Zadaci III iz Fizike elementarnih čestica –
Doktorski studij 2013/14

Str. 46 TASI 2013 lekcija H.E. Logan,
Literatura poblokovima nastave:
9. Fizika Higgsovog sektora

Homework questions

1. Compute the tree-level decay partial width for the Higgs boson into a pair of bottom quarks and show that it is given by

$$\Gamma(h \to b\bar{b}) = \frac{N_c}{8\pi}\frac{m_b^2}{v^2} m_h \left[1 - \frac{4m_b^2}{m_h^2}\right]^{3/2}, \tag{185}$$

where $N_c = 3$ is the number of colours of the b quark and $v = 246$ GeV is the Higgs vacuum expectation value (vev). The Feynman rule for the $hb\bar{b}$ vertex is $-im_b/v$.

2. Imagine that the scalar potential for the Standard Model Higgs field contained a ϕ^6 term, as follows:

$$V(\Phi) = -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2 + \frac{1}{\Lambda^2}(\Phi^\dagger\Phi)^3, \tag{186}$$

where μ^2, λ, and $1/\Lambda^2$ are all positive and Λ has dimensions of mass.

Minimize the potential and eliminate μ^2 in favor of the Higgs vev v. Then find the Higgs mass and the Feynman rules for the hhh and $hhhh$ coupling vertices in terms of v, λ, and $1/\Lambda^2$. You can work in the unitarity gauge and write

$$\Phi = \begin{pmatrix} 0 \\ (h+v)/\sqrt{2} \end{pmatrix}. \tag{187}$$

(Recall that when the $1/\Lambda^2$ term is not there, the Higgs mass is $m_h = \sqrt{2\lambda}v$, the hhh coupling Feynman rule is $-3im_h^2/v$, and the $hhhh$ coupling Feynman rule is $-3im_h^2/v^2$. The idea here is to see whether you can tell that the $1/\Lambda^2$ term is there by measuring m_h and the hhh coupling and comparing to the Standard Model relationship.)

3. The LHC measures rates for Higgs boson production and decay into specific final states, which can be written in the "zero width approximation" as

$$\text{Rate}_{ij} = \sigma_i \times \text{BR}_j = \sigma_i \times \frac{\Gamma_j}{\Gamma_{\text{tot}}}. \tag{188}$$

If there is a new, non-SM decay mode of the Higgs, which is unobservable at the LHC (for example, Higgs decay into light-quark jets, which would be buried under background), all the observable Higgs signal rates can be kept the same by cranking up the production couplings at the same time as the branching ratio to the new final state is increased. All couplings must be increased by the same factor in order to keep the ratios of rates fixed; we can denote that factor by κ, in which case $\sigma_i = \kappa^2\sigma_i^{\text{SM}}$ and $\Gamma_j = \kappa^2\Gamma_j^{\text{SM}}$.

Work out the relationship between the coupling scaling factor κ and the new unobservable decay branching ratio $\mathrm{BR}_{\mathrm{new}}$ that is required to keep all the Higgs signal rates fixed to their SM values. (This relation defines a "flat direction" in the Higgs coupling fit using LHC data. To cut off the flat direction, fits to LHC Higgs data usually assume either that there are no unobservable new decay modes (i.e., $\mathrm{BR}_{\mathrm{new}} = 0$) or that the Higgs couplings to WW and ZZ cannot be larger than their SM values (i.e., $\kappa \leq 1$ in our notation). The latter happens to be true in all models containing only Higgs doublets and/or singlets of $\mathrm{SU}(2)_L$.)

4. The correct ratio for the W and Z masses can (by coincidence) also be generated by a septet (or seven-plet) of $\mathrm{SU}(2)_L$ with hypercharge $Y = 2$ (in the convention $Q = T^3 + Y$). If the vev of the septet is given by (note that v_7 is in the neutral component, $T^3 = -2$, so that $Q = T^3 + Y = 0$)

$$\langle \chi_7 \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ v_7 \\ 0 \end{pmatrix}, \tag{189}$$

work out the gauge boson mass-squared matrix in the W^1, W^2, W^3, B basis and show that it is indeed proportional to the SM case,

$$M_{\mathrm{SM}}^2 = \frac{v_{\mathrm{SM}}^2}{4} \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & -gg' \\ 0 & 0 & -gg' & g'^2 \end{pmatrix}. \tag{190}$$

If the W and Z masses were generated entirely by the septet, what value of v_7 would be needed? (Recall that $M_W = g v_{\mathrm{SM}}/2$ and $M_Z = \sqrt{g^2 + g'^2} v_{\mathrm{SM}}/2$ at tree level for $v_{\mathrm{SM}} = 246$ GeV.)

Could all mass generation in the Standard Model be accomplished by the septet? Why or why not?