Gdje je spin protona ?

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Duboko–neelastično raspršenje

$$\left.\frac{d^2\sigma}{dxdy}\right|_{\mathrm{eN}} = \frac{8\pi M_N E\alpha^2}{Q^4}\cdot\left[xy^2F_1^{eN}(Q^2,x)+\left(1-y-\frac{M_Nxy}{2E}\right)F_2^{eN}\right]$$

- Parton nosi x udjela nukleonskog impulsa.
- Bjorkenova skalirajuća varijabla

$$x=\frac{Q^2}{2\nu M_N},\qquad 0<x\leq 1$$

- Dio energije prenesene nukleonu

$$y=\frac{E-E'}{E},\qquad 0\leq x\leq 1$$

- Veza sa varijablama mjerenim u eksperimentu:

$$Q^2=4EE'\sin^2\left(\frac{\theta}{2}\right)$$

$$\nu=E-E'$$

- Za $Q^2\geq 1$ GeV^2 $F_{1,2}^{eN}$ ne ovise značajno o Q^2 nego samo o x
- Za $Q^2\geq 1$ GeV^2 $F_2^{eN}\approx 2xF_1^{eN}$, Callan–Gross relacija, partoni imaju spin $1/2$
- Približno 50% impulsa nose gluoni ($s\bar{s}$ parovi nose 2.6% impulsa)

Strukturne funkcije sa spinskom zavisnošću

- Za polarizirani hadron imamo dodatni Lorentzov vektor s^μ

$$\begin{aligned}
W_{\mu\nu} &= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1(x, Q^2) \\
&+ \left(P_\mu - q_\mu \frac{q\cdot p}{q^2}\right)\left(P_\nu - q_\nu \frac{q\cdot p}{q^2}\right)\frac{W_2(x,Q^2)}{M^2} \\
&+ i\varepsilon_{\mu\nu\lambda\sigma} q^\lambda s^\sigma M G_1(x,Q^2) \\
&+ i\varepsilon_{\mu\nu\lambda\sigma} q^\lambda (s^\sigma p\cdot P - P^\sigma q\cdot s)\frac{G_2(x,Q^2)}{M}
\end{aligned}$$

- faktor i garantira da $\Gamma^*_\mu(p \leftrightarrow p') = \Gamma_\mu$
- $\varepsilon_{\mu\nu\lambda\sigma}$ osigurava $q^\mu W_{\mu\nu} = q^\nu W_{\mu\nu} = 0$
- dodatni doprinosi mjenjaju predznak ako nukleon promjeni orjentaciju spina. Tako da se oni dokinu kod usrednjavanja po spinu.
- Uvodimo bezimenzionalne funkcije

$$M G_1(x,Q^2) = \frac{g_1(x,Q^2)}{p\cdot q}$$

$$\frac{G_2(x,Q^2)}{M} = \frac{g_2(x,Q^2)}{(p\cdot q)^2}$$

- Da bi mjerili te strukturne funkcije i lepton mora biti polarizirani

$$
\begin{aligned}
L_{\mu\nu} &= \frac{1}{4}\mathrm{Tr}\left[(\not{p}+m)(1-\not{s}\gamma_5)\gamma_\mu(\not{p}'+m)\gamma_\nu\right] \\
&= \frac{1}{4}\left[p_\mu p'_\nu + p'_\nu p_\mu - g_{\mu\nu}(p\cdot p' - m^2)\right] + im\varepsilon_{\mu\nu\alpha\beta}q^\alpha s_e^\beta
\end{aligned}
$$

- kontrakcijom dobivamo dodatni asimetrični dio

$$
\begin{aligned}
L_{\mu\nu}W^{\mu\nu} &= \ldots - 2mM(g_{\lambda\alpha}g_{\sigma\beta} - g_{\lambda\beta}g_{\sigma\alpha})q^\lambda s_p^\sigma q^\alpha s_e^\beta G_1 \\
&- \frac{m}{M}(g_{\lambda\alpha}g_{\sigma\beta} - g_{\lambda\beta}g_{\sigma\alpha})q^\lambda(q\cdot P s_p^\sigma - q\cdot s_p P^\sigma)q^\alpha s_e^\beta G_2
\end{aligned}
$$

- Koji izbor polarizacija odabrati ?
- u sustavu CM imamo

$$
\begin{aligned}
p_\mu &= (E_e, 0, 0, p_e), \qquad P_\mu = (E_p, 0, 0, -P_e) \\
s_e^\mu(l) &= \pm(p_e, 0, 0, E_e), \quad s_p^\mu(l) = \pm(p_p, 0, 0, -E_p) \\
s_{e,p}^\mu(t) &= \pm(0, 1, 0, 0), \qquad s_{e,p}^\mu(t) = \pm(0, 0, 1, 0)
\end{aligned}
$$

- sljedeći produkti su veliki

$$
s_e(l)\cdot s_p(l) \approx \pm 2\frac{E_e E_p}{mM}, \qquad s_e(l)\cdot P \approx \pm 2\frac{E_e E_p}{m}
$$

- raspršujemo longitudinalno polarizirani lepton na longitudinalno ili transverzalno polariziranom nukleonu.
- mjeri se asimetrija za longitudinalno polariziran proton

$$A_1 = \frac{\left[\frac{d^2\sigma}{dE'd\Omega}(\uparrow\uparrow) - \frac{d^2\sigma}{dE'd\Omega}(\uparrow\downarrow)\right]}{\left[\frac{d^2\sigma}{dE'd\Omega}(\uparrow\uparrow) + \frac{d^2\sigma}{dE'd\Omega}(\uparrow\downarrow)\right]}$$

$$A_1 = D\left\{\frac{g_1(x) - \gamma^2 g_1(x)}{F_1(x)} + \eta\gamma\frac{g_1(x) + g_1(x)}{F_1(x)}\right\}$$

$$D = \frac{y(2-y)}{y^2 + 2(1-y)(1+R)}, \qquad \eta = \frac{2\gamma(1-y)}{2-y}$$

$$\gamma = \sqrt{\frac{Q^2 M^2}{\nu^2}}$$

- asimetrija za transverzalnu polarizaciju protona

$$A_2 = \frac{\left[\frac{d^2\sigma}{dE'd\Omega}(\uparrow\rightarrow) - \frac{d^2\sigma}{dE'd\Omega}(\downarrow\rightarrow)\right]}{\left[\frac{d^2\sigma}{dE'd\Omega}(\uparrow\rightarrow) + \frac{d^2\sigma}{dE'd\Omega}(\uparrow\rightarrow)\right]}$$

- u limesu $Q^2 \rightarrow \infty$, $y \rightarrow 1$

$$A_2 = \gamma\frac{g_1(x) + g_1(x)}{F_1(x)}$$

- udarni presjeci za logitudinalnu i transverzalnu polarizaciju trebali bi biti istog reda veličine, tj. $g_2(x)$ nije veći od $g_1(x)$
- $g_2(x)$ je kod longitudinalne polarizacije potisnut za faktor Mx/E. Asimetrija mjeri omjer $g_1(x)$ prema $F_1(x)$.
- možemo li dati $g_1(x)$ praktično značenje kakvo imamo za $F_1(x)$?

$$F_1(x) = \frac{1}{2} \sum_{q=u,d,\bar{u},\bar{d},\ldots} q(x) Q_q^2$$

- $q(x)$ je vjerojatnost nalaženja kvarka sa udjelom impulsa x

$$g_1(x) = \frac{1}{2} \sum_{q=u,d,\bar{u},\bar{d},\ldots} [q^{\uparrow}(x) - q^{\downarrow}(x)] Q_q^2$$

- $q^{\uparrow}(x)$ je vjerojatnost nalaženja kvarka sa udjelom impulsa x polariziranog u istom smjeru kao i cjeli proton.
- procesi koje promatramo

$$e^{\uparrow}(\text{long.}) + p^{\uparrow}, {}^3He^{\uparrow}, d^{\uparrow}(\text{long. ili trans.}) \longrightarrow e' + X$$

$$p + p^{\uparrow}(\text{trans.}) \longrightarrow \gamma + X$$

Björkenovo pravilo sume

- Björken 1966: "Nadam se da će se nešto moći spasiti iz ove bezvrijedne jednakosti."
- Danas: "Björkenovo pravilo sume je rigorozna predikcija algebre struja i QCD-a"
- experiment:

$$\int (g_1^p(x) - g_1^n(x))dx = 0.174^{+0.024}_{-0.012}$$

- teorija:

$$\int (g_1^p(x) - g_1^n(x))dx = 0.181 \pm 0.003$$

- dio udarnog presjeka asimetričnog u spinu dobivamo pomoću optičkog teorema

$$i\gamma_\mu \not{p}' \gamma_\nu \varepsilon^{\mu\nu\alpha\beta} = -2p'^\alpha \gamma^\beta \gamma_5 + 2p'^\beta \gamma^\alpha \gamma_5$$

$$\triangle\sigma \sim \sum_q \langle P, s | Q_q^2 \bar{q} \gamma^\alpha \gamma_5 q | P, s \rangle$$

- za Q_q^2 vrijedi relacija

$$Q_q^2 = \frac{15}{18} + \frac{1}{3}\tau^3 = \begin{cases} 4/9, & \text{u kvark} \\ 1/9, & \text{d kvark} \end{cases}$$

- zahtjevamo

$$\begin{aligned} \triangle\sigma^p - \triangle\sigma^n &\sim \langle P, s|\tau_3 \bar{q}\gamma^\alpha\gamma_5 q|P, s\rangle \\ &\sim \langle P, s|\tau_+ \bar{q}\gamma^\alpha\gamma_5 q|N, s\rangle \end{aligned}$$

- to je matrični element za β raspad neutrona. Radimo vezu sa aksialnim konstantama

$$g_A^\alpha \sim \langle P, s|\bar{q}\gamma^\alpha\gamma_5\lambda^\alpha q|P, s\rangle$$

$$\begin{aligned} g_A^0 &= \triangle u + \triangle d + \triangle s = \triangle\Sigma \sim 0.2 - 0.3 \\ g_A^3 &= \triangle u - \triangle d = F + D = 1.257 \\ g_A^8 &= \triangle u + \triangle d - 2\triangle s = 3F - D = 0.601 \end{aligned}$$

- za $g_1(x, Q^2)$ funkciju protona imamo

$$\Gamma_1^p = \int_0^1 dx g_1^p(x, Q^2) = \frac{1}{2}\left(\frac{4}{9}\triangle u + \frac{1}{9}\triangle d + \frac{1}{9}\triangle s\right)$$

- konačno dobivamo izraz za Björkenovo pravilo sume

$$\int (g_1^p(x) - g_1^n(x))dx = \frac{1}{6}(\triangle u - \triangle d) = \frac{1}{6}\frac{g_A}{g_V}C_{(NS)}$$

$$C_{(NS)} = 1 - \frac{\alpha_s}{\pi} - \frac{43}{12}\left(\frac{\alpha_s}{\pi}\right)^2 - 20.22\left(\frac{\alpha_s}{\pi}\right)^3 + \ldots$$

- striktno vrijedi za $Q^2 \to \infty$

Ellis-Jaffe pravilo sume

- zahtjevaju da $\triangle s = 0$, to vodi na $g_A^0 = g_A^8$

$$\Gamma_1^p = \frac{1}{18}[3F + D + 2\Sigma] \xrightarrow{\Delta s=0} \frac{1}{12} g_A^3 + \frac{5}{35} g_A^8 = 0.181 \pm 0.003$$

$$\Gamma_1^p = 0.176 \pm 0.006 \quad \text{sa korekcijama}$$

- EMC rezultati

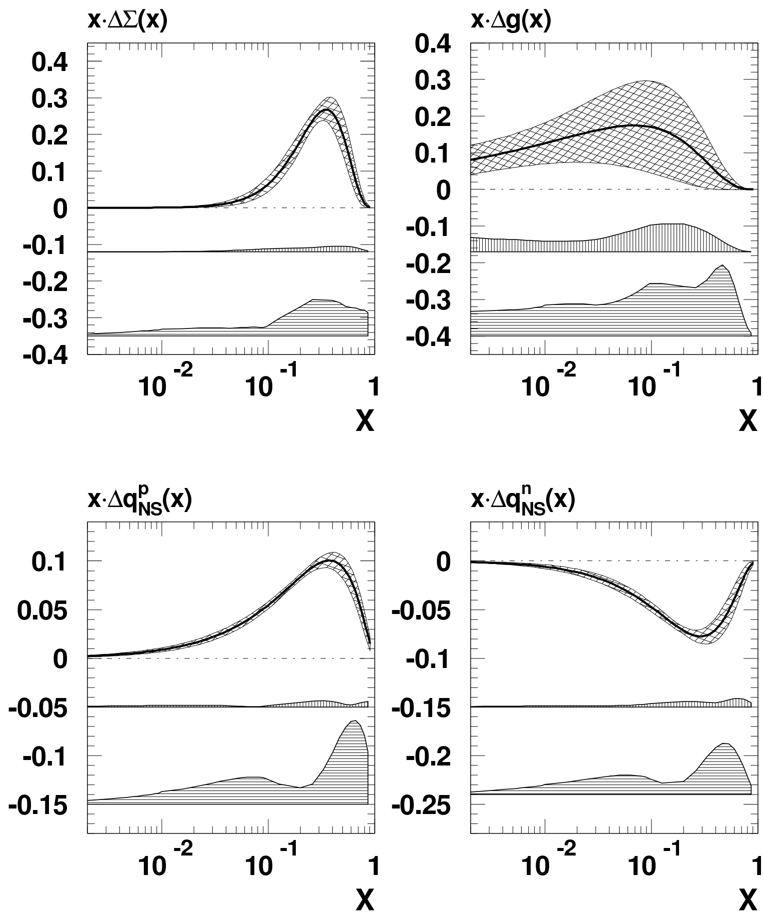
$$\begin{aligned} \Gamma_1^p &= 0.126 \pm 0.010 \\ \Delta u &= 0.77 \\ \Delta d &= -0.49 \\ \Delta s &= -0.15 \\ \Delta\Sigma &= 0.14 \end{aligned}$$

- danas

$$\begin{aligned} \Delta u &= 0.82 \pm 0.06 \\ \Delta d &= -0.44 \pm 0.06 \\ \Delta s &= -0.11 \pm 0.06 \\ \Delta\Sigma &= 0.27 \pm 0.11 \end{aligned}$$

- eksperimentalci više vole

$$\begin{aligned}
\Delta q_{NS}(p) &= \Delta u - \frac{1}{2}\Delta d - \frac{1}{2}\Delta s \\
\Delta q_{NS}(n) &= \Delta d - \frac{1}{2}\Delta u - \frac{1}{2}\Delta s \\
g_1^p &= \frac{2}{9}(\Delta\Sigma + \Delta q_{NS}(p)) \\
g_1^n &= \frac{2}{9}(\Delta\Sigma + \Delta q_{NS}(n))
\end{aligned}$$



- spin protona preko svih doprinosa možemo pisati kao

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta g + L_q + L_g$$

- problem mjerenja ?
- postojanje pravila suma ?
- baždarno invarijantno za gluone je jedino

$$\hat{J}_g = \hat{L}_g + \Delta\hat{g}$$

- J_g za sad izvan dosega eksperimenta
- L_q može biti mjereno
- "Transverzalnost" daje nadu
  - $\delta q_a(x)$ mjeri razliku paralelno-antiparalelno za nukleonski spin
  - daje informaciju koliko je relativistički objekt nukleon, $\delta q(x) = \Delta q(x)$ u ne-relatvističkom slučaju
  - Δq i Δg miješaju se kroz evoluciju po Q^2 ali ne i δq
  - povezana je sa tenzorskim nabojem

$$\bar{q} i \sigma^{0i} \gamma_5 q$$

    i ne veže se sa gluonskim ili $\bar{q}q$ parovima

Quantity	Experiment	Dates
Δg	COMPASS	2003
	HERMES	2002 +
	RHIC	2002-05
	SLAC E-161	2005
Flavour separation	HERMES	2002
	RHIC	2002-04
Λ polarization	RHIC	2002+
Transversity, h_1	COMPASS	2004+
	HERMES	2002-03
	RHIC	2002+
Transversity from e^+e^-	BELLE	2002
DVCS plus meson production	COMPASS	2004+
	HERMES	2004-05
High energy GDH integrand	SLAC E-159	2006

Kraj:

- $\Delta\Sigma \sim 30\%$
- ostalih 70% više ne predstavlja krizu
- ukupni gluonski doprinos 1-2 jedinice spina
  ukupni orbitalni angularni moment -1
- ako se Δg pokaže premalim nova ”kriza”
- Gdje je spin protona ?