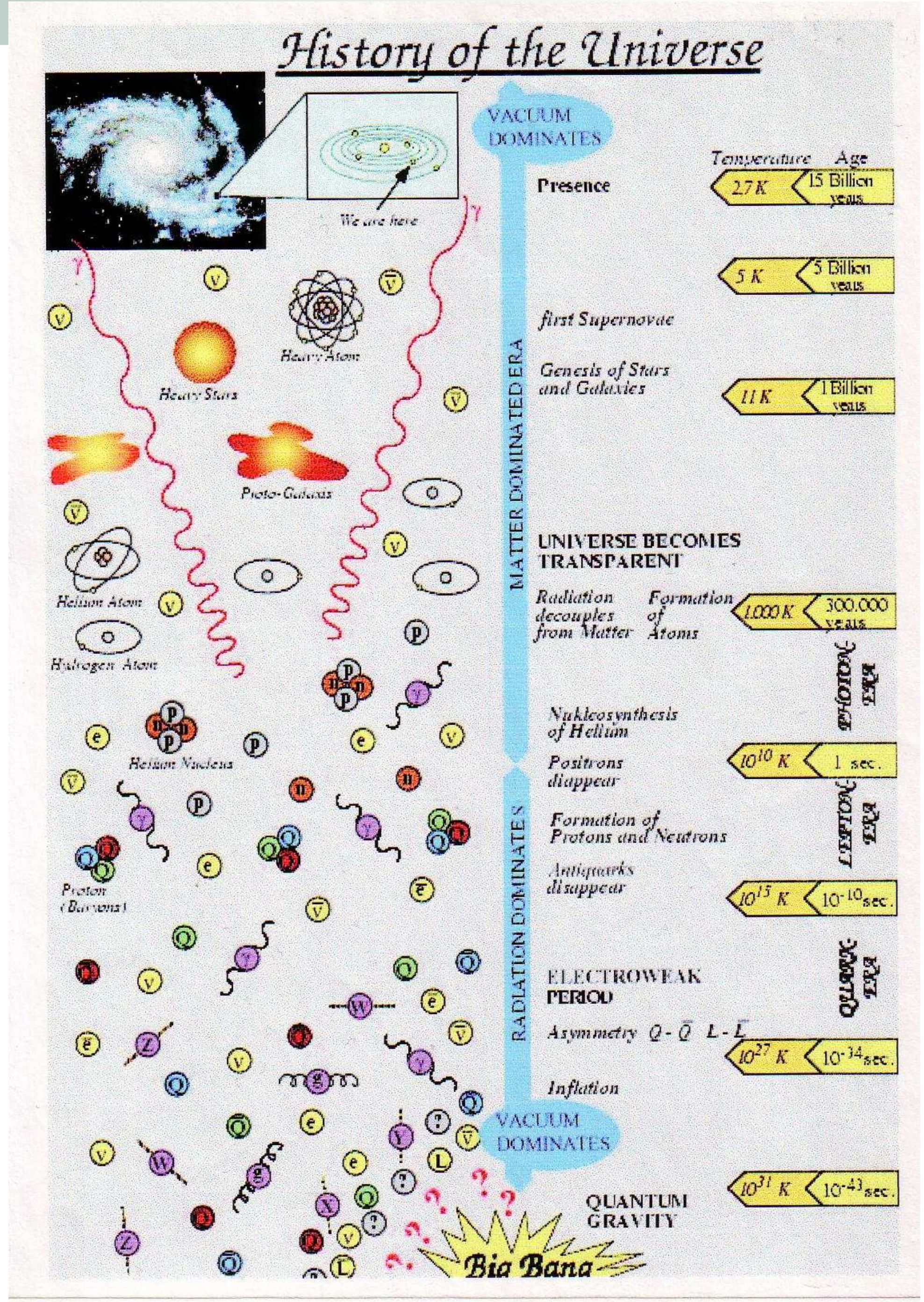
FIZIKALNA

KOZMOLOGIJA

V.

FRIEDMANNOVA JEDNADŽBA IZRAŽENA PARAMETROM GUSTOĆE



Današnjoj eri prethodi era zračenja

◊ **Friedmannove j-be**

– kao specifikacija Einsteinovih BEZ kozmol. člana, na IDEALIZIRANI SVEMIR s **Robertson-Walkerovom metrikom**

kojom izražavamo lijevu stranu Einst. j-be

"LHS" – Einsteinov tenzor

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$

$$G_1^1 \equiv R_1^1 - \frac{1}{2}R = -\frac{1}{c^2}\left(2\frac{\ddot S}{S} + \frac{\dot S^2 + kc^2}{S^2}\right) = G_2^2 = G_3^3$$

$$G_0^0 \equiv R_0^0 - \frac{1}{2}R = -\frac{3}{c^2}\left(\frac{\dot S^2 + kc^2}{S^2}\right)$$

"RHS" – desna strana Einsteinove j-be određena tenzorom energije-impulsa, $T_{\mu\nu}$

$$-\frac{8\pi G}{c^4} T_{\mu\nu} \qquad \begin{cases} T_0^0 = \varepsilon \\ \underbrace{T_1^1 = T_2^2 = T_3^3}_{\text{izotropnost!}} = -p \end{cases}$$

$$2\frac{\ddot S}{S} + \frac{\dot S^2 + kc^2}{S^2} = \frac{8\pi G}{c^2} T_i^i \quad ; \; i = 1,2,3$$

$$\frac{\dot S^2 + kc^2}{S^2} = \frac{8\pi G}{3c^2} T_0^0$$

$$T^\mu{}_\nu = \underbrace{T^\mu{}_\nu}_{\text{tvari}} + \underbrace{T^\mu{}_\nu}_{\substack{\text{zračenja} \\ T_0^0 = \varepsilon}}$$

URL → $\mathrm{diag}(\varepsilon, -\varepsilon/3, -\varepsilon/3, -\varepsilon/3)$

U današnjoj epohi (s faktorom širenja S_0)

gustoća zračenja $\varepsilon_0 \simeq 10^{-13}\ \mathrm{erg\ cm^{-3}}$ $\ll$ gustoća tvari $\rho_0 c^2 \gtrsim 10^{-10}\ \mathrm{erg\ cm^{-3}}$

indicira prijelaz na $S \simeq 10^{-3} S_0$

svemira dominiranog zračenjem → u svemir dominiran materijom

Sugibajući volumen $V = V_0 \left(\frac{S}{S_0}\right)^3$ uz konstantan broj čestica $\Rightarrow \rho = \rho_0 \left(\frac{S_0}{S}\right)^3$

Izbor $\begin{cases} T_0^0 = \rho_0 c^2 \frac{S_0^3}{S^3} \\ T_i^i = 0 \end{cases}$

vodi na Friedmannove j-be

$$\boxed{\begin{aligned} &2\frac{\ddot S}{S} + \frac{\dot S^2 + kc^2}{S^2} = 0 \\ &\frac{\dot S^2 + kc^2}{S^2} = \frac{8\pi G \rho_0}{3} \cdot \frac{S_0^3}{S^3} \end{aligned}}$$

– nisu nezavisne → iz jedne od njih određuje se $S(t)$ za 3 slučaja $(k = 0, 1, -1)$, zasebno.

Uvođenje jednadžbe stanja - uz 2 nezavisne (za 3 nepoznanice)

- Friedmannova j.
- J. Fluida
- J. ubrzanja

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon - \frac{\kappa c^2}{S_0^2 a}$$

$$\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\varepsilon + 3P)$$

$P = w\varepsilon$ kozmološki primjerena linearna relacija

Tri slučaja: w=1/3, 0, -1

◊ Rješenja Friedmann-ovih j-bi

□ uz poznatu **jednadžbu stanja**

$$p = p(\rho) \rightarrow \boxed{p = w\rho c^2 ;\ 0 \le w \le 1}$$

- sadašnja epoha (prašina bez tlaka): $w=0$
- epoha ranog svemira (UR čestica/zrač.): $w=1/3$

i relaciju sadržanu u Friedmann j-bama (oč. tenzora $T^{\nu\mu}_{;\mu}=0$)

$$\boxed{\frac{d}{dS}(\varepsilon S^3) + 3pS^2 = 0}$$

LHS: promjena energije u sugibajućem vol.

$$\boxed{d(\rho c^2 S^3) = -p\, d(S^3)}$$

RHS: tlak × promj. vol.

$$\boxed{\rho \propto S^{-3(1+w)}} \longrightarrow \propto \begin{cases} S^{-3} & \text{tvar} \quad w=0 \\ S^{-4} & \text{zrač.} \quad w=1/3 \\ \text{const.} & \text{vakuum} \quad w=-1 \end{cases}$$

□ Fr. j. $$\boxed{\underbrace{\frac{\dot{S}^2}{S^2}}_{\equiv H^2} + \frac{kc^2}{S^2} = \frac{8\pi G}{3}\rho} \qquad (*)$$

$/H^2 \Rightarrow$

$$1 + \frac{kc^2}{H^2 S^2} = \frac{\rho}{3H^2/8\pi G} \Rightarrow$$

$$k = \frac{S^2 H^2}{c^2}(2q-1) \quad \text{ili} \quad \frac{kc^2}{H^2 S^2} = \frac{\rho}{\frac{3H^2}{8\pi G}} - 1$$

$$\Omega \equiv \rho/\rho_c$$

Uvođenje parametra gustoće

- parametar gustoće

$$\Omega = \rho / \rho_c \;; \quad \rho_c = \frac{3H^2}{8\pi G}$$

$$\rho_{c,0} = 2.76 \cdot 10^{11} h^2 \, M_\odot \, Mpc^{-1}$$

$$-kc^2 = \dot{S}^2 - \frac{8\pi G}{3} \rho S^2 - \frac{\lambda}{3} S^2$$

$$= S^2 H^2 \left[1 - \frac{\rho_m + \rho_r + \rho_\lambda}{\rho_c} \right] \quad \& \quad \Omega_{m,r,\lambda} = \frac{\rho_{m,r,\lambda}}{\rho_c}$$

$$-kc^2 = S^2 H^2 \left[1 - (\Omega_m + \Omega_r + \Omega_\lambda) \right]$$

Evolucija parametra gustoće - za primjer svemirske prašine

Iz ponašanja $\frac{\Omega}{\Omega_0} = \frac{\rho}{\rho_0}\frac{H_0^2}{H^2}$

$\rho_0 \leftarrow (1+z)^3$ $\qquad \rho S^3 = \text{const.}$ za $P=0$

$$\Rightarrow \quad \Omega H^2 = (1+z)^3 \Omega_0 H_0^2$$

$H^2 \leftarrow$ iz Friedmannove j. $\Rightarrow$

$$\Omega = \frac{\Omega_0 (1+z)}{1+\Omega_0 z}$$