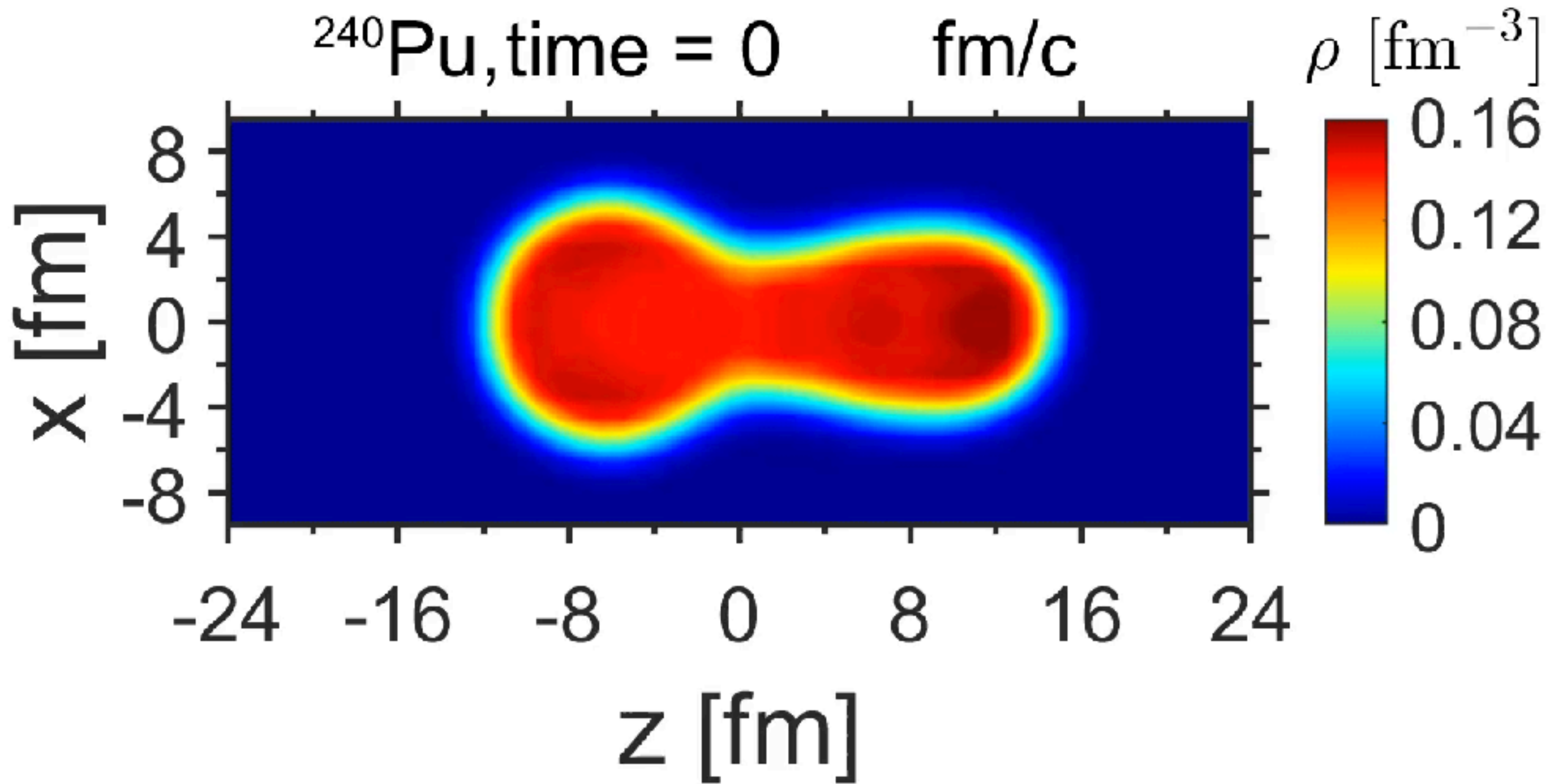
$$\tau_{q\sigma}^{\text{TF}} = \frac{3}{5} (6\pi^2)^{2/3} \rho_{q\sigma}^{5/3}$$

For homogeneous nuclear matter:  $C_{q\sigma} = 1/2$

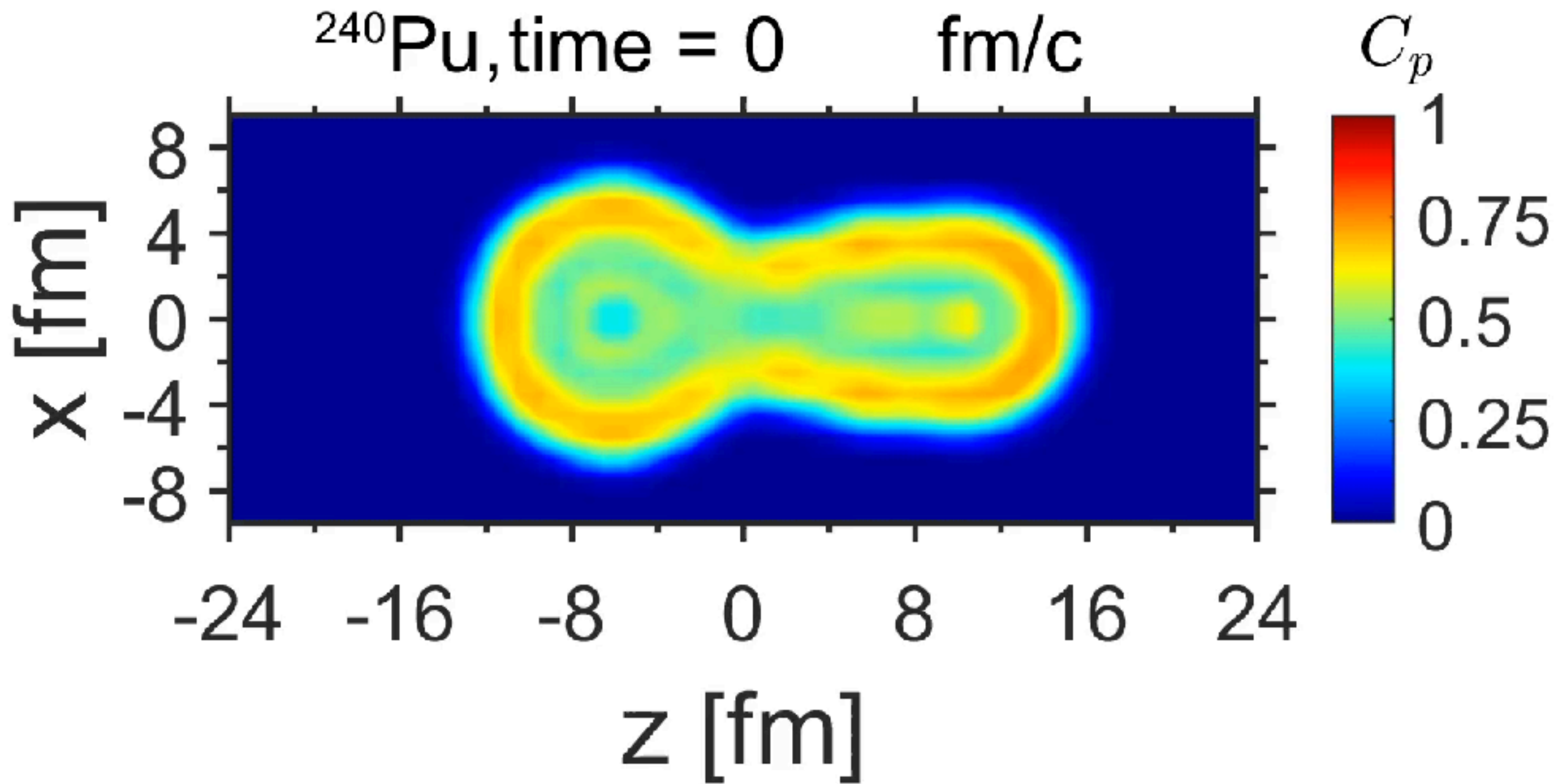
For the  $\alpha$ -cluster of four particles:  $C_{q\sigma}(\vec{r}) \approx 1$



## Trajectory 2



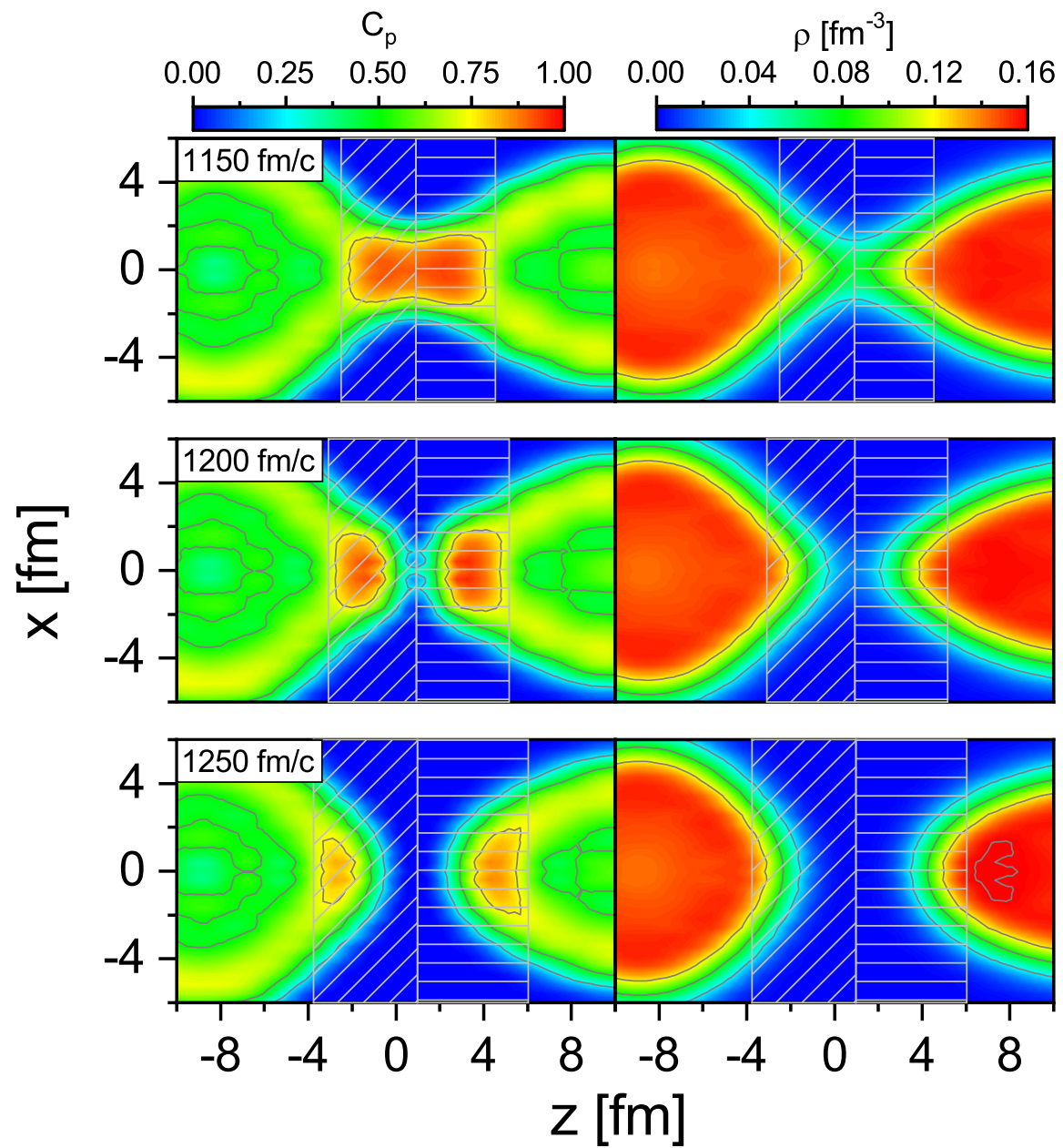
## Trajectory 2



## Trajectory 2

Proton localization

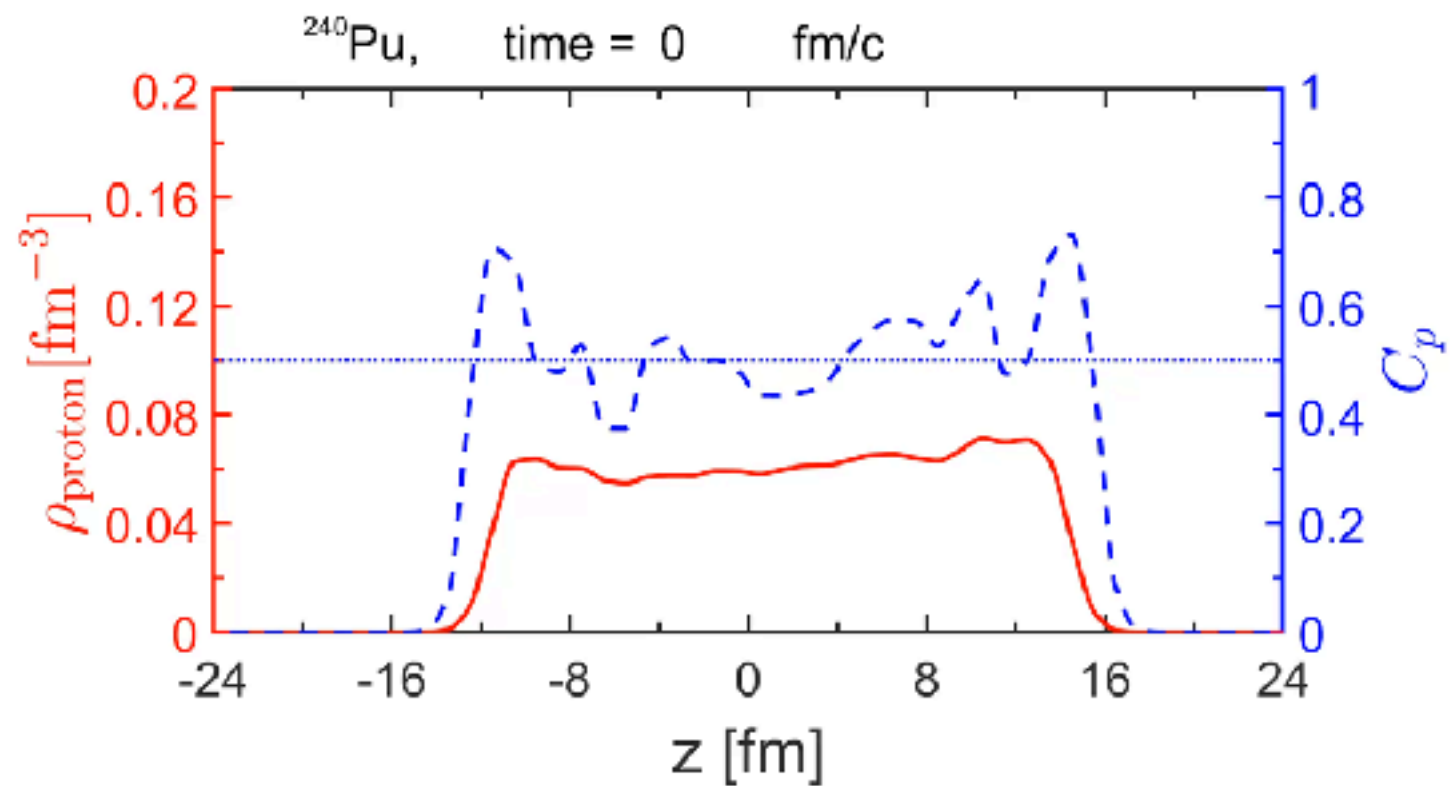
Density



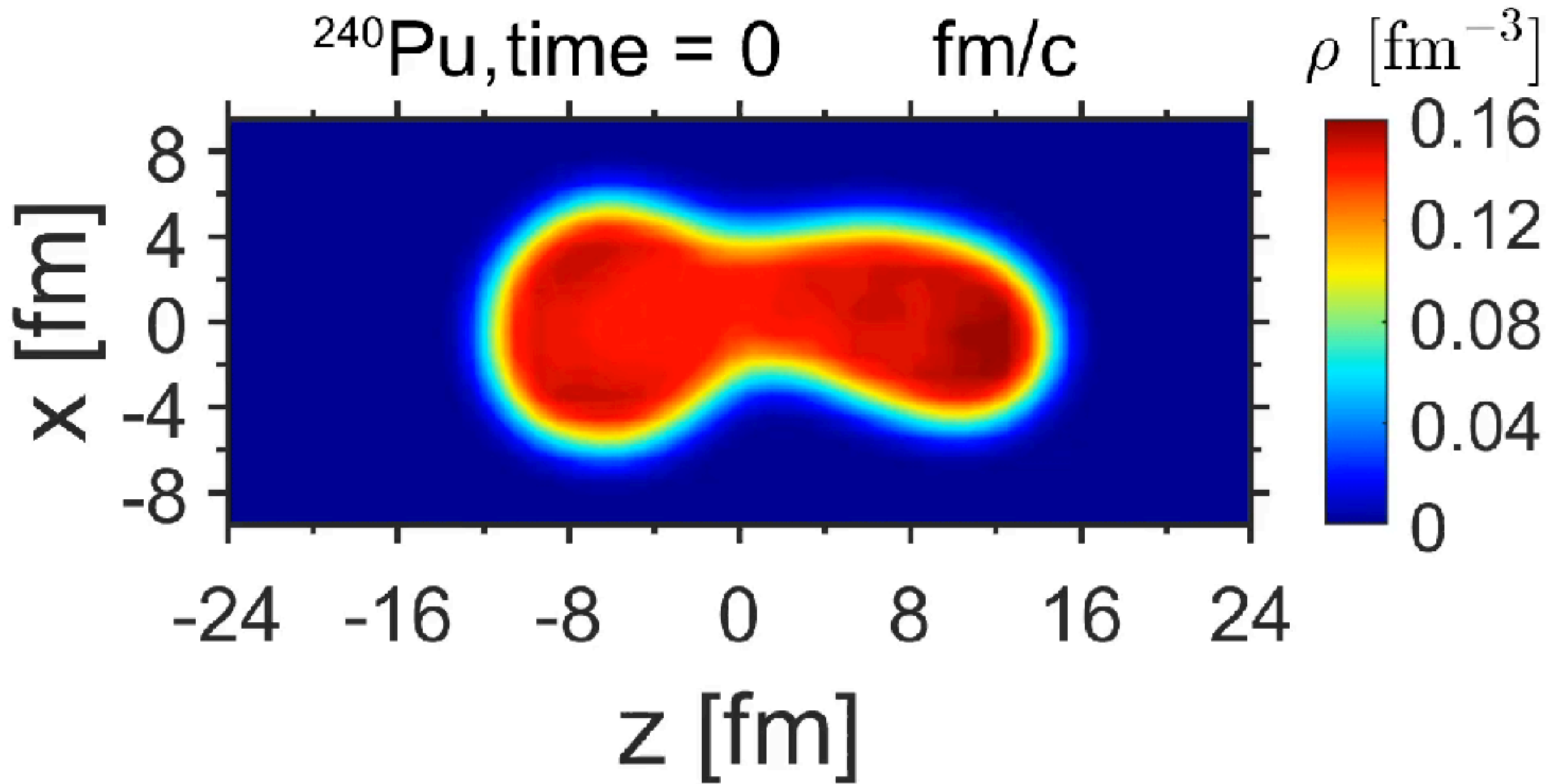
When are these light clusters formed?

What is their structure?

What is their role in the scission mechanism?

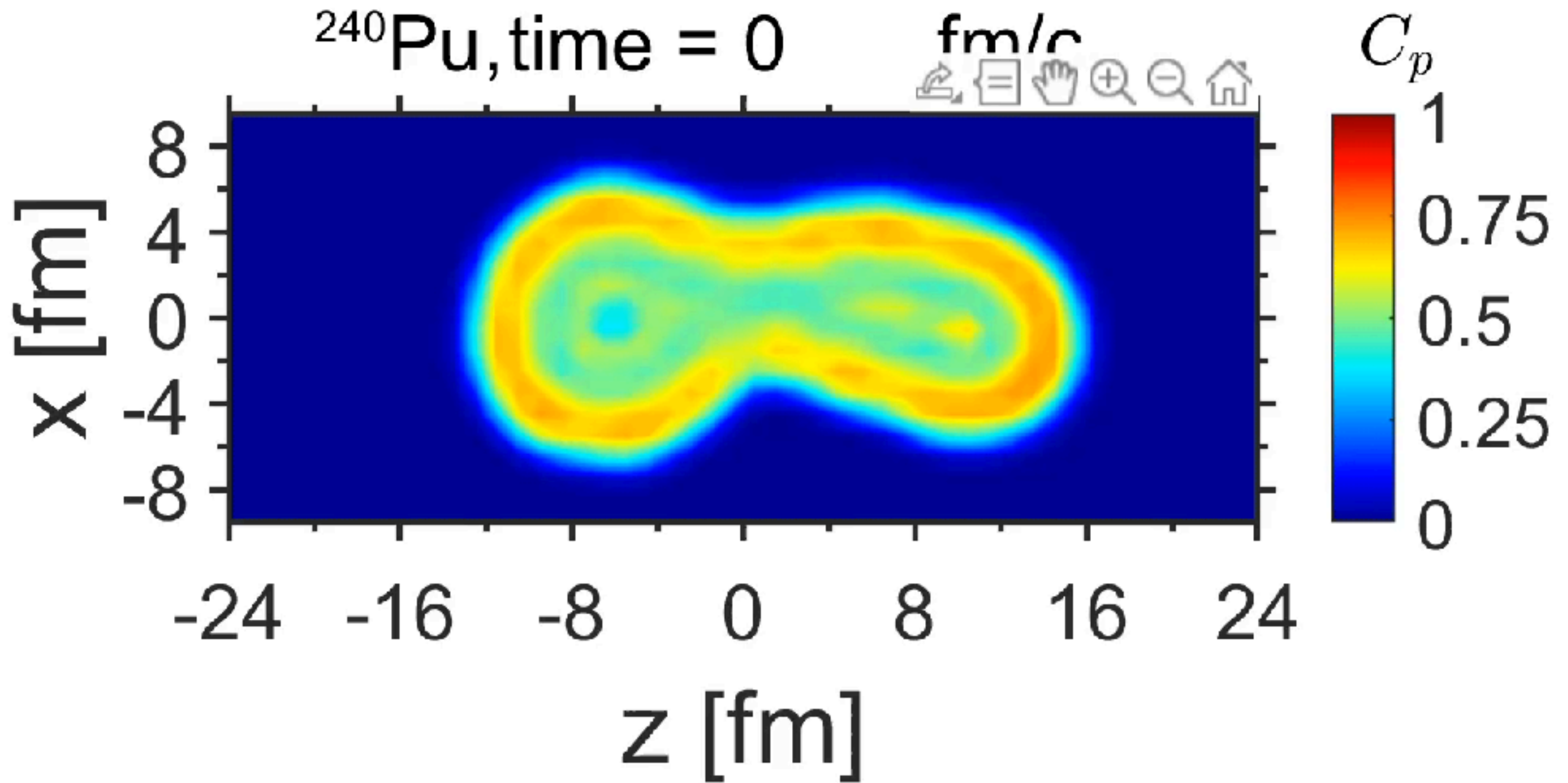


Trajectory 2  
Broken axial symmetry





Trajectory 2  
Broken axial symmetry



## Methods (TDGCM, TDDFT) based on the framework of universal Energy Density Functionals

✓ ...accurate microscopic description of universal collective phenomena (fission) that reflect the organisation of nucleonic matter in finite nuclei.

- Finite temperature effects
- Energy dissipation and TKE of fragments
- Neck formation and scission mechanism
- Ternary fission
- Fragment angular momentum generation
- Symmetry restoration



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<http://bela.phy.hr/quantixlie/hr/>  
<https://strukturnifondovi.hr/>

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