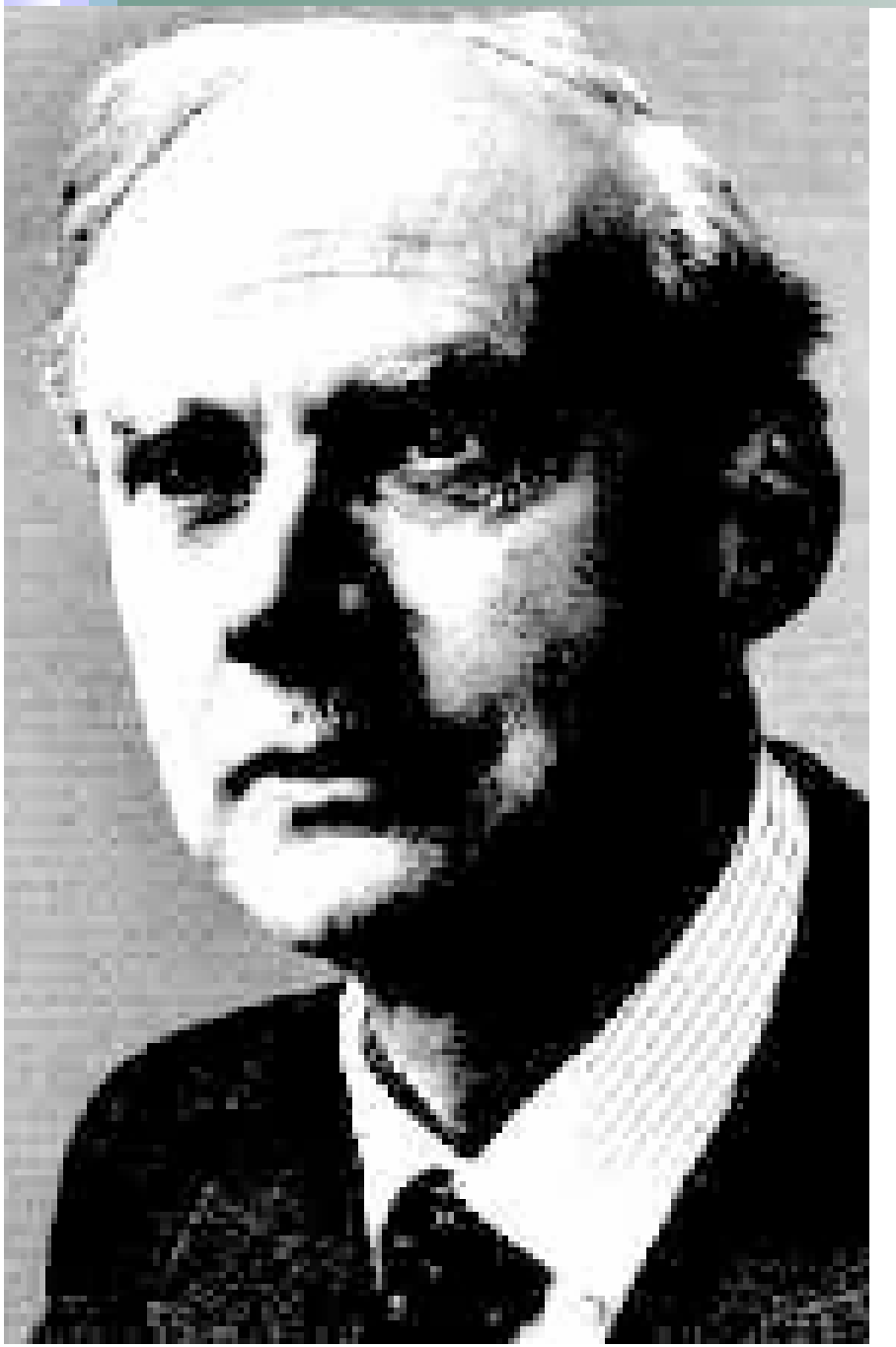


V.DIRACOVA JEDNADŽBA
(FEČ § 2.3.2 str. 81)

- DIRACOVE MATRICE
- RJEŠENJA &
- INTERPRETACIJE NEGATIVNIH ENERGIJA

LINEARIZACIJA KG JEDN.

$$i\frac{\partial\psi(\vec{x},t)}{\partial t} = (-i\vec{\alpha}\cdot\nabla + \beta m)\psi \equiv H\psi$$

$$\begin{aligned}
\left(i\frac{\partial}{\partial t}\right)^2\psi &= (-i\vec{\alpha}\cdot\nabla + \beta m)(-i\vec{\alpha}\cdot\nabla + \beta m)\psi \\
&= \left[-\sum_{i=1}^{3}\alpha_i^2\frac{\partial^2}{(\partial x^i)^2} - \sum_{i>j=1}^{3}(\alpha_i\alpha_j + \alpha_j\alpha_i)\frac{\partial^2}{\partial x^i\partial x^j}\right. \\
&\quad \left. -im\sum_{i=1}^{3}(\alpha_i\beta + \beta\alpha_i)\frac{\partial}{\partial x^i} + \beta^2 m^2\right]\psi\ .
\end{aligned}$$

KOVARIJANTNI ZAPIS I DIRACOVE MATRICE

$$i\frac{\partial \Psi}{\partial t} = H_D \Psi \quad ; \quad H_D = -i\vec{\alpha}\cdot\nabla + \beta m$$

$$\{\alpha_i, \alpha_j\} = 0 \qquad i \neq j$$

$$\{\alpha_i, \beta\} = 0 \quad , \quad \alpha_i^2 = \beta^2 = 1\!\!1$$

$$\gamma^\mu = (\gamma^0, \gamma^i) \quad ,$$

$$\underbrace{\gamma^0 = \beta \quad , \quad \gamma^i = \beta\alpha^i}$$

$$(i\gamma^\mu \partial_\mu - m)\Psi = 0$$

$$\dim \begin{cases} \text{parna} \\ \geq 4 \end{cases}$$

DIRACOVA REPREZENTACIJA GAMA MATRICA

$$\gamma^0 = \beta = \sigma^3 \otimes \underline{I} = \begin{pmatrix} \underline{I} & 0 \\ 0 & -\underline{I} \end{pmatrix}$$

$$\vec{\gamma} = \beta \vec{\alpha} = i\sigma^2 \otimes \vec{\sigma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}$$

(4×4) matrice

$\Rightarrow$ ψ - 4-komponentni

RJEŠENJA DIRACOVE JEDNADŽBE FEČ str. 137

$$\psi^{(1)} = e^{-i(E/\hbar)t} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Electron (spin up)

$$\psi^{(2)} = e^{-i(E/\hbar)t} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Electron (spin down)

$$\psi^{(3)} = e^{+i(E/\hbar)t} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Positron (spin up)

$$\psi^{(4)} = e^{+i(E/\hbar)t} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Positron (spin down)

DIRACOVA INTERPRETACIJA NEGATIVNIH ENERGIJA

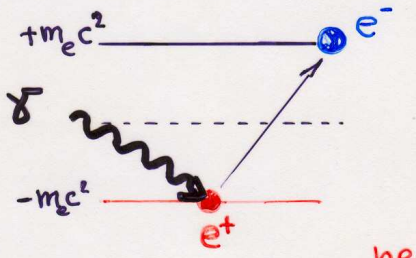
Razlog postojanja antičestica

– u spoju KVANTNE MEHANIKE i SPECIJALNE TEORIJE RELATIVNOSTI

Diracova j-ba za elektron

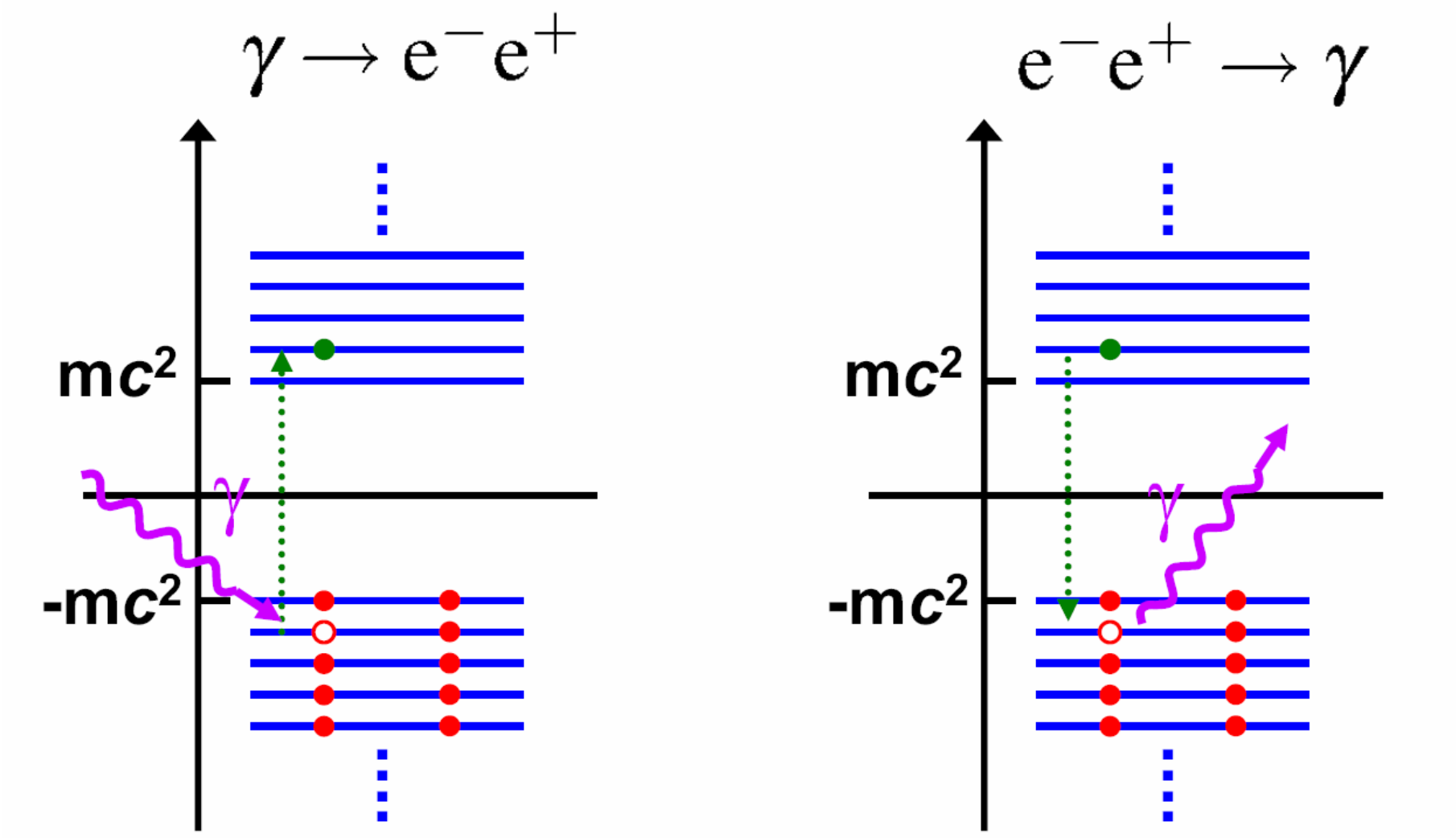
$$\left(i\frac{\partial}{\partial t} + i\vec{\alpha}\cdot\vec{\nabla} - \beta m_e\right)\Psi_e = 0$$

predviđa da njeno rješenje Ψ_e istodobno opisuje pozitron



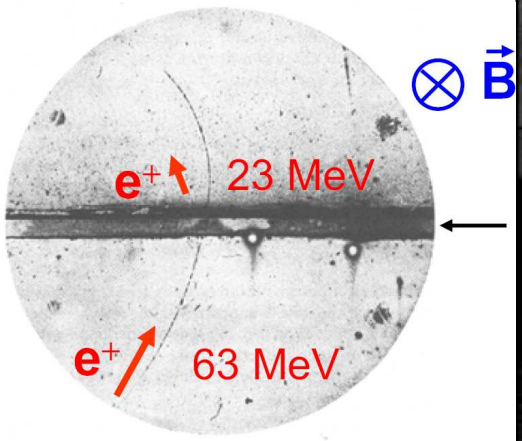
"rupa" u negativnoenergijskom "Diracovom moru" ponaša se kao "antielektron"

POPUNJENO MORE NEGATIVNIH ENERGIJA



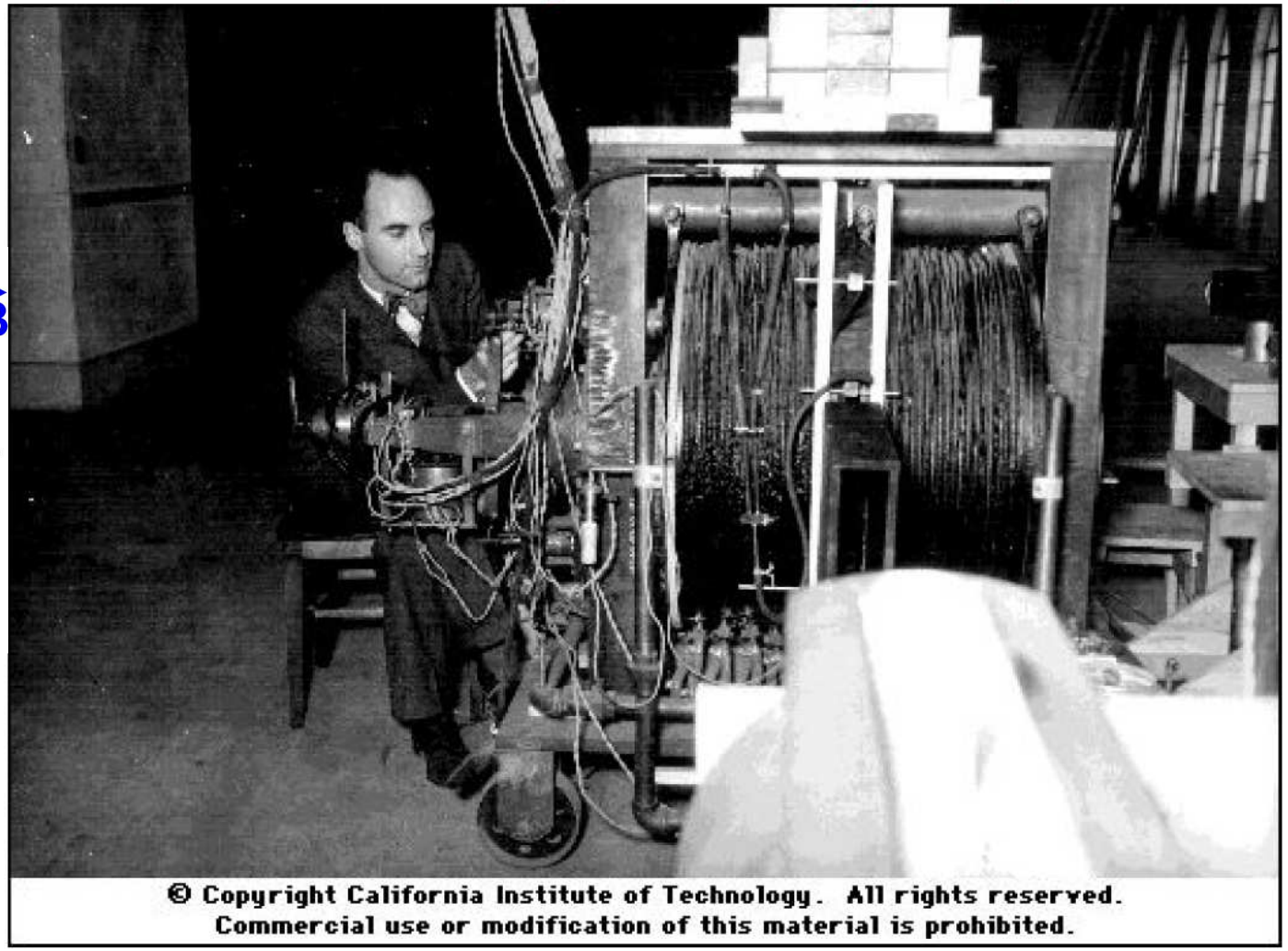
OTKRIĆE POZITRONA

C.D.Anderson, Phys Rev 43 (1933) 491



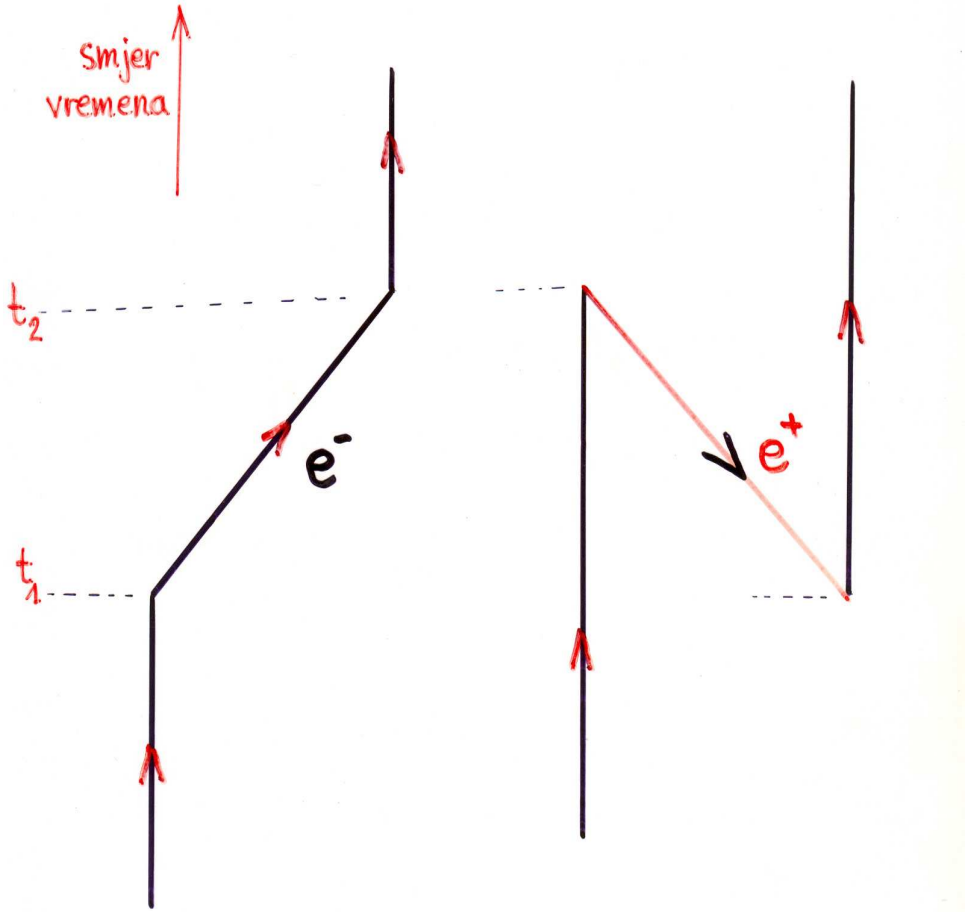
$\otimes \vec{B}$

6 mm olovna ploča



R.P. FEYNMAN

Dva viđenja istog događaja "dvostrukog raspršenja":

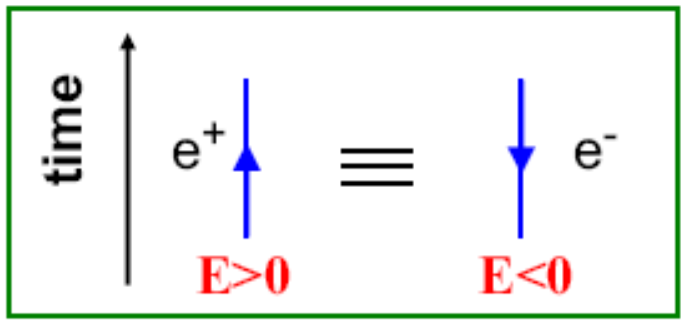


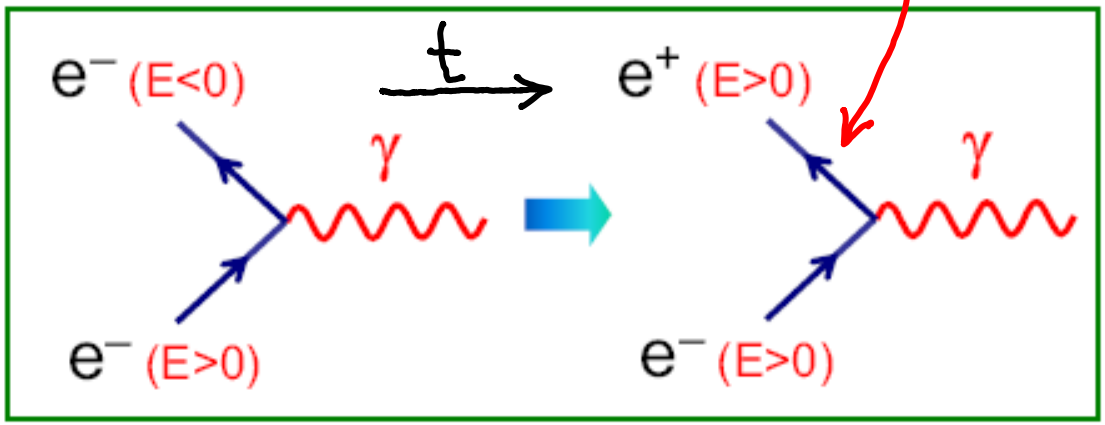
(a) elektron "putuje" između dva raspršenja

(b) pozitron i dva elektrona "putuju" između STVARANJA PARA i PONIŠTENJA PARA

FEYNMAN-STUECKELBERGova INTERPRETACIJA rješenja negativnih energija:

- ČESTICE NEGATIVNIH ENERGIJA KOJE PUTUJU U PROŠLOST
- ANTIČESTICE POZITIVNIH ENERGIJA KOJE PUTUJU U BUDUĆNOST – strelica protivno vremenu označavat će AČ rješenje:



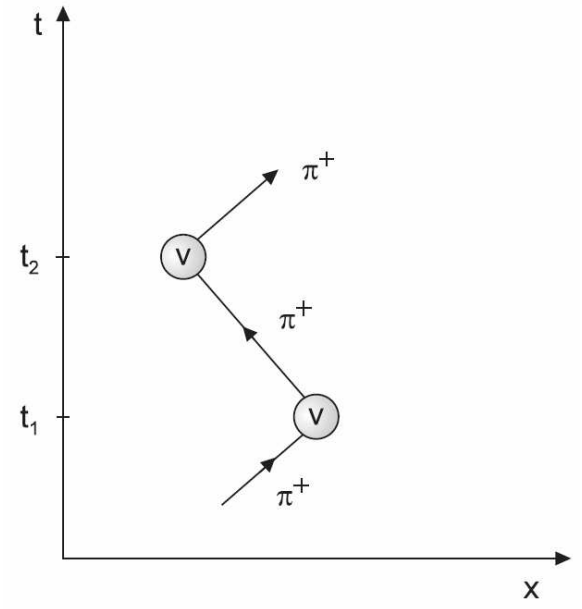


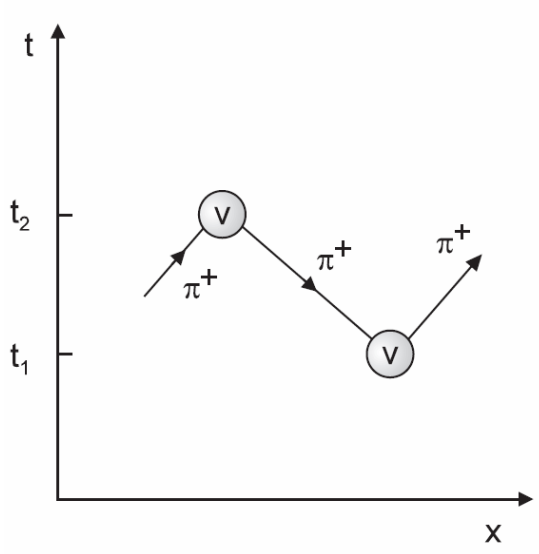
F-S INTERPRETACIJA RADI I ZA BOZONE (dodatna staza piona)

Feynman – Stuckelberg interpretation

t

$\pi^+(E>0)$ e^{-iEt} $\equiv$ $\pi^-(E<0)$ $e^{-i(-E)(-t)}$





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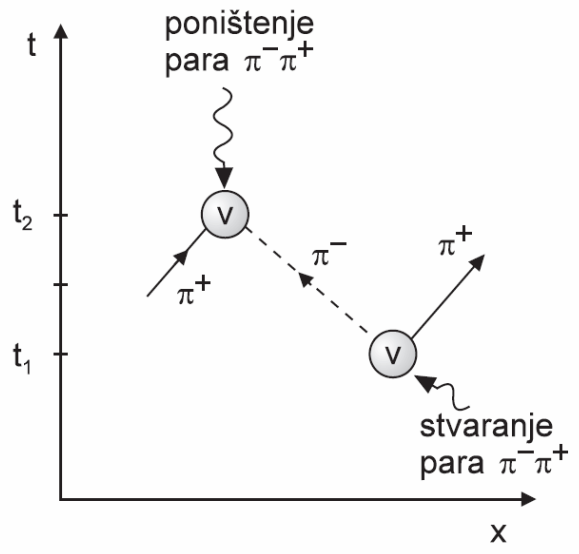

UVODI NOVU STAZU (QM AMPLITUDU)

Two different time orderings giving same observable event :