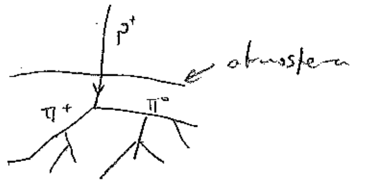
PROCESI U ASTROFIZICI

Kozmičke zrake:

- radijacija vrlo visoke energije
- podrijetlo: - izvan Sunčevog sustava
  - nije u potpunosti objašnjeno (supernove, aktivne galaktičke jezgre?)
- sastav: - primarno: - visokoenergetski protoni i atomske jezgre (90% p, 9% α, 1% e^-, nešto malo e^+, $\bar{p}$)
  - izraz zrake je povijesna pogreška (mislili su da je EM zračenje)
- kada udare u atmosferu stvaraju pljuskove sekundarnih čestica

npr.



- spektar kozmičkih zraka doseže i $3 \cdot 10^{20}$ eV ≈ 50 J (1 eV $= 1.6 \cdot 10^{-19}$ J)
  - teniska loptica brzine $v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2 \cdot 50}{0,055}} = 42$ m/s $= 150$ km/h
  - u usporedbi s LHC-om:

    $E_{LHC} = 7$ TeV $\approx 7 \cdot 10^{12}$ eV

    - protoni 7,8 redova veličine veće energije od LHC-a (laboratorij za FCC?)

  - za svjetlošću nakon 1 godine kasni 46 nm... (za vježbu?)

Zadatak.

Protoni iz kozmičkih zraka se sudaraju s fotonima iz CMB-a, te može doći do reakcije:

$$\gamma + p \rightarrow \Delta^+ \longrightarrow p + \pi^0$$
$$\longrightarrow n + \pi^+$$

Odredite energiju praga za ovaj proces.
Udarni presjek za ovaj proces je $\sigma = 2 \cdot 10^{-28}$ cm², a gustoća fotona CMB $\rho = 400$ cm^{-3}.
Odredite slobodni put protona.

$$s = (p+q)^2 = \underbrace{E^2 + 2Eq + q^2 - \vec{p}^2}_{m_p^2} - 2\vec{p}\vec{q} - \underbrace{\vec{q}^2}_{0} = m_p^2 + 2q(E - |\vec{p}|\cos\theta)$$

$\vec{p}$ - impuls protona
$\vec{q}$ - impuls fotona
E - energija protona
q - —"— fotona

$\Rightarrow$ za UR protone: $E \approx |\vec{p}|$

$$s = m_p^2 + 2qE(1 - \cos\theta)$$

- za čeoni sudar $\cos\theta = -1$

$\Rightarrow s = m_p^2 + 4qE$

- da bi došlo do reakcije $s \geq (m_p + m_\pi)^2 = m_p^2 + 2m_\pi m_p + m_\pi^2$

$$\Rightarrow 4q E_{prag} = m_\pi(2m_p + m_\pi) \Rightarrow E_{prag} = m_\pi \frac{(m_p + \frac{m_\pi}{2})}{2q}$$

- za CMB imamo:

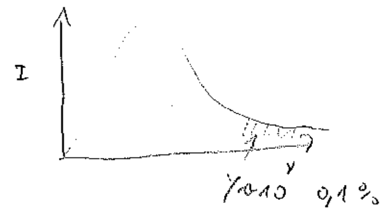
$T = 2{,}73\,K$

$\Rightarrow kT = 2{,}35\cdot 10^{-4}\,eV$ – red veličine energije fotona u CMB koji ima spektar crnog tijela
(Boltzmann $k = 8{,}617\cdot 10^{-5}\,eV\,K^{-1}$)

$\Rightarrow \varepsilon = \underbrace{\gamma}_{faktor} kT$

$$E_{prag} = \frac{2{,}8}{\gamma}\cdot 10^{20}\,eV$$

$$E_{prag} \sim 10^{20}\,eV$$



- energija praga je za ultraenergetske kozmičke zrake zadovoljena.
- zašto ih onda ipak detektiramo, ako bi ovim procesom trebale izgubiti energiju?

$\sigma = 2\cdot 10^{-28}\,cm^2$

$\rho = 400\,cm^{-3}$

iz $\frac{dP}{dx} = \sigma\rho \Rightarrow$ slobodni put $\lambda = \frac{1}{\sigma\rho} = 1{,}25\cdot 10^{25}\,cm \simeq 1{,}3\cdot 10^{7}\,Ly$

- to je veličina galaktičkog klastera!!!
- kozmičke zrake su iz galaktičkog susjedstva

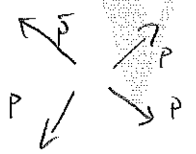
Zadatak

- Energijska raspodjela protona u kozmičkom zračenju je oblika $\frac{dN}{dE} = AE^{-\alpha}$. Ti protoni mogu naletjeti na protone u mirovanju i producirati antiprotone. Izračunajte energijsku raspodjelu antiprotona. Pretpostavite da je udarni presjek za proces neovisan o energiji. Također pretpostavite da antiprotoni dobiju 1/4 ukupne SCM energije i da je proces izotropan u SCM.

$pp \to ppp\bar{p}$

LAB

$\overleftrightarrow{p_1^{\mu}} = (E, p, 0, 0)$ $\quad p_2^{\mu} = (m, 0, 0, 0)$ $\quad \Rightarrow$



SCM

$\overleftrightarrow{p_1'^{\mu}} = (E', \sqrt{E'^2 - m^2}, 0, 0)$ $\quad \overleftrightarrow{p_2'^{\mu}} = (E', -\sqrt{E'^2 - m^2}, 0, 0)$

- koristimo:

$$s = (p_1^{\mu} + p_2^{\mu})^2 = (p_1'^{\mu} + p_2'^{\mu})^2 \Rightarrow \overbrace{E^2 + 2Em + m^2 - p^2}^{m^2} = 4E'^2 \Rightarrow \boxed{E' = \sqrt{\frac{m(E+m)}{2}} \quad p' = \sqrt{E'^2 - m^2} = \sqrt{\frac{m(E-m)}{2}}}$$

- koristimo Lorentzove transformacije:

$$p_x' = \gamma(p_x - \beta E) \qquad -\gamma\beta = \frac{p_x' - \gamma p_x}{E}$$

$$-p_x' = -\gamma\beta m \quad \Rightarrow \quad \Rightarrow -p_x' = (p_x' - \gamma p_x)\frac{m}{E} \Rightarrow -\gamma p_x = -\frac{E p_x'}{m} - p_x' = -\sqrt{\frac{m(E-m)}{2}}\left(\frac{E}{m}+1\right)$$

$$\Rightarrow \gamma = \sqrt{\frac{m(E-m)}{2}}\,\frac{\left(\frac{E}{m}+1\right)}{\sqrt{E^2-m^2}} =$$

$$\boxed{\gamma = \sqrt{\frac{E+m}{2m}}}$$

nakon računa $\Rightarrow$ $\boxed{\gamma\beta = \sqrt{\frac{E-m}{2m}}}$

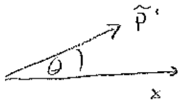
- antiproton odnosi 1/4 ukupne SCM energije

$\tilde E'$ - energija antiprotona u SCM:

$$\tilde E' = \frac{2E'}{4} = \frac{1}{2}\sqrt{\frac{m(E+m)}{2}} = \sqrt{\frac{m(E+m)}{8}}$$

$$\tilde p' = \sqrt{\tilde E'^2 - m^2} = \sqrt{\frac{m(E-7m)}{8}}$$

- u SCM to izgleda:



(boost direction)

$$\vec{\tilde p}' = \tilde p'\cos\theta\,\hat x + \tilde p'\sin\theta\,\hat y$$

- u LAB sistemu imamo

$$\tilde E = \gamma\tilde E' + \beta\gamma\tilde p_x' = \gamma(\tilde E' + \beta\tilde p'\cos\theta)$$

$$= \frac{1}{4}\left[(E+m) + \sqrt{(E-m)(E-7m)}\cos\theta\right] \Rightarrow \cos\theta = \frac{4\tilde E - (E+m)}{\sqrt{(E-m)(E-7m)}}$$

- zanima nas $\frac{dN_{\tilde p}}{d\tilde E}$!!! - broj antiprotona u ovisnosti o njihovoj energiji

- napišimo na početku

$\frac{dN_{\tilde p}}{dE\,d\theta}$ - broj antiprotona produciranih s kutem θ u SCM, i uslijed procesa uzrokovanog protonom energije E u lab sustavu

- zbog neovisnosti udarnog presjeka o energiji mora biti proporcionalan broju protona s energijom E tj. $\frac{dN}{dE}$
- zbog izotropnosti u SCM $\Rightarrow$ $\frac{dN_{\tilde p}}{dE\,d\theta} = C_1\frac{dN}{dE}P(\theta) = \frac{C_1}{\pi}\frac{dN}{dE}$

- povezujemo s $\tilde E$:

$$\frac{dN_{\tilde p}}{dE\,d\tilde E} = \frac{dN_{\tilde p}}{dE\,d\theta}\cdot\frac{d\theta}{d\tilde E} = \frac{C_1}{\pi}\frac{dN}{dE}\cdot\frac{4}{\sqrt{(E-m)(E-7m) - [4\tilde E-(E+m)]^2}}$$

; gdje $\frac{dN}{dE} = AE^{-\alpha}$

$\Rightarrow$ sada je

$$\frac{dN_{\tilde p}}{d\tilde E} = \int_{E_{min}}^{E_{max}} \frac{dN_{\tilde p}}{dE\,d\tilde E}\,dE$$

- koje su granice integracije?

za $\cos\theta = 1$ $\Rightarrow$ $\tilde E = \frac{1}{4}\left[(E+m) + \sqrt{(E-m)(E-7m)}\right] \Rightarrow E_{min} = \frac{8\tilde E^2 - 4\tilde E m + 3m^2}{4\tilde E - 5m}$

za $E \to \infty$ $\tilde E = \frac{1}{4}\left[E + E\cos\theta\right] \Rightarrow E_{max} = +\infty$

ELEKTRON – PROTON RASPRŠENJE

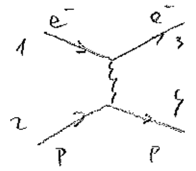
- dvije različite klasifikacije:

I. - Ovisno o prijenosu impulsa
- $q \sim \frac{1}{\lambda}$

a) raspršenje na točkastoj čestici
b) raspršenje na raspodjeli naboja i magnetskog momenta
c) raspršenje na partonima u protonu.

II. - elastično $pe^- \to pe^-$ - neelastično $pe^- \to Xe$ X - hadroni

- slučaj a)
- elastično raspršenje na „elementarnom" protonu



$$\mathcal{M} = \frac{e^2}{q^2}\, \bar{u}_3 \gamma^\mu u_1\, \bar{u}_4 \gamma_\mu u_2$$

$$\overline{|\mathcal{M}|^2} = \frac{m_p^2 e^4}{E_1 E_3 \sin^4(\frac{\theta}{2})}\left[\cos^2\frac{\theta}{2} - \frac{q^2}{2m_p^2}\sin^2\frac{\theta}{2}\right]$$ - u lab sustavu

$$\Rightarrow \left(\frac{d\sigma}{d\Omega}\right)_{lab} = \frac{\alpha^2}{4E_1^2 \sin^4(\frac{\theta}{2})}\frac{E_3}{E_1}\left[\cos^2\frac{\theta}{2} + \frac{Q^2}{2m_p^2}\sin^2\frac{\theta}{2}\right]$$

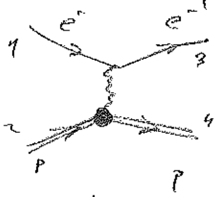
gdje $E_3 = \frac{E_1 m_p}{m_p + E_1(1-\cos\theta)}$; $Q^2 = -q^2 = 2m_p(E_1 - E_3) = \frac{2m_p E_1^2(1-\cos\theta)}{m_p + E_1(1-\cos\theta)}$

- primijetiti da imamo ovisnost o samo jednoj varijabli (u ovom slučaju θ)

- Mottovo raspršenje:
  - raspršenje elektrona spina $1/2$ na statičkom potencijalu
  - $E_1 = E_2$ - nema odboja protona

$$\Rightarrow \left(\frac{d\sigma}{d\Omega}\right)_{LAB} = \frac{\alpha^2}{4E_1^2 \sin^4\frac{\theta}{2}}\cos^2\frac{\theta}{2}$$

- slučaj b)
- elastično raspršenje na protonu sa strukturom.



- i dalje gledamo na proton kao na česticu spina $1/2$, ali ne znamo vrh!
- vrh protona pišemo kao:

$\bar{u}_4(p)\, \Gamma^\mu u_2(p')$
- u slučaju $q <<$, $\Gamma^\mu \to \gamma^\mu$
- Lorentzova simetrija $\Rightarrow$ Γ^μ može biti linearna kombinacija γ^μ, p^μ, p'^μ
- koristeći i očuvanje pariteta dobivamo najopćenitiji mogući formu:

$$\Gamma^\mu(p,p',\gamma^\mu,m,e) = \gamma^\mu L_1(q^2) + \frac{i}{2m_p}\sigma^{\mu\nu}q_\nu L_2(q^2)$$ — L_1 i L_2 su funkcije q^2, m, e

- sad s ovakvim vrhom izračunamo udarni presjek: Rosenbluthova formula

$$\left(\frac{d\sigma}{d\Omega}\right)_{LAB} = \frac{\alpha^2}{4E_1^2\sin^4\frac{\theta}{2}}\cdot\frac{E_3}{E_1}\left[\left(L_1^2 - \frac{q^2}{4m_p^2}L_2^2\right)\cos^2\frac{\theta}{2} - (L_1+L_2)^2\frac{q^2}{2m_p^2}\sin^2\frac{\theta}{2}\right]$$

- iz ovoga se mogu dobiti drugačiji oblici:

$$G_E(q^2) = L_1(q^2) + \frac{q^2}{4m_p^2}L_2(q^2)$$
$$G_M(q^2) = L_1(q^2) + L_2(q^2)$$
$$\Rightarrow \left(\frac{d\sigma}{d\Omega}\right)_{LAB} = \frac{\alpha^2}{4E_1^2\sin^4\frac{\theta}{2}}\frac{E_3}{E_1}\left[\left(\frac{G_E^2+\tau G_M^2}{1+\tau}\right)\cos^2\frac{\theta}{2} + 2\tau G_M^2\sin^2\frac{\theta}{2}\right] \quad ; \tau = \frac{-q^2}{4m_p^2} > 0$$

Zadatak: Izrazite amplitudu za raspršenje nerelativističkog elektrona na potencijalu raspodjele naboja u Bornovoj aproksimaciji!

Bornova aproksimacija podsjetnik:

$$(\nabla^2+k^2)\phi(\vec r) = U(\vec r)\phi(\vec r) \quad ; E = \frac{\hbar^2 k^2}{2m}$$

- opće rješenje

$$\phi(\vec r) = \phi_0(\vec r) + \int d^3r'\, U(\vec r')\phi(\vec r')\, G(\vec r - \vec r') \; ; \phi_0(\vec r) \text{ homogeno} \; ; G \text{ - greenova funkcija}$$

$$(\nabla^2+k^2)G(\vec r-\vec r') = \delta(\vec r) \Rightarrow G(\vec r) = -\frac{1}{4\pi}\frac{e^{ik\vec r}}{r} = -\frac{1}{4\pi}\frac{e^{ikr}}{r}e^{-ik\hat r\vec r'}$$

$$\Rightarrow \phi(\vec r) = \phi_0(\vec r) - \frac{1}{4\pi}\int d^3r'\, U(\vec r')\phi(\vec r')\frac{e^{ik\vec r}}{r} = e^{ikz} + f_k(\theta,\phi)\frac{e^{ikr}}{r} \quad ; \quad f_k(\theta,\phi) = -\frac{1}{4\pi}\int d^3r'\, e^{-ik\hat r\vec r'}U(\vec r')\phi(\vec r')$$

- za $\phi_0(\vec r) = e^{ikz} \Rightarrow$ iterativno $\phi_k(\vec r) = e^{ik\vec r} + \int d^3r'\, G(\vec r-\vec r')U(\vec r')e^{ik\vec r'} + \int d^3r'\int d^3r'' \ldots$

$$V(\vec r) = \int \frac{Q\rho(\vec r')}{4\pi|\vec r-\vec r'|}d^3r'$$

- Bornova aproksimacija za ulazni i izlazni valni funkcije $\psi_i = e^{i(\vec p_i\vec r - Et)}$; $\psi_f = e^{i(\vec p_f\vec r - Et)}$

$$M_{fi} = \langle\psi_f|V(\vec r)|\psi_i\rangle = \int e^{-i\vec p_f\vec r}V(\vec r)e^{i\vec p_i\vec r} = \iint e^{i\vec q\vec r}\frac{Q\rho(\vec r')}{4\pi|\vec r-\vec r'|}d^3r\,d^3r'$$

$$= \iint e^{i\vec q(\vec r-\vec r')}e^{i\vec q\vec r'}\frac{Q\rho(\vec r')}{4\pi|\vec r-\vec r'|}d^3r'd^3r = \int e^{i\vec q\vec R}\frac{Q}{4\pi|\vec R|}d^3R\int\rho(\vec r')e^{i\vec q\vec r'}d^3r'$$

$$\boxed{M_{fi} = M_{fi}^{point}F(\vec q^2)} \; ; \; F(\vec q^2) = \int\rho(\vec r)e^{i\vec q\vec r}d^3r$$

- vratimo se sada na izvod za Rosenbluthovu formulu:
- za $\tau \ll 1$ tj q^2 jako malo

$$\Rightarrow \left(\frac{d\sigma}{d\Omega}\right)_{LAB} = \frac{\alpha^2}{4E_1^2\sin^4\frac{\theta}{2}}\cos^2\frac{\theta}{2}G_E^2 = \left(\frac{d\sigma}{d\Omega}\right)_{Mott}\cdot G_E^2 \Rightarrow G_E \text{ možemo identificirati s } F(\vec q^2) \text{ iz zadatka}$$

tj $G_E(\vec q^2) = \int e^{i\vec q\vec r}\rho(\vec r)d^3r$

- pri prelasku na neelastično raspršenje zgodno je prijeći na nove varijable:

$$Q^2 = -q^2 = -(p_1-p_3)^2 = -2(m_e^2 - E_1E_3 + p_1p_3\cos\theta) \simeq 2E_1E_3(1-\cos\theta) = 4E_1E_3\sin^2\frac{\theta}{2}$$

$$x = \frac{Q^2}{2p_2\cdot q} \quad ; \quad y = \frac{p_2\cdot q}{p_2\cdot p_1} = \frac{m_p(E_1-E_3)}{m_p E_1} = 1 - \frac{E_3}{E_1} \quad ; \quad \nu = \frac{p_2\cdot q}{m_p} = E_1 - E_3$$

- jedino su y i ν međusobno zavisne, ostale su nezavisne.

Zadatak:

U procesu elastičnog raspršenja u LAB sustavu vrijedi

$$Q^2 = 2m_p(E_1 - E_3) = 2m_p E_1 y \quad \text{i} \quad Q^2 = 4E_1E_3\sin^2\frac{\theta}{2}$$

Pretpostavite azimutalnu simetriju i izrazite $\frac{d\sigma}{dQ^2}$ pomoću Q^2 i y.

a) $$\frac{E_1}{E_3}\frac{m_p^2}{Q^2}y^2 = \frac{\cancel{E_1}}{E_3}\frac{m_p^{\cancel{2}}}{2\cancel{m_p}\cancel{E_1}\cancel{y}}y^{\cancel{2}} = \frac{m_p}{2E_3}y = \frac{\cancel{m_p}}{2E_3}\cdot\frac{Q^2}{2\cancel{m_p}E_1} = \frac{1}{\cancel{4E_1E_3}}\cdot\cancel{4E_1E_3}\sin^2\frac{\theta}{2} = \sin^2\frac{\theta}{2}$$

$$\Rightarrow \frac{E_3}{E_1}\cos^2\frac{\theta}{2} = \frac{E_3}{E_1}\left(1-\sin^2\frac{\theta}{2}\right) = \underbrace{\frac{E_3}{E_1}}_{1-y} - \frac{m_p^2}{Q^2}y^2 = 1 - y - \frac{m_p^2}{Q^2}y^2$$

b) $$\frac{d\sigma}{dQ^2} = \frac{d\Omega}{dQ^2}\frac{d\sigma}{d\Omega} = -2\pi\frac{d\cos\theta}{dQ^2}\frac{d\sigma}{d\Omega} \ldots$$

- računamo $\frac{d\cos\theta}{dQ^2}$ imamo $Q^2 = 2E_1E_3(1-\cos\theta) \Rightarrow \cos\theta = 1 - \frac{Q^2}{2E_1E_3} = 1 - \frac{Q^2}{2E_1\left(E_1 - \frac{Q^2}{2m_p}\right)}$

$$\frac{d\cos\theta}{dQ^2} = \frac{-\left(E_3 + \frac{Q^2}{2m_p}\right)}{2E_1E_3^2} = -\frac{\cancel{E_1}}{2\cancel{E_1}E_3^2} = -\frac{1}{2E_3^2}$$

c) kombinirajući a), b) i izraz za $\frac{d\sigma}{d\Omega}$:

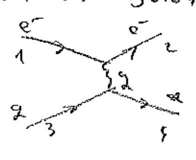
$$\Rightarrow \frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[\frac{G_E^2+\tau G_M^2}{(1+\tau)}\left(1-y-\frac{m_p y^2}{Q^2}\right) + \frac{1}{2}y^2G_M^2\right]$$

- također je praktično uvesti strukturne funkcije: $F_2(Q^2) = x\cdot\frac{G_E^2+\tau G_M^2}{1+\tau}$

$F_1(Q^2) = \frac{G_M^2}{2}$

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1-y-\frac{m_p y^2}{Q^2}\right)\frac{F_2(Q^2)}{x} + y^2F_1(Q^2)\right]$$

- pri prelasku na neelastično raspršenje imamo ovisnost o dvije varijable
- npr E_3 i θ
- može se pokazati da je u tom slučaju potrebno samo napraviti zamjenu $F_1(Q^2) \to F_1(Q^2, x)$, $F_2(Q^2) \to F_2(Q^2, x)$ i $\frac{d^2\sigma}{dQ^2dx}$
- međutim nas zanima duboko neelastično raspršenje.
- virtualni foton interagira s kvarkom unutar protona



$$M = \frac{e^2Q_q}{q^2}\bar{u}_3\gamma^\mu u_1\bar{u}_4\gamma_\mu u_2 \qquad Q_q = \frac{2}{3}, \frac{1}{3}$$

$$\overline{|M|^2} = \frac{2e^4Q_q^2}{t^2}\left[s^2+u^2\right] \quad \text{u URL}$$

$$= 8e^4Q_q^2\frac{1+\frac{1}{4}(1+\cos\theta)^2}{(1-\cos\theta)^2} \quad \text{u SCM}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{SCM} = \frac{1}{64\pi^2 s}\frac{p_f}{p_i}\overline{|M_{fi}|^2} = \frac{e^4Q_q^2}{8\pi^2 s}\frac{1+\frac{1}{4}(1+\cos\theta)^2}{(1-\cos\theta)^2}$$

- izvodimo $\frac{d\sigma}{dQ^2}$; $Q^2 = 2E^2(1-\cos\theta) = \frac{s}{2}(1-\cos\theta) \Rightarrow \cos\theta = \left(1-\frac{2Q^2}{s}\right)$

$$\frac{d\sigma}{dQ^2} = \frac{d\sigma}{d\Omega}\frac{d\Omega}{dQ^2} = \frac{d\sigma}{d\Omega}(-2\pi)\frac{d\cos\theta}{dQ^2} = \frac{4\pi}{s}\frac{d\sigma}{d\Omega} = \frac{2\pi\alpha^2Q_q^2}{Q^4}\left[1+\left(1-\frac{Q^2}{s}\right)^2\right]$$

- sada zamislimo da je naš kvark u protonu.
- prebacimo se u sustav u kojem proton ima veliki impuls: $m_p \to 0$

$P_2 = (E_2, 0, 0, E_2)$

- kvark nosi udio ξ od ukupnog impulsa protona: $P_q = (\xi E_2, 0, 0, \xi E_2)$
  - vrijedi: $m_q^2 = (\xi P_2 + q)^2 = m_q^2 + 2\xi P_2 q + q^2 \Rightarrow \xi = \frac{-q^2}{2P_2 q} = x$
- izrazimo kinematičke veličine za raspršenje na kvarku s onima za raspršenje na protonu.

$$S_q = (\xi P_2 + P_1)^2 \approx 2P_1 P_2 \xi = \xi s = xs$$

$$Y_q = \frac{\xi P_2 q}{\xi P_2 P_1} = Y$$

$$x_q = \frac{Q^2}{2\xi P_2 q} = 1$$ jer na kvarku imamo elastično raspršenje $\Rightarrow q = P_3 - P_2 \Rightarrow P_3^2 = (q + P_2)^2 \Rightarrow m_p^2 = q^2 + 2qP_2 + m_p^2$

$$q^2 = -2qP_2$$

$$Q^2 = (S_q - m_q^2)x_q Y_q = S_q Y_q \quad \text{za } m_q \to 0$$

- sada naš izraz za udarni presjek za raspršenje na kvarku unutar protona koji nosi udio x impulsa:

$$\frac{d\sigma}{dQ^2} = \frac{2\pi\alpha Q_q^2}{Q^4}\left[1 + \left(1 - \frac{Q^2}{S_q}\right)^2\right] = \frac{4\pi\alpha Q_q^2}{Q^4}\left[(1-Y) + \frac{Y^2}{2}\right]$$

- ovo izgleda kao izraz za neelastično raspršenje uz $m_p \ll Q^2$ i bez strukturnih funkcija
- za sada imamo kvark koji nosi točno udio x od ukupnog impulsa (x se skriva u Y)
- međutim postoji neka vjerojatnost da kvark nosi udio x.
- uvodimo partonske distribucijske funkcije (PDF)
- npr. za up kvark u protonu:

$u^p(x)\,dx$

predstavlja broj up kvarkova u protonu s udjelom impulsa između x i $x+dx$

- sada udarni presjek za raspršenje na kvarku okusa i s nabojem eQ_i i udjelom momenta $\{x, x+dx\}$ glasi

$$\frac{d\sigma}{dQ^2} = \frac{2\pi\alpha}{Q^4}\left[(1-Y) + \frac{Y^2}{2}\right] Q_i^2 q_i^p(x)\,dx$$; gdje je $q_i^p(x)$ PDF za okus i

- izraz za duboko neelastično raspršenje dobijemo sumacijom po svim okusima kvarkova:

$$\frac{d\sigma}{dx\,dQ^2} = \frac{2\pi\alpha}{Q^4}\left[(1-Y) + \frac{Y^2}{2}\right]\sum_i Q_i^2 q_i^p(x)$$

$$\Rightarrow F_2^{ep}(x, Q^2) = 2x F_1^{ep}(x, Q^2) = x\sum_i Q_i^2 q_i^p(x)$$

- primjećujemo dvije stvari:
  - u partonskom modelu strukturne funkcije F_1 i F_2 ne ovise o Q^2 (Bjorkenovo skaliranje)
  - Callan-Gross relacija
    $F_2^{ep}(x) = 2x F_1^{ep}(x)$

- isto možemo imati i za neutron.
- suma po i ide po valentnim kvarkovima i kvarkovima mora.
- za proton npr.

$$\int_0^1 u_V^p(x)\,dx = 2 \; ; \int_0^1 d_V^p(x)\,dx = 1$$

- ovo i izospinska simetrija nam govore:
$d_V^n(x) = u_V^p(x)$; $u_V^n(x) = d_V^p(x)$

- za neutron

$$\int_0^1 u_V^n(x)\,dx = 1 \; ; \int_0^1 d_V^n(x)\,dx = 2$$

- kvarkovi mora i prva aproksimacija:

$$u_S(x) = \bar{u}_S(x) = d_S(x) = \bar{d}_S(x) = S_S(x) = \bar{S}_S(x)$$

- jer nastaju istim mehanizmom
- pretpostavljamo da imaju približno istu masu
- teži kvarkovi su potisnuti

Zadatak.

Mjerenja elektron-deuteron i elektron-proton raspršenja daju:

$$\frac{\int_0^1 F_2^D(x)\,dx}{\int_0^1 F_2^p(x)\,dx} = 0{,}84$$

gdje je $F_2^D(x)$ strukturna funkcija deuterona po nukleonu

odredite omjer impulsa kojeg u protonu nose down kvarkovi/antikvarkovi i up kvarkovi/antikvarkovi.

$$F_2^D(x) = \frac{F_2^p(x)+F_2^n(x)}{2}$$

$$F_2^p(x) = x\left[\left(\tfrac{2}{3}\right)^2 u_v^p(x) + \left(\tfrac{1}{3}\right)^2 d_v^p(x) + \left(\tfrac{2}{3}\right)^2\left[u_s^p(x)+\bar{u}_s^p(x)\right] + \left(\tfrac{1}{3}\right)^2\left[d_s^p(x)+\bar{d}_s^p(x)\right]\right]$$

$$F_2^n(x) = x\left[\left(\tfrac{2}{3}\right)^2 u_v^n(x) + \left(\tfrac{1}{3}\right)^2 d_v^n(x) + \left(\tfrac{2}{3}\right)^2\left[u_s^n(x)+\bar{u}_s^n(x)\right] + \left(\tfrac{1}{3}\right)^2\left[d_s^n(x)+\bar{d}_s^n(x)\right]\right]$$

- koristimo:

$$\left.\begin{aligned} u_v^p(x) &= d_v^n(x) \\ d_v^p(x) &= u_v^n(x) \end{aligned}\right\} \text{ izospinska simetrija ..}$$

$$\left.\begin{aligned} u_s^p(x) = \bar{u}_s^p(x) &= d_s^n(x) = \bar{d}_s^n(x) \\ d_s^p(x) &= \dots \end{aligned}\right\} \text{ nastaju isto vakuumom u protonu i neutronu.}$$

$$\Rightarrow \quad F_2^n(x) = x\left[\left(\tfrac{2}{3}\right)^2 d_v^p(x) + \left(\tfrac{1}{3}\right)^2 u_v^p(x) + \left(\tfrac{2}{3}\right)^2\left[d_s^p(x)+\bar{d}_s^p(x)\right] + \left(\tfrac{1}{3}\right)^2\left[u_s^p(x)+\bar{u}_s^p(x)\right]\right]$$

- impuls nosen down kvarkom/antikvarkom:

$$\int_0^1 x\left[d_v^p(x)+d_s^p(x)+\bar{d}_s^p(x)\right]dx = f_d$$

- impuls nosen up kvarkom/antikvarkom:

$$\int_0^1 x\left[u_v^p(x)+u_s^p(x)+\bar{u}_s^p(x)\right]dx = f_u$$

$$\Rightarrow \quad \int_0^1 F_2^D(x)\,dx = \frac{1}{2}\left[\frac{4}{9}f_u + \frac{1}{9}f_d + \frac{4}{9}f_d + \frac{1}{9}f_u\right] = \frac{5}{18}\left[f_u+f_d\right]$$

$$\int_0^1 F_2^p(x)\,dx = \frac{4}{9}f_u + \frac{1}{9}f_d$$

$$\Rightarrow \quad \frac{\frac{5}{18}\left[f_u+f_d\right]}{\frac{1}{9}\left[4f_u+f_d\right]} = 0{,}84 \quad \Big/ \cdot \frac{18}{f_u} \quad \Rightarrow \quad 0{,}84 = \frac{5\left[1+\frac{f_d}{f_u}\right]}{8+2\frac{f_d}{f_u}} \quad \Rightarrow \quad \frac{f_d}{f_u} = \frac{8\cdot 0{,}84-5}{5-2\cdot 0{,}84} = 0{,}52$$

- u protonu up kvarkovi/antikvarkovi nose dvostruko više impulsa.

Zadatak

Izvedite Gottfriedovu relaciju

$$\int_0^1 \frac{dx}{x}\left[F_2^p(x)-F_2^n(x)\right] = \frac{1}{3}$$

Uključite doprinos strange kvarkova/antikvarkova.

$$F_2^p(x) = x\left[\left(\tfrac{2}{3}\right)^2\left(u_v^p(x)+u_s(x)+\bar{u}_s(x)\right) + \left(\tfrac{1}{3}\right)^2\left(d_v^p(x)+d_s(x)+\bar{d}_s(x)\right) + \left(\tfrac{1}{3}\right)^2\left(s_s(x)+\bar{s}_s(x)\right)\right]$$

- za neutron:

$u_v^n = d_v^p$

$d_v^n = u_v^p$

- u prvoj aproksimaciji
svi kvarkovi mora se javljaju približno jednakom frekvencijom
$u_s(x) = \bar{u}_s(x) = d_s(x) = \bar{d}_s(x) = s_s(x) = \bar{s}_s(x)$

$$\Rightarrow \quad F_2^p(x) = x\left[\frac{4}{9}u_v^p(x) + \frac{1}{9}d_v^p(x) + \frac{12}{9}S_s(x)\right]$$

$$F_2^n(x) = x\left[\frac{4}{9}d_v^p(x) + \frac{1}{9}u_v^p(x) + \frac{12}{9}S_s(x)\right]$$

$$\frac{1}{x}\left[F_2^p(x) - F_2^n(x)\right] = \frac{1}{3}\left[u_v^p(x) - d_v^p(x)\right]$$

$$\int\frac{dx}{x}\left[F_2^p(x) - F_2^n(x)\right] = \frac{1}{3}\int dx\left[u_v^p(x) - d_v^p(x)\right] = \frac{1}{3}[2-1] = \underline{\frac{1}{3}}$$

KVANTNA KROMODINAMIKA

- BAŽDARNA SIMETRIJA
- pogledajmo Lagrangian Diracovog polja: ..

$$\mathcal{L} = \bar{\Psi}(x)(i\gamma^\mu\partial_\mu - m)\Psi(x)$$

podsjetnik:
Euler-Lagrange jednadžbe nam daju Diracovu jednadžbu:
$$\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\bar{\Psi})}\right) - \frac{\partial\mathcal{L}}{\partial\bar{\Psi}} = 0 \Rightarrow (i\gamma^\mu\partial_\mu - m)\Psi = 0$$

- pretpostavimo da postoji fundamentalna simetrija svemira na lokalne promjene faze:

$$\Psi(x) \to e^{i\alpha(x)}\Psi(x) \quad ; \quad \bar{\Psi}(x) \to e^{-i\alpha(x)}\bar{\Psi}(x)$$

- Diracov Lagrangian nije invarijantan na ovakve transformacije
- smeta nam derivacija:

$$\partial_\mu\Psi(x) \to \partial_\mu\left(e^{i\alpha(x)}\Psi(x)\right) = i(\partial_\mu\alpha(x))\Psi(x) + e^{i\alpha(x)}\partial_\mu\Psi(x)$$

- kako bi učinili Lagrangian invarijantnim uvodimo kovarijantnu derivaciju:

$$D_\mu = \partial_\mu - ieA_\mu(x)$$

- gdje se $A_\mu(x)$ transformira kao:

$$A_\mu(x) \to A_\mu(x) + \frac{1}{e}\partial_\mu\alpha(x)$$

- sada se $D_\mu\Psi(x)$ transformira kao

$$D_\mu\Psi(x) \to e^{i\alpha(x)}D_\mu\Psi(x)$$

- novo vektorsko polje $A_\mu(x)$ nije ništa drugo nego elektromagnetsko polje
- Lagrangianu još dodajemo kinetički dio za fotonsko polje A_μ:

$$\frac{1}{4}F^{\mu\nu}F_{\mu\nu} \quad ; \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$\Rightarrow$ Ukupni Lagrangian QED-a:

$$\mathcal{L}_{QED} = \bar{\Psi}(i\gamma^\mu\partial_\mu - m)\Psi + e\bar{\Psi}\gamma^\mu\Psi A_\mu - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

- QED je invarijantan na lokalne promjene faze (U(1) simetrija)

- slično kao što je QED izgrađen na lokalnoj $U(1)$ simetriji QCD je izgrađen na lokalnoj $SU(3)$ simetriji:
- transformacija je:

$$\psi(x) \to e^{i\alpha_a(x)\hat{T}_a}\psi(x)$$

gdje se podrazumjeva suma po a.

- Matrice $\hat{T}_a$ su definirane s

$$\hat{T}_a = \frac{\lambda_a}{2} \; ; \; a = 1,2,\dots,8$$

$$\lambda^1 = \begin{pmatrix} 0&1&0\\1&0&0\\0&0&0 \end{pmatrix} ; \; \lambda^2 = \begin{pmatrix} 0&-i&0\\i&0&0\\0&0&0 \end{pmatrix} ; \; \lambda^3 = \begin{pmatrix} 1&0&0\\0&-1&0\\0&0&0 \end{pmatrix} ; \; \lambda^4 = \begin{pmatrix} 0&0&1\\0&0&0\\1&0&0 \end{pmatrix} ; \; \lambda^5 = \begin{pmatrix} 0&0&-i\\0&0&0\\i&0&0 \end{pmatrix} ; \; \lambda^6 = \begin{pmatrix} 0&0&0\\0&0&1\\0&1&0 \end{pmatrix} ;$$

$$\lambda^7 = \begin{pmatrix} 0&0&0\\0&0&-i\\0&i&0 \end{pmatrix} ; \; \lambda^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1&0&0\\0&1&0\\0&0&-2 \end{pmatrix}$$

- $\psi(x)$ sada očito ima tri komponente $\psi(x) = \begin{pmatrix} \psi_1\\ \psi_2\\ \psi_3 \end{pmatrix}$ gdje su ψ_1, ψ_2, ψ_3 obični Diracovi spinori koji označavaju nove stupnjeve slobode (boju)
- analogno s QED-om kovarijantna derivacija koja će lagrangian učiniti invarijantnim na $SU(3)$ je:

$$D_\mu = \partial_\mu + i g_s \hat{T}^a G^a_\mu \quad ; \text{ gdje se } G^a_\mu \text{ transformira kao: } G^a_\mu \to G^a_\mu + \frac{1}{g_s}\partial_\mu \alpha_a - f_{abc}\alpha_b G^c_\mu$$

- član s f_{abc} se javlja jer generatori $SU(3)$ ne komutiraju već imamo:

$$[T_a, T_b] = i f_{abc} T_c$$

- QCD Lagrangian:

$$\mathcal{L}_{QCD} = \bar{\Psi}(i\gamma^\mu\partial_\mu - m)\Psi - g_s(\bar{\Psi}\gamma^\mu \hat{T}_a \Psi)G^a_\mu - \frac{1}{4}G^a_{\mu\nu}G_a^{\mu\nu}$$

- analogno s QED-om možemo iz Lagrangiana išcitati Feynmanova pravila:
- Vrh:

$$-i g_s \gamma^\mu \frac{1}{2}\lambda^a$$

- vanjske linije:

$c_i u^s(p)$
$c_i v^s(p)$

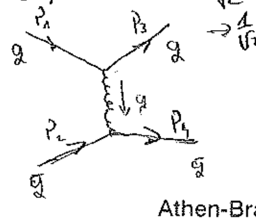
$c_i^\dagger \bar{u}^s(p)$
$c_i^\dagger \bar{v}^s(p)$

$c_i = r, g, b \; ; \; r = \begin{pmatrix}1\\0\\0\end{pmatrix} ; \; g = \begin{pmatrix}0\\1\\0\end{pmatrix} ; \; b = \begin{pmatrix}0\\0\\1\end{pmatrix}$

- gluonski propagator:

$$-\frac{i g_{\mu\nu}}{q^2}\delta_{ab}$$

__Zadatak__ Odredite potencijal koji opisuje kratkodosežnu kvark-kvark interakciju za konfiguraciju $\frac{1}{\sqrt{2}}(rg + gr)$!!! općenito:



$\to \frac{1}{\sqrt{2}}(-\|-)$

$$-iM = \bar{u}_3 c_3^\dagger \left(-\frac{i g_s}{2}\lambda^a\gamma^\mu\right)u_1 c_1 \left(-\frac{i g_{\mu\nu}\delta^{ab}}{q^2}\right)\bar{u}_4 c_4^\dagger \left(-\frac{i g_s}{2}\lambda^b\gamma^\nu\right)u_2 c_2$$

$$M = -\frac{g_s^2}{4q^2}\bar{u}_3\gamma^\mu u_1 \bar{u}_4\gamma_\mu u_2 \; c_3^\dagger\lambda^a c_1 \, c_4^\dagger\lambda^a c_2$$

Bojni faktor je $f = \frac{1}{4}(c_3^\dagger\lambda^a c_1)(c_4^\dagger\lambda^a c_2)$ - predstavlja razliku u odnosu na QED

- za elektrodinamiku je potencijal između dva elektrona:

$$V(r) = +\frac{e^2}{r}$$ - analogno za QCD $V(r) = +f\,\frac{g_s^2}{r}$

- treba izračunati f za $\frac{1}{\sqrt{2}}(rg+gr) \to \frac{1}{\sqrt{2}}(rg+gr)$

$$f = \left(\frac{1}{\sqrt{2}}\right)^2 \left\{ f(\overset{1\,2}{rg} \to \overset{3\,4}{rg}) + f(rg \to gr) + f(gr \to rg) + f(gr \to gr) \right\}$$

$$f(rg \to rg) = \frac{1}{4}\left[(1\,0\,0)\lambda^a \begin{pmatrix}1\\0\\0\end{pmatrix}\right]\left[(0\,1\,0)\lambda^a \begin{pmatrix}0\\1\\0\end{pmatrix}\right] = \frac{1}{4}\lambda^a_{11}\lambda^a_{22} = \frac{1}{4}\left\{\lambda^3_{11}\lambda^3_{22} + \lambda^8_{11}\lambda^8_{22}\right\} = \frac{1}{4}\left\{1(-1) + \frac{1}{\sqrt{3}}\cdot\frac{1}{\sqrt{3}}\right\} = -\frac{1}{6}$$

$$f(gr \to gr) = f(rg \to rg)$$

$$f(rg \to gr) = \frac{1}{4}\left[(0\,1\,0)\lambda^a \begin{pmatrix}1\\0\\0\end{pmatrix}\right]\left[(1\,0\,0)\lambda^a \begin{pmatrix}0\\1\\0\end{pmatrix}\right] = \frac{1}{4}\lambda^a_{12}\lambda^a_{21} = \frac{1}{4}\left\{\lambda^1_{12}\lambda^1_{21} + \lambda^2_{12}\lambda^2_{21}\right\} = \frac{1}{4}\left\{1\cdot 1 + i(-i)\right\} = \frac{1}{2}$$

$$f(gr \to rg) = f(rg \to gr)$$

$$\Rightarrow \quad f = \frac{1}{2}\left\{-\frac{2}{6} + 1\right\} = \frac{1}{3}$$

$\Rightarrow$ Potencijal:

$$V(r) = \frac{1}{3}\frac{g_s^2}{r}$$ - odbojan potencijal

<u>Zadatak</u> Odredite bojni faktor za QCD raspršenje $gb \to gr$.

$$f = \frac{1}{4}(c_3^\dagger \lambda^a c_1)(c_4^\dagger \lambda^a c_2) =$$

$$= \frac{1}{4}\left[(0\,1\,0)\lambda^a \begin{pmatrix}0\\1\\0\end{pmatrix}\right]\left[(1\,0\,0)\lambda^a \begin{pmatrix}0\\0\\1\end{pmatrix}\right] = \frac{1}{4}\left[\lambda^a_{22}\lambda^a_{31}\right] = 0$$

- Ovakvo raspršenje je nemoguće!!!
Očuvanje boje.

SLABA SILA

	QED	QCD	SLABA SILA
djeluje na	el. naboj	naboj boje	okus
medijatori (prijenosnici)	foton (γ) bezmasen	gluoni (g) bezmaseni	$W^{\pm}$, Z masivni
osjećaju	e, μ, τ, kvarkovi, $W^{\pm}$	kvarkovi, gluoni	svi fermioni, $W^{\pm}$, Z
baždarna grupa simetrija	$U(1)$	$SU(3)$	- nakon elektroslabog ujedinjenja: $SU(2) \times U(1)_Y \rightarrow U(1)_{EM}$
potencijal	$V = -\frac{\alpha}{r}$	$V = -\frac{\alpha_s}{r} + kr$	$V = -\frac{\alpha_w}{r} e^{-M_w r}$
paritet	očuvan	očuvan	narušen

- zašto ime slaba sila?
- neutrino postulirao Pauli kao rješenje nedostatka energije u β raspadima

$n \rightarrow p + e^- + \bar{\nu}_e$ → neopažena čestica (jer slabo interagira s materijom)

- koliko je slaba slaba sila?

<u>Sunce:</u>
- proizvodi $2 \cdot 10^{38}$ neutrina/s
- na udaljenosti od $1au = 150 \cdot 10^9 m$ to odgovara toku:
  $flux = 7 \cdot 10^{10}$ neutrina/cm²s
- za usporedbu: 10^{45} fotona/s
  $flux = 10^{16}$ fotona/cm²s

<u>Žarulja:</u>
- snage $P = 40W$,
- efikasnost $\eta = 10\%$
- foton $\lambda = 550 nm \Rightarrow$
  $E_\gamma = \frac{hc}{\lambda} = 3{,}6 \cdot 10^{-19} J$
- $\Rightarrow 1{,}1 \cdot 10^{19}$ fotona/s
- na $d = 10m$
  $flux = 8{,}8 \cdot 10^{11}$ fotona/cm²s

<u>Nuklearke</u>
$\sim 10^{21}$ neutrina/s

<u>CMB</u>
330 ν/cm³

<u>Ljudsko tijelo</u>
- sadrži oko 20mg radioaktivnog $^{40}K \Rightarrow 5 \cdot 10^3$ neutrina/s

- toliko puno neutrina oko nas
- zašto ih ne vidimo?
- debljina olova potrebna da zaustavi česticu energije $E = 1MeV$
  - p $d = 0{,}1mm$
  - e^- $d = 10mm$
  - ν $d = 10 Ly$
- zato od $\sim 10^{15}$ neutrina/s koji prolaze kroz naše tijelo samo nekoliko njih u životu interagira s nama
- za detekciju neutrina treba ogroman i jeftin detektor
  $\Rightarrow$ Superkamiokande
  - 1000 m pod zemljom (tako da ništa osim neutrina ne dođe do detektora)
  - 50 000 tona ultra-čiste vode
  - neutrino interagira s elektronom ili jezgrom
  - produkcija visokoenergetske čestice $\Rightarrow$ Cerenkovljevo zračenje
  - 11146 fotomultiplikatora

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- zašto je slaba sila tako slaba?
  - imamo masivne prijenosnike sile $W^{\pm}, Z$
  - Yukawa potencijal:

$$V(r) \sim \frac{1}{r} e^{-M_W r} \quad ; M_W = 80,4\,GeV \quad ; M_Z = 91,26\,GeV \Rightarrow \text{doseg } d \simeq 10^{-18}\,m$$

Zadatak

Pomoću Bornove aproksimacije odredite ponašanje udarnog presjeka za slabe procese.

$$V(r) = -\alpha_W \frac{e^{-Mr}}{r}$$

- Bornova aproksimacija kaže $\sigma \sim |V(q)|^2$
- treba nam Fourierov transformat Yukawa potencijala

$$V(q) = \int d^3r\, e^{-i\vec{q}\cdot\vec{r}} V(r) =$$

$$= \int_0^{2\pi} d\varphi \int_0^{\infty} r^2 V(r) dr \int_0^{\pi} e^{iqr\cos\theta} \sin\theta\, d\theta \ldots$$

$$= 2\pi \int_0^{\infty} r^2 V(r) dr \int_{-1}^{1} e^{iqr\cos\theta} d(\cos\theta) = 2\pi \int_0^{\infty} r V(r) \left. \frac{e^{iqrx}}{iqr} \right|_{x=-1}^{1} dr =$$

$$= 2\pi \int_0^{\infty} r^2 V(r) \frac{e^{iqr} - e^{-iqr}}{iqr} dr = \frac{4\pi}{q} \int_0^{\infty} r \sin(qr) V(r) dr =$$

$$= -\frac{4\pi\alpha_W}{q} \int_0^{\infty} \sin(qr) e^{-Mr} dr = -\frac{4\pi\alpha_W}{q} \int_0^{\infty} \frac{e^{(-M+iq)r} - e^{(-M-iq)r}}{2i} dr =$$

$$= -\frac{2\pi\alpha_W}{iq} \left[\frac{e^{(-M+iq)r}}{-M+iq} - \frac{e^{(-M-iq)r}}{-M-iq} \right]_{r=0}^{\infty} = -\frac{2\pi\alpha_W}{iq} \left[\frac{-1}{-M+iq} - \frac{-1}{-M-iq} \right] = -\frac{2\pi\alpha_W}{iq} \left[\frac{+M+iq \cancel{-} M+iq}{M^2+q^2} \right] =$$

$$= -\frac{2\pi\alpha_W}{iq} \frac{(+2iq)}{M^2+q^2} = \frac{-4\pi\alpha_W}{M^2+q^2}$$

- gledamo dva slučaja:

$$q^2 \ll M^2 \Rightarrow \sigma \sim \frac{1}{M^4} \quad \text{umjesto } \sigma \sim \frac{1}{q^4} \text{ za QED}$$

$$\Rightarrow V(q) = \frac{4\pi\alpha_W}{M^2} = \text{const} \Rightarrow V(r) = \delta(r) \text{ - točkasta interakcija}$$

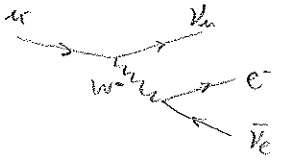
$$q^2 \gg M^2 \Rightarrow \sigma \sim \frac{1}{q^4} \quad \text{isto kao za QED}$$

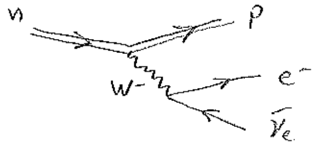
$$V(q) = \frac{4\pi\alpha_W}{q^2} \Rightarrow V(r) = -\frac{\alpha_W}{r} \text{ - Coulombov potencijal}$$

- konstanta α_W, odnosno jakost vezanja g_W nije mala sama po sebi $\alpha_W = \frac{1}{30} > \frac{1}{137} = \alpha_{QED}$
- slaba sila je jača od elektromagnetizma, ali zbog masivnih prijenosnika sile ima jako kratak doseg, i kao posljedicu toga mali udarni presjek.
- na visokim energijama $q \gg M$ slaba sila je usporediva s elektromagnetizmom

- gledamo dva slaba procesa:

$\mu^- \to \nu_\mu e^- \bar{\nu}_e$ $\quad\quad$ $n \to p e^- \bar{\nu}_e$





$\tau = 2{,}2 \cdot 10^{-6}\,s$ $\quad\quad$ $\tau = 885{,}7\,s$

- Pitanje: Zašto je tako velika razlika u vremenima života miona i neutrona, a procesi su slični? Isti faktori u vrhovima, propagator W bozona, spinovi vanjskih linija...

- u prvom semestru smo za raspad oblika $A \to 1+2+3+\ldots+n$ pokazali

$$d\Gamma(i\to f) = \frac{|M_{fi}|^2}{2E_A}\cdot(2\pi)^4\,\delta^4(p_f-p_i)\prod_{j=1}^{n}\frac{d^3p_j}{(2\pi)^3\,2E_j}$$

- u slučaju $n=2$ δ-funkcije „pojedu" 4 stupnja slobode faznog prostora. Ostanu 2 stupnja slobode za koje uzimamo θ i ϕ, odnosno $d\Omega$, a iznosi impulsa, tj. energije su u potpunosti određeni
- za slučaj 3 čestice u konačnom stanju, $n=3$ (kao za gornje procese), broj stupnjeva slobode je $9-4=5$.

- iznosi impulsa nisu u potpunosti određeni zakonima sačuvanja
- potrebno je integrirati preko faznog prostora zadovoljavajući zakone sačuvanja.
- probajmo na prste odrediti omjer širina raspada
- tražimo maksimalni impuls koji odnosi elektron u oba raspada

$m_p = 938{,}27\,MeV$
$m_n = 939{,}57\,MeV$
$m_\mu = 105{,}67\,MeV$
$m_e = 0{,}511\,MeV$

- gledamo
$n \to (p\bar{\nu}_e)e^-$

$P_n = (m_n, \vec{0})$; $P_e = (E_e, \vec{p}_e)$; $P_{p\bar{\nu}_e} = (E_p + E_{\bar{\nu}_e}, \vec{p}_p + \vec{p}_{\bar{\nu}_e}) = (E_2, \vec{p}_2)$ - definicija E_2 i $\vec{p}_2$

$\Rightarrow \quad (m_n - E_2)^2 = E_e^2 = \vec{p}_e^{\,2} - m_e^2$

$\Rightarrow \quad \boxed{|\vec{p}_e| = \sqrt{(m_n - E_2)^2 - m_e^2}}$

- da bi našli maksimum od $|\vec{p}_e|$, treba naći minimum od E_2
- definiramo m_2 kao: $m_2^2 = E_2^2 - \vec{p}_2^{\,2}$ - efektivna masa sustava $p, \bar{\nu}_e$
- imamo:
$\vec{p}_2 = -\vec{p}_e$

$\Rightarrow \quad \cancel{E_2^2} = m_2^2 + \vec{p}_e^{\,2} = m_2^2 + (m_n - E_2)^2 - m_e^2 = m_2^2 + m_n^2 - 2m_nE_2 + \cancel{E_2^2} - m_e^2$

$\Rightarrow \quad \boxed{E_2 = \frac{m_n^2 + m_2^2 - m_e^2}{2m_n}}$

- za minimum od E_2 treba naći minimum od m_2

$m_2^2 = E_2^2 - \vec{p}_2^{\,2} = (E_p+E_\nu)^2 - (\vec{p}_p+\vec{p}_\nu)^2$

$= \underbrace{E_p^2 + 2E_pE_\nu + E_\nu^2 - \vec{p}_p^{\,2}}_{m_p^2} - 2|\vec{p}_p||\vec{p}_\nu|\cos\theta - \underbrace{\vec{p}_\nu^{\,2}}_{m_\nu^2=0}$

$= m_p^2 + 2E_\nu(E_p - |\vec{p}_p|\cos\theta)$

- $|\vec{p}_p| \le E_p$, te je stoga minimum m_2^2 za $E_\nu = 0$

$\Rightarrow \boxed{m^2_{2min} = m_p^2}$

$\Rightarrow \boxed{E_{2min} = \frac{m_n^2+m_p^2-m_e^2}{2m_n}}$

$\Rightarrow \quad |\vec{p}_e|_{max} = \sqrt{\left(m_n - \frac{m_n^2+m_p^2-m_e^2}{2m_n}\right)^2 - m_e^2} = \sqrt{\left(\frac{m_n^2-m_p^2+m_e^2}{2m_n}\right)^2 - m_e^2}$

- za proces $n \to p e^- \bar{\nu}_e$ dakle imamo $|\vec{p}_e|_{max} = 1{,}19\,MeV$
- a za $\mu^- \to \nu_\mu e^- \bar{\nu}_e$ račun ide analogno $|\vec{p}_e|_{max} = 52{,}8\,MeV$
- kako sada procjeniti širinu raspada?
- dimenzionalna analiza.

$[\Gamma] = [T]^{-1} = [M]$

- u prošlom zadatku smo dobili:

$\Gamma \sim \frac{1}{M_W^4}$ - dolazi od W propagatora

- u brojnik treba ići nešto dimenzije $[M]^5$
- uzmimo da je to naš $|\vec{p}_e|_{max}$

$\frac{\tau_n}{\tau_\mu} = \frac{\Gamma_\mu}{\Gamma_n} = \left(\frac{52{,}8}{1{,}19}\right)^5 = 1{,}72\cdot 10^8$

- eksperimentalno:

$\frac{\tau_n}{\tau_\mu} = 4{,}03\cdot 10^8$

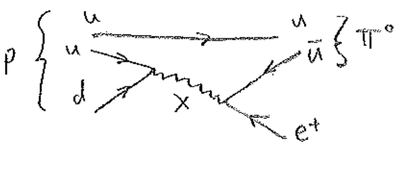
- dobar red veličine
- ovaj primjer pokazuje općenito pravilo potiskivanja faznim prostorom

Zadatak

U GUT (Grand Unified Theory) modelima postoji baždarni bozon koji može pretvarati kvark u lepton.

U tom slučaju moguć je raspad protona $p \to \pi^0 e^+$.

Ako znamo eksperimentalnu granicu $\tau(p) > 2\cdot 10^{29}$ godina, procjenite donju granicu za masu takvog bozona.



$M_{fi} \sim \frac{1}{m_X^2} \Rightarrow |M_{fi}|^2 \sim \frac{1}{m_X^4}$

$\Rightarrow \Gamma \sim \frac{1}{m_X^4}|\vec{p}_e|^5 \sim \frac{1}{m_X^4} m_p^5 \qquad \tau = \frac{1}{\Gamma} \sim \frac{m_X^4}{m_p^5}$

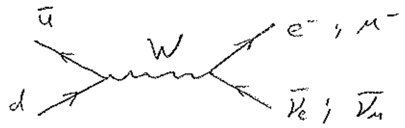
$\tau_{min}(p) = 2\cdot 10^{29}$ godina $= 6{,}31\cdot 10^{36}\,s \approx 1\cdot 10^{52}\,eV^{-1}$.

$\Rightarrow m_X^4 > \tau_{min} m_p^5 = \tau_{min}(10^9)^5 \approx 1\cdot 10^{97}\,eV^4 \Rightarrow m_X > 1{,}8\cdot 10^{15}\,GeV$

- pogledajmo sada raspad piona

$\pi^- \to e^- \bar{\nu}_e$

$\pi^- \to \mu^- \bar{\nu}_\mu$



- procjenimo omjer širina raspada...

$$\Gamma \sim \frac{|p^*|^5}{m_W^4} \qquad |p^*| = \frac{1}{2m_\pi}\sqrt{\left[m_\pi^2 - (m_\mu + m_\nu)^2\right]\left[m_\pi^2 - (m_\mu - m_\nu)^2\right]}$$

$$m_\nu \to 0$$

$$|p^*| = \frac{1}{2m_\pi}\left(m_\pi^2 - m_\mu^2\right) = \frac{m_\pi}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)$$

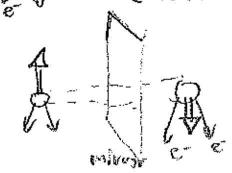
$$\frac{\Gamma_e}{\Gamma_\mu} = \frac{\left(1 - \frac{m_e^2}{m_\pi^2}\right)^5}{\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^5} \approx 70$$

- međutim eksperimentalno: $\frac{\Gamma_e}{\Gamma_\mu} = 1{,}230 \cdot 10^{-4}$ Što se dogodilo???

- potisnuće helicitetom!
- uzrok je neočuvanje pariteta u slabim procesima
- Wu eksperiment

$^{60}Co \to {}^{60}Ni + e^- + \bar{\nu}_e + 2\gamma$

- imamo narušenje pariteta
- zrcalni proces nije opažen



- u QED-u i QCD-u je paritet očuvan (to znamo iz eksperimenta)
- očuvanje pariteta se vidi i iz oblika amplitude

$$M \sim j_{12} \cdot j_{34}$$

- j_{12} i j_{34} su struje dane izrazima:

$$j_{12} = \bar{u}_1 \gamma^\mu u_2$$
$$j_{34} = \bar{u}_3 \gamma^\nu u_4$$

- operator pariteta je $P = \gamma^0 \Rightarrow P^{-1} = \gamma^0$
- pogledajmo kako se amplituda transformira na operaciju pariteta

$$j_{12} \xrightarrow{P} j_{12}' = \bar{u}_1' \gamma^\mu u_2' = \bar{u}_1 P^{-1} \gamma^\mu P u_2 = \bar{u}_1 \gamma^0 \gamma^\mu \gamma^0 u_2 = \begin{cases} \mu = 0 & \bar{u}_1 \gamma^0 u_2 \\ \mu = 1,2,3 & -\bar{u}_1 \gamma^\mu u_2 \end{cases}$$

- struja se na paritet transformira kao vektor
- slijedi da se amplituda M transformira kao skalar (odnosno imamo invarijantnost amplitude na paritet)
- očito ovakav tip struje ne može narušiti paritet!
- kakve još struje možemo imati?
- općeniti oblik $j_{12} = \bar{u}_1 \Gamma u_2$
- sve takve oblike je moguće prikazati pomoću bilinearnih kovarijanti (1. semestar)

$\bar{\Psi}\Psi$ - skalar $\quad \bar{\Psi}\gamma^5\Psi$ - pseudo skalar $\quad$ - vodi na vezanje s bozonom spina $s=0$

$\bar{\Psi}\gamma^\mu\Psi$ - vektor $\quad \bar{\Psi}\gamma^\mu\gamma^5\Psi$ - pseudovektor $\quad$ — " — $\quad s=1$

$\bar{\Psi}\sigma^{\mu\nu}\Psi$ - tenzor; $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$ $\quad$ — " — $\quad s=2$

- znamo da su prijenosnici slabe sile vektorski $(s=1)$ bozoni $W^{\pm}, Z$
- iz toga slijedi da je najopćenitiji oblik struje za slabu silu:

$$j_{12} = \bar{u}_1 (g_V \gamma^\mu + g_A \gamma^\mu \gamma^5) u_2 = (g_V j_{12}^V + g_A j_{12}^A)$$

Zadatak

Nađite za koje vrijednosti g_V i g_A imamo maksimalno narušenje pariteta!

- amplituda je dana kao:

$$\mathcal{M} \sim j_{12} \cdot j_{34} = (g_V j_{12}^V + g_A j_{12}^A)(g_V j_{34}^V + g_A j_{34}^A) = g_V^2 j_{12}^V \cdot j_{34}^V + g_A^2 j_{12}^A \cdot j_{34}^A + g_V g_A (j_{12}^V j_{34}^A + j_{12}^A j_{34}^V)$$

- kako se amplituda transformira na paritet?
- prvi član je isti kao za QED, QCD:

$$j_{12}^V \cdot j_{34}^V \xrightarrow{P} j_{12}^V \cdot j_{34}^V$$

- kako se transformira j_{12}^A

$$j_{12}^A = \bar{u}_1 \gamma^\mu \gamma^5 u_2 \xrightarrow{P} j_{12}'^A = \bar{u}_1' \gamma^\mu \gamma^5 u_2' = \bar{u}_1 \gamma^0 \gamma^\mu \gamma^5 \gamma^0 u_2 = -\bar{u}_1 \gamma^0 \gamma^\mu \gamma^0 \gamma^5 u_2 = \begin{cases} -\bar{u}_1 \gamma^\mu \gamma^5 u_2 & \text{za } \mu = 0 \\ \bar{u}_1 \gamma^\mu \gamma^5 u_2 & \text{za } \mu = 1,2,3 \end{cases}$$

- iz toga vidimo:

$$j_{12}^A j_{34}^A \xrightarrow{P} j_{12}^A j_{34}^A$$

- ono što mijenja paritet su miješani članovi tipa:

$$j_{12}^V j_{34}^A \xrightarrow{P} -j_{12}^V j_{34}^A$$

$$\mathcal{M} \xrightarrow{P} \mathcal{M}' \sim g_V^2 j_{12}^V j_{34}^V + g_A^2 j_{12}^A j_{34}^A - g_V g_A (j_{12}^V j_{34}^A + j_{12}^A j_{34}^V)$$

- relativna jakost člana koji narušava paritet je: $\dfrac{g_V g_A}{g_V^2 + g_A^2}$
- narušenje nema za $g_V = 0$ ili $g_A = 0$, a maksimalno je za $|g_V| = |g_A|$
- za maksimalno narušenje pariteta imamo dvije mogućnosti: 1' $j_{12} = \bar{u}_1 \gamma^\mu (1 + \gamma^5) u_2$

2' $j_{12} = \bar{u}_1 \gamma^\mu (1 - \gamma^5) u_2$

- iz eksperimenta znamo da je druga opcija ispravna za nabijenu slabu interakciju (izmjenu $W^{\pm}$)
- vrh za izmjenu $W^{\pm}$ bozona je: $\boxed{-\frac{i g_W}{\sqrt{2}} \frac{1}{2} \gamma^\mu (1 - \gamma^5)}$

Zadatak

Pokažite da su operatori $P_R = \frac{1}{2}(1 + \gamma^5)$ i $P_L = \frac{1}{2}(1 - \gamma^5)$ projektori! [koristimo $\gamma^5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$]

1) $P_L + P_R = 1$!

$$P_L + P_R = \frac{1}{2}[1 + \cancel{\gamma^5} + 1 - \cancel{\gamma^5}] = 1$$

2) $P_R^2 = P_R$; $P_L^2 = P_L$

$$P_R^2 = \frac{1}{4}(1 + \gamma^5)(1 + \gamma^5) = \frac{1}{4}(1 + 2\gamma^5 + \underbrace{\gamma^5 \gamma^5}_{1}) = \frac{1}{4}(2 + 2\gamma^5) = \frac{1}{2}(1 + \gamma^5) = P_R$$

$$P_L^2 = \frac{1}{4}(1 - 2\gamma^5 + \gamma^5 \gamma^5) = \frac{1}{2}(1 - \gamma^5) = P_L$$

3) $P_R P_L = 0$

$$P_R P_L = \frac{1}{4}(1 + \gamma^5)(1 - \gamma^5) = \frac{1}{4}(1 - \gamma^5 \gamma^5) = 0$$

- P_R i P_L su takozvani projektori kiralnosti!

- u vrhu za slabu silu nam se javlja P_L
- zanima nas kakva su to stanja koja on isprojicira (stanja koja ulaze u slabu interakciju)
- povezat ćemo stanja kiralnosti i heliciteta!
- helicitet je projekcija spina na impuls čestice
- operator heliciteta je stoga dan izrazom:

$$\hat{h} = \frac{\hat{\Sigma}\cdot\hat{p}}{2p} = \frac{1}{2p}\begin{pmatrix}\vec{\sigma}\cdot\hat{p} & 0\\ 0 & \vec{\sigma}\cdot\hat{p}\end{pmatrix}$$

- za naše spinore (općenita rješenja Diracove jednadžbe) smo dobili

$$u_{1,2}(p) = \begin{pmatrix}\chi_{1,2}\\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi_{1,2}\end{pmatrix} \quad ; \quad v_{1,2}(p) = \begin{pmatrix}-\frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi_{1,2}\\ \chi_{1,2}\end{pmatrix}$$

- nadimo svojstvena stanja operatora heliciteta.
- djelujemo s $\hat{h}$ na $u_{1,2}$:

$$\hat{h}u_{1,2} = \lambda u_{1,2} \Rightarrow \begin{aligned}&\vec{\sigma}\cdot\vec{p}\,\chi_{1,2} = 2p\lambda\chi_{1,2}\\ &\frac{1}{2p}\frac{p^2}{E+m}\chi_{1,2} = \lambda\frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi_{1,2}\end{aligned}$$

- prva jednadžba nam omogućava da $u_{1,2}$ pišemo kao $u_{1,2}(p) = \begin{pmatrix}\chi_{1,2}\\ \frac{2\lambda p}{E+m}\chi_{1,2}\end{pmatrix}$
- kombinacija prve i druge daje $\chi_{1,2} = 4\lambda^2\chi_{1,2}$
- slijedi da su svojstvene vrijednosti $\lambda_{1,2} = \pm\frac{1}{2}$ (baš kako i očekujemo)
- označimo sa $u_\uparrow$ ($u_\downarrow$) stanje s $h = +\frac{1}{2}$ ($h = -\frac{1}{2}$)
- imamo:

$$u_\uparrow = \begin{pmatrix}\chi_\uparrow\\ \frac{p}{E+m}\chi_\uparrow\end{pmatrix} \quad ; \quad u_\downarrow = \begin{pmatrix}\chi_\downarrow\\ -\frac{p}{E+m}\chi_\downarrow\end{pmatrix} \quad \text{gdje} \quad \begin{aligned}&\vec{\sigma}\cdot\vec{p}\,\chi_\uparrow = p\chi_\uparrow\\ &\vec{\sigma}\cdot\vec{p}\,\chi_\downarrow = -p\chi_\downarrow\end{aligned}$$

- slično:

$$v_\uparrow = \begin{pmatrix}-\frac{p}{E+m}\chi_\uparrow\\ \chi_\uparrow\end{pmatrix} \quad ; \quad v_\downarrow = \begin{pmatrix}\frac{p}{E+m}\chi_\downarrow\\ \chi_\downarrow\end{pmatrix}$$

- sada možemo svojstvena stanja heliciteta raspisati preko svojstvenih stanja kiralnosti!
- za $u_\uparrow$:

$$P_L u_\uparrow = \frac{1}{2}(1-\gamma^5)\begin{pmatrix}\chi_\uparrow\\ \frac{p}{E+m}\chi_\uparrow\end{pmatrix} = \frac{1}{2}\left[\begin{pmatrix}\chi_\uparrow\\ \frac{p}{E+m}\chi_\uparrow\end{pmatrix} - \begin{pmatrix}\frac{p}{E+m}\chi_\uparrow\\ \chi_\uparrow\end{pmatrix}\right] = \frac{1}{2}\left(1-\frac{p}{E+m}\right)\begin{pmatrix}\chi_\uparrow\\ -\chi_\uparrow\end{pmatrix}$$

$$P_R u_\uparrow = \frac{1}{2}(1+\gamma^5)\begin{pmatrix}\chi_\uparrow\\ \frac{p}{E+m}\chi_\uparrow\end{pmatrix} = \frac{1}{2}\left[\begin{pmatrix}\chi_\uparrow\\ \frac{p}{E+m}\chi_\uparrow\end{pmatrix} + \begin{pmatrix}\frac{p}{E+m}\chi_\uparrow\\ \chi_\uparrow\end{pmatrix}\right] = \frac{1}{2}\left(1+\frac{p}{E+m}\right)\begin{pmatrix}\chi_\uparrow\\ \chi_\uparrow\end{pmatrix}$$

- što su $\begin{pmatrix}\chi_\uparrow\\ -\chi_\uparrow\end{pmatrix}$ i $\begin{pmatrix}\chi_\uparrow\\ \chi_\uparrow\end{pmatrix}$?
- to su upravo svojstvena stanja kiralnosti! Označimo ih s $u_L = \begin{pmatrix}\chi_\uparrow\\ -\chi_\uparrow\end{pmatrix}$ i $u_R = \begin{pmatrix}\chi_\uparrow\\ \chi_\uparrow\end{pmatrix}$
- očito vrijedi

$$\gamma^5 u_L = \begin{pmatrix}0 & \mathbb{1}\\ \mathbb{1} & 0\end{pmatrix}\begin{pmatrix}\chi_\uparrow\\ -\chi_\uparrow\end{pmatrix} = -\begin{pmatrix}\chi_\uparrow\\ -\chi_\uparrow\end{pmatrix} = -u_L \Rightarrow \begin{aligned}&P_L u_L = \frac{1}{2}(1-\gamma^5)u_L = u_L\\ &P_R u_L = 0\end{aligned}$$

$$\gamma^5 u_R = u_R \Rightarrow \begin{aligned}&P_L u_R = 0\\ &P_R u_R = 1\end{aligned}$$

- sada možemo pisati:

$$u_\uparrow = P_L u_\uparrow + P_R u_\uparrow = \frac{1}{2}\left(1-\frac{p}{E+m}\right)u_L + \frac{1}{2}\left(1+\frac{p}{E+m}\right)u_R$$

- i slično možemo dobiti:

$$u_\downarrow = \frac{1}{2}\left(1+\frac{p}{E+m}\right)u_L + \frac{1}{2}\left(1-\frac{p}{E+m}\right)u_R$$

gdje sada $P_L v_R = v_R$, $P_R v_R = 0$, $P_L v_L = 0$, $P_L v_R = v_R$

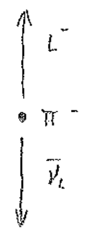
$$v_\uparrow = \frac{1}{2}\left(1+\frac{p}{E+m}\right)v_R + \frac{1}{2}\left(1-\frac{p}{E+m}\right)v_L$$

$$v_\downarrow = \frac{1}{2}\left(1-\frac{p}{E+m}\right)v_R + \frac{1}{2}\left(1+\frac{p}{E+m}\right)v_L$$

- vidimo da za $m \to 0$ jedan član uvijek nestaje i stanja heliciteta su ista kao stanja kiralnosti
- znamo da nam se u slaboj interakciji javljaju samo u_L i v_R (samo lijeve čestice i desne antičestice)
- ako invertiramo gornje izraze:

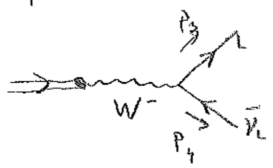
$$u_L = \frac{\frac{1}{2}\left(1-\frac{p}{E+m}\right)u_\uparrow - \frac{1}{2}\left(1+\frac{p}{E+m}\right)u_\downarrow}{\frac{p}{E+m}} \underset{m\to 0}{=} -u_\downarrow + \frac{m}{2E}u_\uparrow$$

- vidimo da su čestična stanja desnog heliciteta potisnuta s faktorom $\frac{m}{2E}$
- vratimo se na naš raspad piona.



$m_\pi = 139,6$ MeV
$m_\mu = 105,7$ MeV
$m_e = 0,5$ MeV
$J(\pi^-) = 0$

- antineutrino je bezmasen. Budući da u slaboj interakciji sudjeluju samo desne antičestice (desne kiralnosti) i lijeve čestice (lijeve kiralnosti), antineutrino je opisan spinorom $v_R = v_\uparrow$
- stoga lepton mora također biti u stanju desnog heliciteta kako bi imali očuvanje angularnog momenta.
- amplitudu za raspad možemo opisati dijagramom



$$\mathcal{M} = \left[\frac{g_W}{\sqrt{2}}\bar{u}_\uparrow(p_3)\gamma^\mu v_\uparrow(p_4)\right]\left[\frac{g_{\mu\nu}}{M_W^2}\right]\left[\frac{g_W}{\sqrt{2}} f_\pi p_\pi^\nu\right]$$

- prvi faktor je leptonska struja, drugi faktor dolazi od propagatora W bozona $\frac{-g_{\mu\nu}}{q^2 - M_W^2}$, zadnji faktor je nepoznata pionska struja. Budući da je pion spina 0, jedina vektorska veličina u struji može biti impuls piona, te je treći faktor najopćenitiji izraz za pionsku struju.

$$\mathcal{M} = \frac{g_W^2}{4M_W^2} f_\pi p_\pi^\mu \bar{u}_\uparrow \gamma_\mu v_\uparrow$$

- radimo u SCM pa je $p_\pi^\mu = (m_\pi, 0, 0, 0)$!

$$\mathcal{M} = \frac{g_W^2}{4M_W^2} f_\pi m_\pi \bar{u}_\uparrow \gamma^0 v_\uparrow = \frac{g_W^2}{4M_W^2} f_\pi m_\pi u_\uparrow^\dagger v_R$$

- pogledajmo faktor $u_\uparrow^\dagger v_R$

$$u_\uparrow^\dagger v_R = \left[\frac{1}{2}\left(1-\frac{p}{E+m}\right)u_L^\dagger + \frac{1}{2}\left(1+\frac{p}{E+m}\right)u_R^\dagger\right]v_R$$

$u_L^\dagger = (P_L u)^\dagger = u^\dagger P_L^\dagger = u^\dagger P_L \Rightarrow u_L^\dagger P_L = u_L^\dagger$

$u_R^\dagger = u^\dagger_R P_R$

$v_R = P_L v_R$

$\Rightarrow \quad u_\pi^\dagger v_R = \frac{1}{2}\left(1-\frac{p}{E+m}\right) u_L^\dagger P_L P_L v_R + \frac{1}{2}\left(1+\frac{p}{E+m}\right) u_R^\dagger \underbrace{P_R P_L}_{0} v_R = \frac{1}{2}\left(1-\frac{p}{E+m}\right) u_L^\dagger v_R$

- za u_L i v_R smo imali $u_L = \begin{pmatrix}\chi_1\\-\chi_1\end{pmatrix}$ i $v_R = \begin{pmatrix}\chi_1\\-\chi_1\end{pmatrix}$ gdje je $\vec\sigma\cdot\hat p\,\chi_1 = p\chi_1$
- u produktu $u_L^\dagger v_R$ treba uzeti u obzir Lorentz kovarijantnu normalizaciju spinora
$N = \sqrt{E+m}$

$\Rightarrow \quad u_L^\dagger v_R = \sqrt{E_L+m_L}\sqrt{E_\nu+m_\nu} = \sqrt{E_L+m_L}\sqrt{p}$

- konačno za amplitudu dobivamo:

$$M = \frac{g_w^2}{8M_w^2} f_\pi m_\pi \left(1-\frac{p}{E_L+m_L}\right)\sqrt{E_L+m_L}\sqrt{p}$$

- izrazi za E i p su određeni zakonima sačuvanja.

$m_\pi = E_L + E_\nu$ $\qquad m_L^2 = E_L^2 - \vec p_L^{\,2} = E_L^2 - p_L^2$

$0 = \vec p_L + \vec p_\nu$ $\qquad m_\pi = E_L + |\vec p_L|$

$\Rightarrow |\vec p_L| = E_\nu$ $\qquad m_L^2 = E_L^2 - (m_\pi - E_L)^2 = \cancel{E_L^2} - m_\pi^2 + 2m_\pi E_L - \cancel{E_L^2}$

$\Rightarrow E_L = \frac{m_L^2+m_\pi^2}{2m_\pi}$; $|\vec p_L| = m_\pi - E_L = \frac{m_\pi^2-m_L^2}{2m_\pi}$

- uvrstimo E_L i p_L u M:

$$M = \frac{g_w^2}{8M_w^2} f_\pi m_\pi \left(1 - \frac{m_\pi^2-m_L^2}{(m_L+m_\pi)^2}\right) \frac{m_\pi+m_L}{\sqrt{2m_\pi}} \cdot \left(\frac{m_\pi^2-m_L^2}{2m_\pi}\right)^{1/2} = \frac{g_w^2}{8M_w^2} f_\pi m_\pi \left(1-\frac{m_\pi-m_L}{m_\pi+m_L}\right) \frac{m_\pi+m_L}{\sqrt{2m_\pi}}\,\frac{(m_\pi^2-m_L^2)^{1/2}}{\sqrt{2m_\pi}} =$$

$$= \frac{g_w^2}{8M_w^2} f_\pi m_\pi \frac{2m_L}{m_\pi+m_L}\,\frac{m_\pi+m_L}{2m_\pi}(m_\pi^2-m_L^2)^{1/2} = \frac{g_w^2}{8M_w^2} f_\pi m_L (m_\pi^2-m_L^2)^{1/2}$$

- kvadrat amplitude:

$$\overline{|M|^2} = \frac{g_w^4}{64M_w^4} f_\pi^2 m_L^2 (m_\pi^2-m_L^2)$$

- konačno diferencijalna širina raspada je:

$$d\Gamma = \frac{1}{2m_\pi}\overline{|M|^2}\, dLips = \frac{p_L}{32\pi^2 m_\pi^2}\overline{|M|^2}\, d\Omega = \frac{m_\pi^2-m_L^2}{64\pi^2 m_\pi^3}\overline{|M|^2}\, d\Omega$$

- $\overline{|M|^2}$ nema kutne ovisnosti pa integral po $d\Omega$ daje 4π.

$$\Gamma = \frac{g_w^4}{64M_w^4} f_\pi^2 \frac{m_\pi^2-m_L^2}{64\pi^2 m_\pi^3} m_L^2(m_\pi^2-m_L^2)4\pi = \frac{g_w^4 f_\pi^2}{32^2 M_w^4 m_\pi^3\pi} m_L^2(m_\pi^2-m_L^2)^2$$

$$= \frac{g_w^4 f_\pi^2 m_\pi}{32^2 M_w^4\pi} m_L^2\left(1-\frac{m_L^2}{m_\pi^2}\right)^2$$

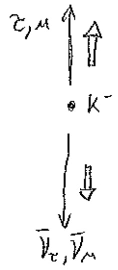
- možemo procijeniti omjer raspada:

$$\frac{\Gamma(\pi^-\to e^-\bar\nu_e)}{\Gamma(\pi^-\to\mu^-\bar\nu_\mu)} = \frac{m_e^2\left(1-\frac{m_e^2}{m_\pi^2}\right)^2}{m_\mu^2\left(1-\frac{m_\mu^2}{m_\pi^2}\right)^2} = 1{,}23\cdot10^{-4}$$, a eksp $1{,}23\cdot10^{-4}$

- drugi primjer

$|K^-\rangle = |\bar u s\rangle$; $J(K^-) = 0$

- dva raspada: $K^- \to \tau \bar\nu_\tau$
$K^- \to \mu \bar\nu_\mu$

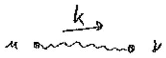


- τ, μ je natjeran u stanje „krivog" heliciteta te su procesi potisnuti faktorom $\sim \left(\frac{m}{E}\right)^2$,
- proces s lakšim leptonom μ je jače potisnut.
- potisnuće helicitetom.

Feynmanova pravila za slabi sektor

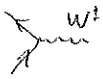
- propagator masivnog vektorskog bozona

$$\frac{-i\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{M^2}\right)}{k^2 - M^2}$$



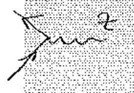
- vrh:
  - za W bozon (nabijena struja)

  $$\frac{-ig_W}{2\sqrt{2}}\gamma^\mu(1-\gamma_5)$$



  - za Z bozon (neutralna struja)

  $$\frac{-ig_W}{2\cos\theta_W}\gamma^\mu(c_V - c_A\gamma_5)$$; $\cos\theta_W = \frac{g_W}{\sqrt{g_W^2 + g'^2}}$ gdje je g_W SU(2), a g' U(1) konstanta vezanja

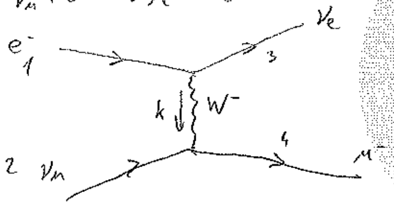


  - za neutrine $c_V = c_A = \frac{1}{2}$
  - za e, μ, τ $c_A = \frac{1}{2}$, $c_V = -\frac{1}{2} + 2\sin^2\theta_W$; $\sin^2\theta_W = 0{,}23$
  - za kvarkove opet drugačije

Primjer

Inverzni mionski raspad

$\nu_\mu + e^- \to \mu^- + \nu_e$



$$-i\mathcal{M} = \bar u_3 \frac{-ig_W}{2\sqrt{2}}\gamma^\mu(1-\gamma_5)u_1 \frac{-i\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{M_W^2}\right)}{k^2 - M_W^2} \bar u_4 \frac{-ig_W}{2\sqrt{2}}\gamma^\nu(1-\gamma_5)u_2$$

$$\mathcal{M} = -\frac{g_W^2}{8}\bar u_3\gamma^\mu(1-\gamma_5)u_1 \frac{\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{M_W^2}\right)}{k^2 - M_W^2}\bar u_4\gamma^\nu(1-\gamma_5)u_2$$

- kako postupiti u vezi s propagatorom?
- imamo dva režima:
$k^2 \gg M_W^2$ i $k^2 \ll M_W^2$
- kako znamo koliki je k^2?

$$k^2 = t = (p_1 - p_3)^2 \underset{URL}{\simeq} -2E^2(1-\cos\theta)$$

$$s = 4E^2 \Rightarrow t \approx \frac{s}{2}(\cos\theta - 1) \Rightarrow |k^2| = |t| \le s$$

- pogledajmo sada član u propagatoru s faktorom $\frac{k_\mu k_\nu}{M_W^2}$
- imamo:

$$\bar u_3\gamma^\mu(1-\gamma_5)u_1 \cdot k_\mu = \bar u_3 \not{k}(1-\gamma_5)u_1 =$$

$$= \bar u_3(\not{p}_1 - \not{p}_3)(1-\gamma_5)u_1 = \bar u_3(1+\gamma_5)\not{p}_1 u_1 - \bar u_3\not{p}_3(1-\gamma_5)u_1 =$$

$$\left[\begin{array}{l}\text{iskoristimo Diracovu jednadžbu:} \\ (\not{p} - m)u = 0 \quad \bar u(\not{p} - m) = 0\end{array}\right] = \bar u_3\left[(1+\gamma_5)m_1 - m_3(1-\gamma_5)\right]u_1$$

- ukupno imamo:

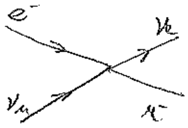
$$\bar{u}_3\gamma^\mu(1-\gamma_5)u_1\frac{k_\mu k_\nu}{M_W^2}\,\bar{u}_4\gamma^\nu(1-\gamma_5)u_2 =$$

$$= \frac{1}{M_W^2}\,\bar{u}_3\left[(1+\gamma_5)m_1 - m_3(1-\gamma_5)\right]u_1\,\bar{u}_4\left[m_4(1-\gamma_5)-(1+\gamma_5)m_2\right]u_2 \sim \frac{m_i m_j}{M_W^2}$$

- zanemarivo za sve fermione osim za top kvark $m_t \approx 180\,\mathrm{GeV}$, $m_W \approx 80\,\mathrm{GeV}$
- u slučaju $s \ll M_W^2$ možemo pisati propagator:

$$\frac{i g_{\mu\nu}}{M_W^2} \rightarrow \quad \mathcal{M} = \frac{g_W^2}{8M_W^2}\,\bar{u}_3\gamma^\mu(1-\gamma_5)u_1\,\bar{u}_4\gamma^\nu(1-\gamma_5)u_2$$

- efektivno smo dobili Fermijevu struja×struja teoriju s $\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8M_W^2}$



- za slučaj $s \gtrsim M_W^2$

$$\mathcal{M} = -\frac{g_W^2}{8}\,\frac{1}{k^2-M_W^2}\,\bar{u}_3\gamma^\mu(1-\gamma_5)u_1\,\bar{u}_4\gamma_\mu(1-\gamma_5)u_2$$

$$\mathcal{M}^\dagger = -\frac{g_W^2}{8}\,\frac{1}{k^2-M_W^2}\left[\bar{u}_3\gamma^\mu(1-\gamma_5)u_1\right]^\dagger\left[\bar{u}_4\gamma_\mu(1-\gamma_5)u_2\right]^\dagger$$

$$\left[\bar{u}_3\gamma^\mu(1-\gamma_5)u_1\right]^\dagger = u_1^\dagger(1-\gamma_5^\dagger)\gamma^{\mu\dagger}\gamma^{0\dagger}u_3 = u_1^\dagger(1-\gamma_5)\gamma^0\gamma^\mu\underbrace{\gamma^0\gamma^0}_{1}u_3 = u_1^\dagger(1-\gamma_5)\gamma^0\gamma^\mu u_3 = u_1^\dagger\gamma^0(1+\gamma_5)\gamma^\mu u_3 =$$

$$= \bar{u}_1\gamma^\mu(1-\gamma_5)u_3$$

$$\Rightarrow \mathcal{M}^\dagger = -\frac{g_W^2}{8}\,\frac{1}{k^2-M_W^2}\,\bar{u}_1\gamma^\nu(1-\gamma_5)u_3\,\bar{u}_2\gamma_\nu(1-\gamma_5)u_4$$

$$|\mathcal{M}|^2 = \frac{g_W^4}{64}\,\frac{1}{(k^2-M_W^2)^2}\,\bar{u}_1\gamma^\nu(1-\gamma_5)u_3\,\bar{u}_2\gamma_\nu(1-\gamma_5)u_4\,\bar{u}_3\gamma^\mu(1-\gamma_5)u_1\,\bar{u}_4\gamma_\mu(1-\gamma_5)u_2 =$$

- Casimirov trik:

$$|\mathcal{M}|^2 = \frac{g_W^4}{64}\,\frac{1}{(k^2-M_W^2)^2}\,\mathrm{Tr}\left[\gamma^\nu(1-\gamma_5)u_3\bar{u}_3\gamma^\mu(1-\gamma_5)u_1\bar{u}_1\right]\mathrm{Tr}\left[\gamma_\nu(1-\gamma_5)u_4\bar{u}_4\gamma_\mu(1-\gamma_5)u_2\bar{u}_2\right]$$

- usrednjavamo po početnim i sumiramo po konačnim spinovima!
- faktor za usrednjavanje je $\frac{1}{2}$, a ne $\frac{1}{4}$!!! Neutrino dolazi u samo jednom stanju spina.

$$\overline{|\mathcal{M}|^2} = \frac{1}{2}\sum_{spin}|\mathcal{M}|^2 = \frac{g_W^4}{128}\,\frac{1}{(k^2-M_W^2)^2}\sum_{spin}\mathrm{Tr}\left[\gamma^\nu(1-\gamma_5)u_3\bar{u}_3\gamma^\mu(1-\gamma_5)u_1\bar{u}_1\right]\mathrm{Tr}\left[\gamma_\nu(1-\gamma_5)u_4\bar{u}_4\gamma_\mu(1-\gamma_5)u_2\bar{u}_2\right]$$

$$= \frac{g_W^4}{128}\,\frac{1}{(k^2-M_W^2)^2}\,\mathrm{Tr}\left[\gamma^\nu(1-\gamma_5)\not{p}_3\gamma^\mu(1-\gamma_5)\not{p}_1\right]\mathrm{Tr}\left[\gamma_\nu(1-\gamma_5)\not{p}_4\gamma_\mu(1-\gamma_5)\not{p}_2\right] =$$

$$\overset{\text{zadaća}}{\underset{\text{u 1. semestru}}{=}} \frac{g_W^4}{128}\,\frac{1}{(k^2-M_W^2)^2}\,256\cdot(p_3\cdot p_4)(p_1\cdot p_2) = \frac{2g_W^4}{(k^2-M_W^2)^2}\,(p_1\cdot p_2)(p_3\cdot p_4)$$

- u URL:

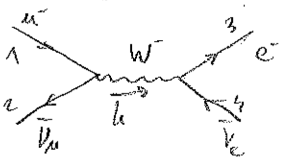
$$p_1\cdot p_2 = p_3\cdot p_4 \approx \frac{s}{2}$$

$$k^2 = t \approx -2|\vec{p}_1||\vec{p}_3|(1-\cos\theta) = 2|\vec{p}_1||\vec{p}_3|(\cos\theta-1)$$

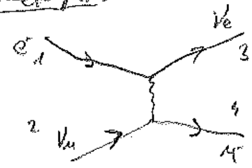
$$\Rightarrow \overline{|\mathcal{M}|^2} = \frac{2g_W^4}{\left[2E^2(\cos\theta-1)-M_W^2\right]^2}\,\frac{s^2}{4} \qquad \Rightarrow \left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{1}{64\pi^2 s}\,\frac{p_f}{p_i}\,\overline{|\mathcal{M}|^2} = \frac{g_W^4}{128\pi^2}\,\frac{4E^2}{\left[2E^2(\cos\theta-1)-M_W^2\right]^2}$$

Primjer

$\mu^- + \bar{\nu}_\mu \to e^- + \bar{\nu}_e$



-lako za izračunati, ali još jednostavnije je iskoristiti križnu simetriju:



$\nu_\mu + e^- \to \mu^- + \nu_e$ $\qquad$ $\mu^- + \bar{\nu}_\mu \to e^- + \bar{\nu}_e$

$$s = (p_1+p_2)^2 = (p_e + p_{\nu_\mu})^2 \Rightarrow (p_e - p_{\bar{\nu}_\mu})^2 = (p_3 - p_2)^2 = u$$

$$t = (p_1 - p_3)^2 = (p_e - p_{\nu_e})^2 \Rightarrow (p_e + p_{\bar{\nu}_e})^2 = (p_3 + p_4)^2 = s$$

$$u \Rightarrow t$$

$$\overline{|M|^2} = \frac{2 g_w^4}{(s - M_w^2)^2} \cdot \frac{u^2}{4}$$

- s ne ovisi o kutu. Sada možemo za $s \gg M_w^2$ pisati $\overline{|M|^2} = \frac{2g_w^4}{s^2} \cdot \frac{u^2}{4}$

Zadatak

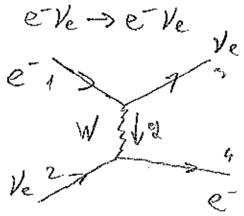
Razmatramo tri procesa: $e^-\nu_e \to e^-\nu_e$, $e^-\bar{\nu}_e \to e^-\bar{\nu}_e$; $e^-\nu_\mu \to e^-\nu_\mu$.

a) Nacrtajte sve dijagrame u vodećem redu i napišite izraz za amplitudu.

b) Za koji od tih procesa očekujete maksimum u $\sigma(\sqrt{s})$ i za koju vrijednost $\sqrt{s}$

a)

1. $e^-\nu_e \to e^-\nu_e$



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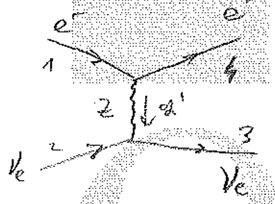


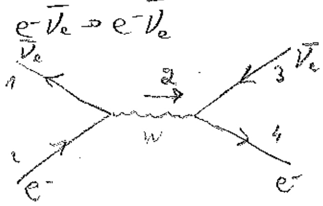
$q^2 = (p_1 - p_3)^2 = t$

$q'^2 = (p_1 - p_4)^2 = u$

$$-i\mathcal{M} = \bar{u}_3 \frac{-ig_w}{2\sqrt{2}} \gamma^\mu (1-\gamma_5) u_1 \frac{-i[g_{\mu\nu} - q_\mu q_\nu / M_W^2]}{q^2 - M_W^2} \bar{u}_4 \frac{-ig_w}{2\sqrt{2}} \gamma^\nu (1-\gamma_5) u_2 +$$

$$+ \bar{u}_4 \frac{-ig_w}{2\cos\theta_w} \gamma^\mu (c_V^e - c_A^e \gamma^5) u_1 \frac{-i[g_{\mu\nu} - q'_\mu q'_\nu / M_Z^2]}{q'^2 - M_Z^2} \bar{u}_3 \frac{-ig_w}{2\cos\theta_w} \gamma^\nu (c_V^\nu - c_A^\nu \gamma^5) u_2$$

2. $e^-\bar{\nu}_e \to e^-\bar{\nu}_e$



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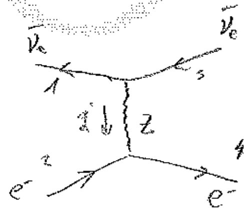
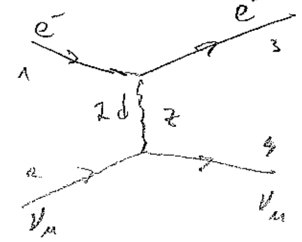


$q^2 = (p_1 + p_2)^2 = s$

$q'^2 = (p_1 - p_3)^2 = t$

$$i\mathcal{M} = \bar{v}_1 \frac{-ig_w}{2\sqrt{2}} \gamma^\mu (1-\gamma_5) u_2 \frac{-i[g_{\mu\nu} - q_\mu q_\nu / M_W^2]}{q^2 - M_W^2} \bar{u}_4 \frac{-ig_w}{2\sqrt{2}} \gamma^\nu (1-\gamma_5) v_3 +$$

$$+ \bar{v}_1 \frac{-ig_w}{2\cos\theta_w} \gamma^\mu (c_V^\nu - c_A^\nu \gamma^5) v_3 \frac{-i[g_{\mu\nu} - q'_\mu q'_\nu / M_Z^2]}{q'^2 - M_Z^2} \bar{u}_4 \frac{-ig_w}{2\cos\theta_w} \gamma^\nu (c_V^e - c_A^e \gamma_5) u_2$$

3. 

$$q^2 = (p_1 - p_3)^2 = t$$

$$-i\mathcal{M} = \bar{u}_3 \frac{-ig_w}{2\cos\theta_w}\gamma^\mu (c_V^e - c_A^e\gamma^5) u_1 \frac{-i[g_{\mu\nu} - q_\mu q_\nu / M_Z^2]}{q^2 - M_Z^2} \bar{u}_4 \frac{-ig_w}{2\cos\theta_w}\gamma^\nu (c_V^\nu - c_A^\nu \gamma_5) u_2$$

b) Za proces 2. imamo maksimum u $s = M_W^2$ zbog propagatora $\sim \frac{1}{s - M_W^2}$

Zadatak

Slaba neutralna struja je opisana članom interakcije:

$$\mathcal{L}_I = -\frac{g}{\cos\theta_w}(\bar{\chi}_L \gamma_\mu \tau_3 \chi_L - \sin^2\theta_w \bar{\Psi}\gamma^\mu Q \Psi) Z_\mu$$

i obično se rastavlja na vektorsku i aksijalnu komponentu:

$$-\frac{g}{\cos\theta_w}\bar{\Psi}^f \gamma_\mu \frac{1}{2}(c_V^f - c_A^f\gamma^5)\Psi^f$$

Odredite c_V^f i c_A^f za sve fermione.

$$\chi_L = \begin{pmatrix} \psi_L^{f_1} \\ \psi_L^{f_2} \end{pmatrix}$$

$$j_\mu^3 = \bar{\chi}_L \gamma_\mu \tau_3 \chi_L = \bar{\Psi}^f \gamma_\mu \underbrace{\tfrac{1}{2}(1-\gamma_5)}_{P_L} T_3 \Psi^f$$

↳ $\tau_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ ↳ treća komponenta slabog izospina

$$j_\mu^{e.m.} = \bar{\Psi}^f \gamma^\mu Q \Psi^f$$

↳ el. naboj

$$\Rightarrow j_\mu^{N.C.} = j_\mu^3 - \sin^2\theta_w j_\mu^{e.m.}$$

$$= \bar{\Psi}^f \gamma^\mu \left[\tfrac{1}{2}(1-\gamma_5)T_3 - \sin^2\theta_w Q\right]\Psi^f$$

$$= \bar{\Psi}^f \gamma^\mu \tfrac{1}{2}\left[c_V^f - c_A^f \gamma_5\right]\Psi^f$$

$$\Rightarrow \boxed{\begin{array}{l} c_V^f = T_3 - 2\sin^2\theta_w Q \\ c_A^f = T_3 \end{array}}$$

⇒

	T^3	Q	C_A	C_V
ν_e, ν_μ, ν_τ	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$
e^-, μ^-, τ^-	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	$-\frac{1}{2} + 2\sin^2\theta_w \simeq -0{,}032$
u, c, t	$\frac{1}{2}$	$+\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{2} - \frac{4}{3}\sin^2\theta_w \simeq 0{,}188$
d, s, b	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{2} + \frac{2}{3}\sin^2\theta_w \simeq -0{,}344$

za $\sin^2\theta_w = 0{,}234$

Zadatak

Raspad D mezona:

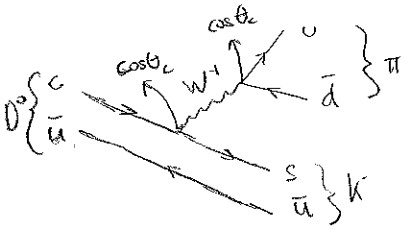
Imamo dva moda raspada D^0:

$D^0 \to K^- \pi^+$

$D^0 \to K^+ \pi^-$

$(D^0 = c\bar{u},\ K^+ = \bar{s}u,\ \pi^+ = u\bar{d})$

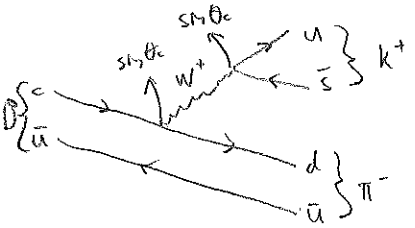
Nacrtajte Feynmanove dijagrame: objasnite zašto je jedan od ta dva raspada potisnut. Usporedite s eksp. podacima!



$\Rightarrow \Gamma \sim \cos^4\theta_c$

$\theta_c \simeq 13^\circ$

$$\Rightarrow \frac{\Gamma(D^0 \to K^-\pi^+)}{\Gamma(D^0 \to K^+\pi^-)} \simeq \frac{\cos^4\theta_c}{\sin^4\theta_c} = 3{,}5\cdot 10^2$$



$\Rightarrow \Gamma \sim \sin^4\theta_c$

Eksp.

Particle Data Group

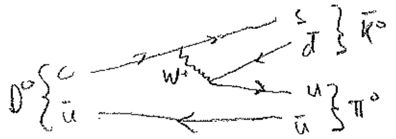
$BR(D^0 \to K^-\pi^+) = 3{,}89\ \%$

$BR(D^0 \to K^+\pi^-) = 1{,}48\cdot 10^{-4}$

$$\Rightarrow \frac{\Gamma(K^-\pi^+)}{\Gamma(K^+\pi^-)} = \frac{0{,}0389}{1{,}48\cdot 10^{-4}} = 2{,}6\cdot 10^2$$

- potisnuće Cabibbovim kutem!
- još jedan zanimljiv mod raspada:

$D^0 \to \bar{K}^0 \pi^0 \quad (\bar{K}^0 = s\bar{d})\ ;\ \pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$



$\Gamma \sim \cos^4\theta_c$ - reklo bi se isto kao i za $D^0 \to K^-\pi^+$

- ali $\Gamma = BR(D^0 \to \bar{K}^0\pi^0) = 1{,}13\cdot 10^{-2} \Rightarrow \frac{\Gamma(D^0 \to \bar{K}^0\pi^0)}{\Gamma(D^0 \to K^-\pi^+)} = \frac{1{,}13\cdot 10^{-2}}{3{,}89\cdot 10^{-2}} = 0{,}29 \approx \frac{1}{2}$
- objašnjenje:
  - D^0 je skalar u prostoru boje (svojstvo sile vezanih stanja kvarkova (barioni i mezoni))
  - K^0 i π^0 su skalari u prostoru boje
  - s ima boju kao i c (slaba sila ne mijenja boju)
  - $\bar{u}$ ima boju isto kao i početni $\bar{u}$

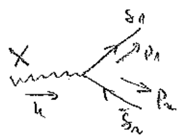
  } $\bar{d}$ i u moraju fiksirati boju!!!
- u raspadu $D^0 \to K^-\pi^+$ i $D^0 \to K^+\pi^-$ mogli su nastati kvarkovi bilo koje boje! $\Rightarrow$ faktor 3

Zadatak

Širina raspada vektorskog bozona

$X \to f_1 \bar{f}_2 \quad X = (W^\pm, Z)$; vrh ćemo pisati $-i g_X \gamma^\mu \frac{1}{2}(c_V - c_A\gamma^5)$; - za $W^\pm \Rightarrow g_X = \frac{g_W}{\sqrt{2}}$; $c_V = c_A = 1$

- za $Z \Rightarrow g_X = \frac{g_W}{\cos\theta_W}$; $c_V = c_A =$ nešto



$$-i\mathcal{M} = \bar{u}_1 (-i) g_X \gamma^\mu \frac{1}{2}(c_V - c_A\gamma^5) v_2 \; \varepsilon^\lambda_\mu(k)$$

$$\mathcal{M} = \frac{g_X}{2} \bar{u}_1 \gamma^\mu (c_V - c_A\gamma^5) v_2 \; \varepsilon^\lambda_\mu(k)$$

$$\mathcal{M}^\dagger = \frac{g_X}{2} \bar{v}_2 \gamma^\mu (c_V - c_A\gamma^5) u_1 \; \varepsilon^{\lambda *}_\mu(k)$$

$$|\mathcal{M}|^2 = \frac{g_X^2}{4} \varepsilon^{\lambda *}_\nu(k)\varepsilon^\lambda_\mu(k)\, \bar{v}_2 \gamma^\nu (c_V - c_A\gamma^5) u_1 \bar{u}_1 \gamma^\mu (c_V - c_A\gamma^5) v_2$$

$$\overline{|\mathcal{M}|^2} = \frac{g_X^2}{4}\frac{1}{3}\sum_{s,\lambda} \varepsilon^{\lambda *}_\nu(k)\varepsilon^\lambda_\mu(k)\, \bar{v}_2 \gamma^\nu (c_V - c_A\gamma^5) u_1 \bar{u}_1 \gamma^\mu (c_V - c_A\gamma^5) v_2$$

po spinovima vektorskog bozona

$$\sum_\lambda \varepsilon^\lambda_\mu(k)\varepsilon^{\lambda *}_\nu(k) = -g_{\mu\nu} + \frac{k_\mu k_\nu}{M_X^2}$$

- Casimirov trik

$$\sum_s \bar{v}_2 \gamma^\nu (c_V - c_A\gamma^5) u_1 \bar{u}_1 \gamma^\mu (c_V - c_A\gamma^5) v_2 = \mathrm{Tr}[(\not{p}_1 + \overset{0 \text{ URL}}{m_1})\gamma^\mu (c_V - c_A\gamma^5)(\not{p}_2 - \overset{0 \text{ URL}}{m_2})\gamma^\nu (c_V - c_A\gamma^5)] =$$

$$= c_V^2 \mathrm{Tr}[\not{p}_1\gamma^\mu \not{p}_2 \gamma^\nu] + c_A^2 \mathrm{Tr}[\not{p}_1\gamma^\mu \underbrace{\gamma^5 \not{p}_2 \gamma^\nu \gamma^5}_{(\gamma^5)^2 = 1}] - c_V c_A \left(\mathrm{Tr}[\not{p}_1 \gamma^\mu \gamma^5 \not{p}_2 \gamma^\nu] + \mathrm{Tr}[\not{p}_1\gamma^\mu \not{p}_2 \gamma^\nu \gamma^5]\right)$$

$$= (c_V^2 + c_A^2)\mathrm{Tr}[\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu] - 2c_V c_A \mathrm{Tr}[\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu\gamma^5] = (c_V^2 + c_A^2)[4(p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - (p_1\cdot p_2)g^{\mu\nu})] - 2c_V c_A(-4i\epsilon^{\sigma\mu\beta\nu} p_{1\sigma} p_{2\beta})$$

$$\overline{|\mathcal{M}|^2} = \frac{g_X^2}{4}\frac{1}{3}\left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{M_X^2}\right)\left[(c_V^2 + c_A^2)4\cdot(p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - (p_1\cdot p_2)g^{\mu\nu}) + 8i c_V c_A \epsilon^{\sigma\mu\beta\nu} p_{1\sigma} p_{2\beta}\right] =$$

$$= \frac{g_X^2}{12}(c_V^2 + c_A^2)4\cdot\left(-2p_1\cdot p_2 + 4p_1\cdot p_2 + \frac{2(p_1\cdot k)(p_2\cdot k)}{M_X^2} - \frac{(p_1\cdot p_2)k^2}{M_X^2}\right)$$

$$= \frac{g_X^2}{3}(c_V^2 + c_A^2)\left(p_1\cdot p_2 + \frac{2(p_1\cdot k)(p_2\cdot k)}{M_X^2}\right)$$

tu koristimo $g_{\mu\nu}g^{\mu\nu} = 4$

$k^2 = M_X^2$

i $g_{\mu\nu}\epsilon^{\sigma\mu\beta\nu} = 0$; $k_\mu k_\nu \epsilon^{\sigma\mu\beta\nu} = 0$

simetričan, antisimetričan; simetrično, antisimetrično

u SCM imamo: (URL)

$$k^\mu = (M_X, 0, 0, 0)$$

$$p_1^\mu = \left(\frac{M_X}{2}, 0, 0, \frac{M_X}{2}\right) \Rightarrow p_1\cdot k = p_2\cdot k = \frac{M_X^2}{2}$$

$$p_2^\mu = \left(\frac{M_X}{2}, 0, 0, -\frac{M_X}{2}\right) \qquad p_1\cdot p_2 = 2\left(\frac{M_X}{2}\right)^2 = \frac{M_X^2}{2}$$

$$\overline{|\mathcal{M}|^2} = \frac{g_X^2}{3}(c_V^2 + c_A^2)\left(\frac{M_X^2}{2} + \frac{M_X^2}{2}\right) = \frac{g_X^2}{3}(c_V^2 + c_A^2)M_X^2$$

$$\Gamma = \frac{p^*}{32\pi^2 M_X^2}\int \overline{|\mathcal{M}|^2}\, d\Omega = \frac{p^*}{8\pi M_X^2}\overline{|\mathcal{M}|^2} = \frac{\frac{M_X}{2}}{8\pi M_X^2}\frac{g_X^2}{3}(c_V^2 + c_A^2)M_X^2 = \frac{g_X^2 M_X}{48\pi}(c_V^2 + c_A^2)$$

$$\Rightarrow \Gamma(W^\pm \to f\bar{f}) = \frac{g_W^2 M_W}{2\cdot 48\pi}\cdot(1+1) = \frac{g_W^2 M_W}{48\pi}$$

$$\Gamma(Z \to f\bar{f}) = \frac{g_W^2 M_Z}{\cos^2\theta_W \cdot 48\pi}(c_V^2 + c_A^2)$$

c_V i c_A ovise o izlaznim česticama

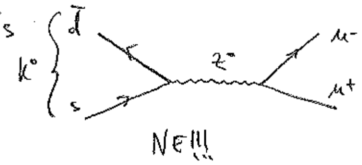
- do sada smo radili četiri vrste potisnuća (faznim prostorom, helicitetom, Cabibbo, i bojom)

- ostaje nam još GIM-ov mehanizam

GIM-ov mehanizam

$K^0 \to \mu^+ \mu^-$

$K^0 = \bar{d}s$

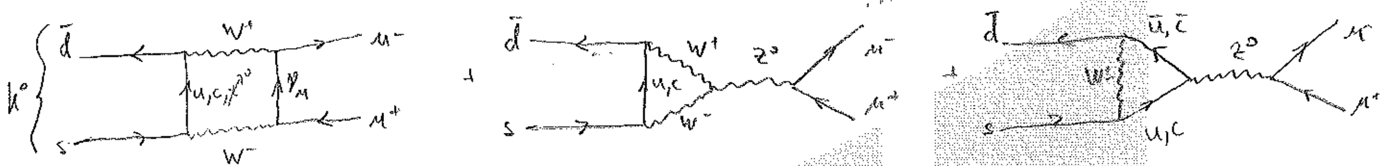


- ovo ne može ići
- FCNC (flavour changing neutral currents) ne postoje u granastim dijagramima

$\begin{pmatrix} u \\ d' \end{pmatrix} W^{\pm}$ - $W^{\pm}$ bozoni mogu mijenjati okus zbog CKM matrice
- d' je dano s $\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$ gdje su d i s svojstvena stanja mase, a d', s' svojstvena stanja $SU(2)$ grupe.

$\begin{pmatrix} u \\ d' \end{pmatrix} Z$ - Z^0 bozoni ne mijenjaju okus (nema prelaska među generacijama)

- potrebno je ići na dijagrame višeg reda!
- dijagrami s jednom petljom:



- dijagrami višeg reda! Potisnuti! Ali to još nije dovoljno.

$\begin{pmatrix} d \\ s \end{pmatrix} = \begin{pmatrix} \cos\theta_c & -\sin\theta_c \\ \sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d' \\ s' \end{pmatrix}$

- prvi i drugi dijagram:
  - ako se propagira u; $M \sim \sin\theta_c \cos\theta_c$
  - ako se propagira c; $M \sim \cos\theta_c(-\sin\theta_c)$ $\Rightarrow$ kraćenje (nije potpuno jer u i c imaju različite mase)
- treći dijagram:
  - ako imamo $u\bar{u}$; $M \sim \cos\theta_c \sin\theta_c$
  - ako imamo $c\bar{c}$; $M \sim -\sin\theta_c \cos\theta_c$ $\Rightarrow$ opet kraćenje

Higgsovo polje

- QED i QCD imaju baždarnu simetriju izgrađenu na grupama $U(1)_{em}$ i $SU(3)_{QCD}$
- to znači da su svi članovi u Lagrangianu teorije invarijantni na takve transformacije polja
- jedna od posljedica baždarne simetrije je i bezmasenost prijenosnika sila (fotona i gluona)
- u slaboj teoriji imamo očito nekakve dublete $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \begin{pmatrix} u \\ d \end{pmatrix}_L \ldots$; imamo tri prijenosnika sila, dva od kojih „prebacuju" čestice iz jednog člana dubleta u drugi
- sve to jako podsjeća na $SU(2)$ baždarnu simetriju!!!
- međutim imamo dva problema: mase članova dubleta i mase prijenosnika sila
- pogledajmo pobliže član mase elektrona u Lagrangianu:

$$-m\bar{\Psi}^e\Psi^e = -m(\bar{\Psi}^e_L + \bar{\Psi}^e_R)(\Psi^e_L + \Psi^e_R) = -m(\bar{\Psi}^e_L\Psi^e_R + \bar{\Psi}^e_R\Psi^e_L)$$

jer $\bar{\Psi}_L = \Psi_L^{\dagger}\gamma^0 = \left[\frac{1}{2}(1-\gamma^5)\Psi\right]^{\dagger}\gamma^0 = \Psi^{\dagger}\frac{1}{2}(1-\gamma^5)\gamma^0 = \Psi^{\dagger}\gamma^0\frac{1}{2}(1+\gamma^5) = \bar{\Psi}P_R$, pa $\bar{\Psi}_L\Psi_L = \bar{\Psi}P_R P_L \Psi = 0$

- međutim Ψ^e_L je član dubleta $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$, no Ψ^e_R nije.
- ovakav član mase ne bi bio invarijantan na $SU(2)$ transformacije
- isto vrijedi i za član mase $W^{\pm}, Z^0$ bozona!
- npr. $Z_\mu Z^\mu \xrightarrow{SU(2)} (Z_\mu + \frac{1}{g_W}\partial_\mu\alpha_z - \varepsilon_{zwv}\alpha_w W^v_\mu)(\cdots)$ - očito nije invarijantno

- ako zahtjevamo da je $SU(2)_L$ egzaktna baždarna simetrija moramo riješiti problem masenih članova.
- uvodimo novo polje (kompleksni higgsov dublet):

$$\phi = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$ -dublet na $SU(2)$
-skalar (ne može biti lijevi ili desni)

- pišemo sve $SU(2)$ dopuštene članove u lagrangianu (članove mase ne!)
- između ostalog javlja se član

$$Y_e\left[\bar{L}\phi e_R^- + \bar{e}_R^- \phi^+ L\right] \quad ; \text{ gdje je } L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$$

- sada ide ona priča s „mexican hat" potencijalom polja ϕ.
- budući da imamo baždarno invarijantan Lagrangian možemo napraviti rotaciju tako da dobijemo: $\phi \to \phi' = \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$; $h(x)$ je sad realno skalarno polje.

- gornji član nam daje: $Y_e\left[\bar{e}_L^-(v+h(x))e_R^- + \bar{e}_R^-(v+h(x))e_L\right] = Y_e v(\bar{e}_L^- e_R^- + \bar{e}_R^- e_L^-) + Y_e h(x)(\bar{e}_L^- e_R^- + \bar{e}_R^- e_L^-) =$

$$= \underbrace{Y_e v\bar{e}e}_{\text{član mase}} + Y_e h(x)\bar{e}e$$

(no lagrangian je i dalje $SU(2)$ invarijantan)

- za mase dobivamo:

$$m_{W^\pm}^2 = \frac{g_W^2 v^2}{4}$$

$$m_Z^2 = \frac{g_W^2 v^2}{4\cos^2\theta_W}$$

$$m_f = Y_e v$$

$\hookrightarrow$ ovo neznamo unaprijed

- vidimo također da je vezanje na higgs-a proporcionalno masi.
- Feynmanova pravila za higgsovo polje: ($v = 246\,GeV$)
  - propagator:

$$\frac{i}{p^2 - m_h^2 + i\varepsilon}$$

- vrhovi:

$$i\frac{g_W^2}{2} g_{\mu\nu} = 2i\frac{M_W^2}{v^2} g_{\mu\nu}$$

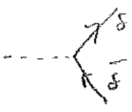
$$i\frac{g_W^2 v}{2} = 2i\frac{M_W^2}{v}$$

$$i\frac{g_W^2}{2\cos^2\theta_W} g_{\mu\nu} = 2i\frac{M_Z^2}{v^2} g_{\mu\nu}$$

$$i\frac{g^2}{2\cos^2\theta_W} v = 2i\frac{M_Z^2}{v}$$

$$-i6\lambda v = -i3\frac{m_h^2}{v}$$

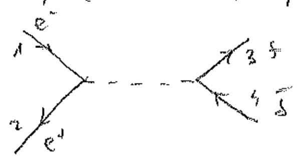
$$-i6\lambda = -i3\frac{m_h^2}{v^2}$$



$$-iY_f = -i\frac{m_f}{v} = -i\frac{g}{2m_W} m_f$$

Zadatak:

Proučite proces: $e^+e^- \to h \to f\bar{f}$! Nađite udarni presjek za ovaj proces u SCM! Odredite omjer: $\dfrac{\sigma(e^+e^- \to h \to b\bar{b})}{\sigma(e^+e^- \to h \to \tau^-\tau^+)}$ ($m_b = 4{,}7\,GeV$, $m_\tau = 1{,}786\,GeV$)! Koja kvalna stanja doprinose?



$$-i\mathcal{M} = \bar{v}_2\left(-i\frac{m_e}{v}\right)u_1 \frac{i}{p^2-m_h^2}\bar{u}_3\left(-i\frac{m_b}{v}\right)v_4$$

$$\mathcal{M} = \frac{m_e m_b}{v^2}\bar{v}_2 u_1 \frac{1}{p^2-m_h^2}\bar{u}_3 v_4$$

$$\overline{|\mathcal{M}|^2} = \frac{1}{4}\sum_s \left(\frac{m_e m_b}{v^2}\right)^2\left(\frac{1}{s-m_h^2}\right)^2 \bar{v}_4 u_3 \bar{u}_1 v_2 \bar{v}_2 u_1 \bar{u}_3 u_4 =$$

$$= \frac{1}{4}\left(\frac{m_e m_b}{v^2}\right)^2\left(\frac{1}{s-m_h^2}\right)^2 \sum_s \mathrm{Tr}\left[u_1\bar{u}_1 v_2\bar{v}_2\right]\mathrm{Tr}\left[u_3\bar{u}_3 v_4\bar{v}_4\right] =$$

$$= \frac{1}{4}\left(\frac{m_e m_b}{v^2}\right)^2\left(\frac{1}{s-m_h^2}\right)^2 \mathrm{Tr}\left[(\not{p}_1+m_e)(\not{p}_2-m_e)\right]\mathrm{Tr}\left[(\not{p}_3+m_b)(\not{p}_4-m_b)\right] =$$

$$= \frac{1}{4}\left(\frac{m_e m_b}{v^2}\right)^2\left(\frac{1}{s-m_h^2}\right)^2 \cdot\left[4p_1\cdot p_2-4m_e^2\right]\left[4p_3\cdot p_4-4m_b^2\right] =$$

$$= \left(\frac{2m_e m_b}{v^2(s-m_h^2)}\right)^2\cdot\left(\frac{s}{2}-2m_e^2\right)\left(\frac{s}{2}-2m_b^2\right)$$

$$\boxed{p_1\cdot p_2 = \frac{s-m_1^2-m_2^2}{2}}$$

scm

$$\boxed{\begin{aligned} p_f^2 &= \frac{s}{4}-m_b^2 \Rightarrow p_f = \sqrt{\frac{s}{4}-m_b^2} \\ p_i &= \sqrt{\frac{s}{4}-m_e^2}\end{aligned}}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{1}{64\pi^2 s}\frac{|\vec{p}_f|}{|\vec{p}_i|}\overline{|\mathcal{M}|^2} =$$

$$= \frac{1}{64\pi^2 s}\left(\frac{2m_e m_b}{v^2(s-m_h^2)}\right)^2\left(\frac{s}{2}-2m_b^2\right)^{3/2}\sqrt{\frac{s}{2}-2m_e^2}$$

- nema kutne ovisnosti pa je udarni presjek:

$$\sigma = 4\pi\left(\frac{d\sigma}{d\Omega}\right)_{CM}$$

- odredimo omjer

faktor boje

$$\frac{\sigma(e^+e^-\to h\to b\bar{b})}{\sigma(e^+e^-\to h\to \tau^-\tau^+)} = 3\left(\frac{m_b}{m_\tau}\right)^2\frac{\left(\frac{s}{2}-2m_b^2\right)^{3/2}}{\left(\frac{s}{2}-2m_\tau^2\right)^{3/2}} \approx \text{URL} = 3\left(\frac{m_b}{m_\tau}\right)^2 = 20{,}91$$

- veća vjerojatnost raspada u teže čestice

- od svih mogućih kiralnih stanja (npr. u početnom stanju $e_L^-e_R^+, e_L^-e_L^+, e_R^-e_L^+, e_R^-e_R^+$) doprinose:

$$\bar{\Psi}_L h\Psi_R + \bar{\Psi}_R h\Psi_L$$

poništava desne čestice

poništava desne antičestice

$\Rightarrow$ u početnom stanju ($e_R^-e_R^+$ i $e_L^-e_L^+$)
u konačnom stanju ($f_R\bar{f}_R$ i $f_L\bar{f}_L$)

- to se vidi na kutnoj ovisnosti dif. udarnog presjeka!

$e^- \longrightarrow \cdot \longleftarrow e^+$

- higgs mora imati spin 0 (skalarna čestica)
- nema preferiranog smjera (nema kutne ovisnosti)