# III. Kvantna kromodinamika

## QCD KAO NEABELOVA BAŽDARNA TEORIJA

- **ZATOČENJE KVARKOVA I GLUONA svodi se na ZATOČENJE BOJE**
- **OPĆENITE LOKALNE SU(N) TRANSFORMACIJE**
- **BOJNI FAKTORI I PRIVLAČNI KANALI**

# KVANTNA KROMODINAMIKA MIKROSKOPSKA TEORIJA JAKE SILE (dinamika boje kvarkova, i=1,2,3)

Spin- $\frac{1}{2}$ quarks: $\psi^i = u^i, d^i, s^i, \ldots$(flavor)

- with new quantum number– color
- gauge principle leads to eight new

Spin- 1 gluons: $A^a$ with $a = \{1, \ldots, 8\}$

# PRINCIP LOKALNE SIMETRIJE - za QED, FEČ 3.2, STR.127

U(1) baždarna invarijantnost

$\mathcal{L}_0 = \bar{\psi}(i\partial\!\!\!/ - m)\psi$

$\psi(x) \to \psi'(x) = e^{-i\alpha(x)}\psi(x)$, $e^{-i\alpha(x)} \equiv S(x)$

$\bar{\psi}(x) \to \bar{\psi}'(x) = \bar{\psi}(x) S^{\dagger}(x)$ (*)

$\Rightarrow \delta\mathcal{L}_0 = \bar{\psi}\gamma^{\mu}\psi\,\partial_{\mu}\alpha(x)$

kompenziran dodavanjem interaktivnog člana

$\mathcal{L}_0 \to \mathcal{L}_{tot} = \mathcal{L}_0 - q\bar{\psi}\gamma^{\mu}\psi A_{\mu}$

$A_{\mu}(x) \to A'_{\mu}(x) = A_{\mu}(x) + \frac{1}{q}\partial_{\mu}\alpha(x)$ (**)

$\equiv A_{\mu}(x) + \frac{i}{q}(\partial_{\mu}S(x))S^{\dagger}(x)$

Kinetički član e.m. (baždarnog) polja

$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$

$F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$

invarijantan je na (**)

# QCD DJELOVANJE SLIČNO ELEKTRODINAMIČKOM

$$S_{\mathrm{QCD}} = \int d^4x \left[ \overline{\psi}_j (i\partial\!\!\!/ + gA\!\!\!/ - m_q)^j_i \psi^i - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 \right]$$

But there is a gluonic self coupling

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\ f^{abc} A^b_\mu A^c_\nu$$

and the interaction strength is different:

$$\alpha_s = \frac{g^2}{4\pi} = O(1) \quad \text{instead of} \quad \alpha_{QED} = \frac{e^2}{4\pi} \approx 1/137$$

# PRINCIP LOKALNE SIMETRIJE - za tri "bojna naboja" QCD

SU(3) transformacija

$$\psi(x) \rightarrow e^{-igT^a\Theta^a(x)}\,\psi(x) \quad (*)$$

$$T^a = \frac{\lambda^a}{2}$$

$$S(x) = e^{-ig\vec{T}\cdot\vec{\Theta}(x)}$$

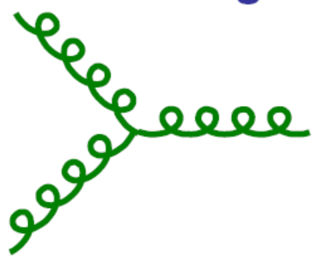
- zahtijeva postojanje 8 baždarnih polja $A^a_\mu$ $(a = 1, 2, \ldots, 8)$
- $\partial_\mu \rightarrow D_\mu = \partial_\mu - ig\vec{T}\cdot\vec{A}_\mu(x) \equiv \partial_\mu + g\,G_\mu(x)$

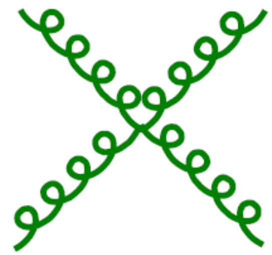
$$G_\mu(x) = \frac{1}{i}\vec{T}\cdot\vec{A}_\mu(x)$$

# GLUONSKE SAMOINTERAKCIJE

★ Two new vertices (no QED analogues)

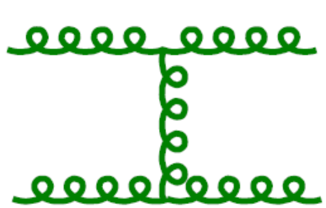
triple-gluon vertex

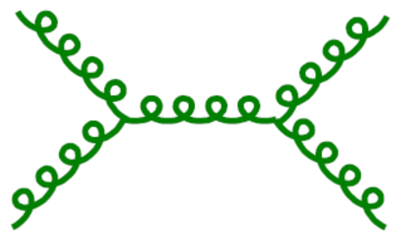




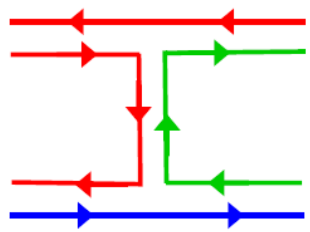
quartic-gluon vertex

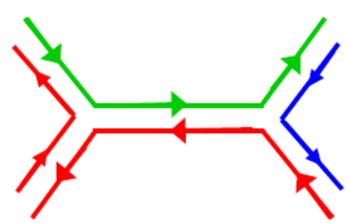
★ In addition to quark-quark scattering, therefore can have gluon-gluon scattering





e.g. possible way of arranging the colour flow



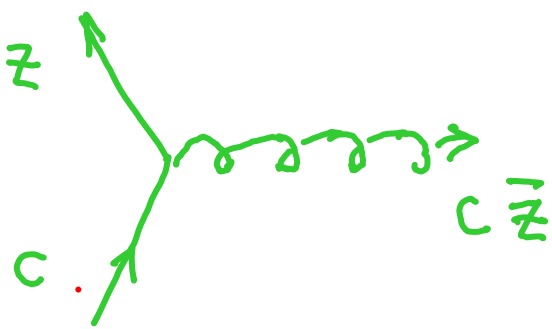


# TRANSFORMACIJA BOJE SE DOGAĐA U KVARKOVSKO-GLUONSKOM VRHU

$$c: \; |1\rangle \rightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

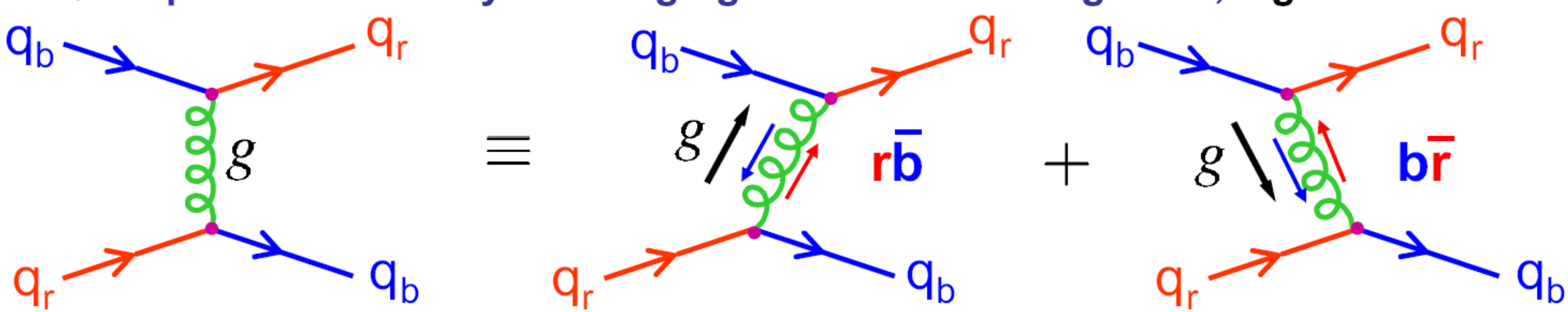
$$z: \; |2\rangle \rightarrow \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$p: \; |3\rangle \rightarrow \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

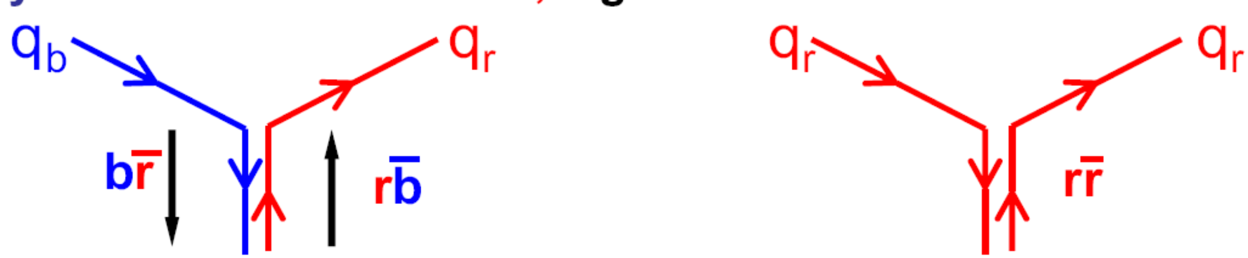



$$(1, 0, 0) \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

In QCD quarks interact by exchanging virtual massless gluons, **e.g.**

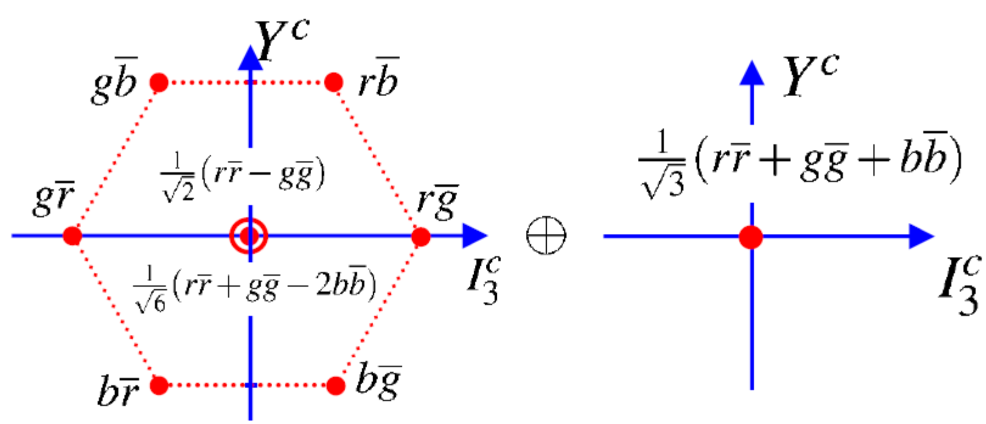



Gluons carry colour and anti-colour, **e.g.**




Gluon colour wave-functions (colour + anti-colour) are the same as those obtained for mesons (also colour + anti-colour)

⇒ OCTET + "COLOURLESS" SINGLET



# DAJE 6 BOJNO NABIJENIH GLUONA

$c\bar{z}$ $\quad \lambda_{12} = \frac{1}{2}(\lambda_1 + i\lambda_2) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & & 0 \\ 0 & & \end{pmatrix}$ & h.c.

$c\bar{p}$ $\quad \lambda_{45} = \frac{1}{2}(\lambda_4 + i\lambda_5) = \begin{pmatrix} 0 & 0 & 1 \\ & & 0 \\ & & 0 \end{pmatrix}$

$z\bar{p}$ $\quad \lambda_{67} = \frac{1}{2}(\lambda_6 + i\lambda_7) = \begin{pmatrix} 0 & 0 & 0 \\ & & 1 \\ & & 0 \end{pmatrix}$

- 2 BOJNO NEUTRALNIH GLUONA

$c\bar{c} - z\bar{z}$ $\quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

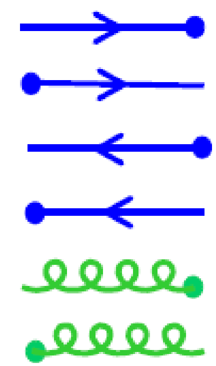
$c\bar{c} + z\bar{z} - 2p\bar{p}$ $\quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$

# FEYNMANOVA PRAVILA QCD-a

**External Lines**

spin 1/2
- incoming quark $u(p)$
- outgoing quark $\bar{u}(p)$
- incoming anti-quark $\bar{v}(p)$
- outgoing anti-quark $v(p)$

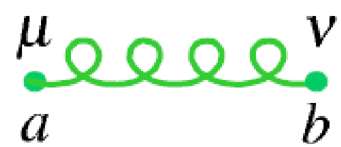
spin 1
- incoming gluon $\varepsilon^{\mu}(p)$
- outgoing gluon $\varepsilon^{\mu}(p)^{*}$



**Internal Lines (propagators)**

spin 1 gluon

$$\frac{-ig_{\mu\nu}}{q^2}\delta^{ab}$$

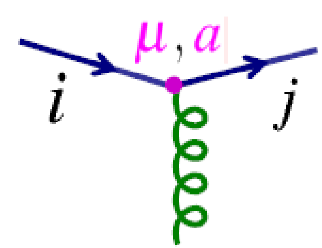



a, b = 1,2,…,8 are gluon colour indices

**Vertex Factors**

spin 1/2 quark

$$-\frac{1}{2}ig_s\lambda^{a}_{ji}\gamma^{\mu}$$




i, j = 1,2,3 are quark colours,

$\lambda^{a}$ a = 1,2,..8 are the Gell-Mann SU(3) matrices

+ 3 gluon and 4 gluon interaction vertices

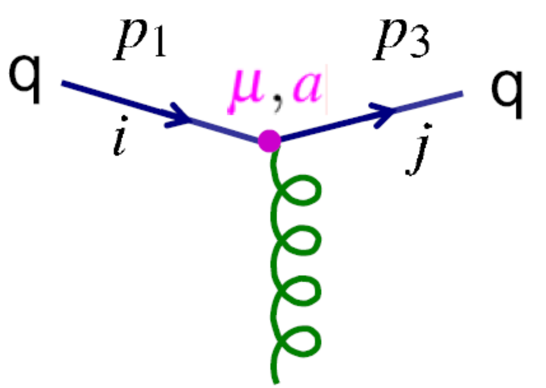
Matrix Element $-iM$ = product of all factors

# BOJNI FAKTORI KVARKA

- Look at the QCD vertex factors in more detail. Representing the colour part of the fermion wave-functions by:

$$r = c_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad g = c_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad b = c_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

- Particle wave-functions $u(p) \longrightarrow c_i u(p)$




- The QCD qqg vertex can then be written:

$$\overline{u}(p_3) c_j^\dagger \{-\tfrac{1}{2} i g_s \lambda^a \gamma^\mu\} c_i u(p_1)$$

- For which the colour part is

$$c_j^\dagger \lambda^a c_i = c_j^\dagger \begin{pmatrix} \lambda^a_{1i} \\ \lambda^a_{2i} \\ \lambda^a_{3i} \end{pmatrix} = \lambda^a_{ji}$$

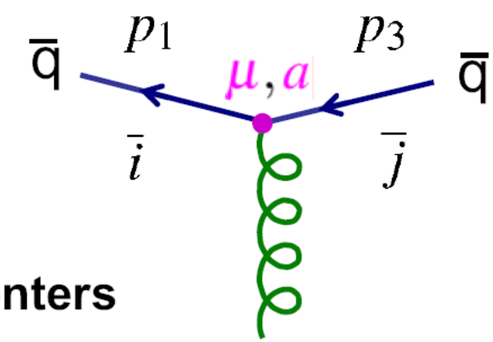
- Hence

$$\overline{u}(p_3) c_j^\dagger \{-\tfrac{1}{2} i g_s \lambda^a \gamma^\mu\} c_i u(p_1) \equiv \overline{u}(p_3) \{-\tfrac{1}{2} i g_s \lambda^a_{ji} \gamma^\mu\} u(p_1)$$

# & ANTI-KVARKA

★ Now consider the anti-quark vertex

- The QCD $\bar{q}\bar{q}g$ vertex is:




$$\overline{v}(p_1)c_i^{\dagger}\{-\tfrac{1}{2}ig_s\lambda^a\gamma^{\mu}\}c_j v(p_3)$$

**Note that the incoming anti-particle now enters on the LHS of the expression**

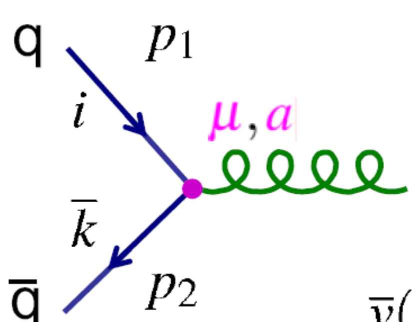
- For which the colour part is

$$c_i^{\dagger}\lambda^a c_j = c_i^{\dagger}\begin{pmatrix}\lambda^a_{1j}\\ \lambda^a_{2j}\\ \lambda^a_{3j}\end{pmatrix} = \lambda^a_{ij}$$

i.e indices *ij* swapped compared compared to quark case

- Hence $\overline{v}(p_1)c_i^{\dagger}\{-\tfrac{1}{2}ig_s\lambda^a\gamma^{\mu}\}c_j v(p_3) \equiv \overline{v}(p_1)\{-\tfrac{1}{2}ig_s\lambda^a_{ij}\gamma^{\mu}\}v(p_3)$

★ Finally we can consider the annihilation vertex



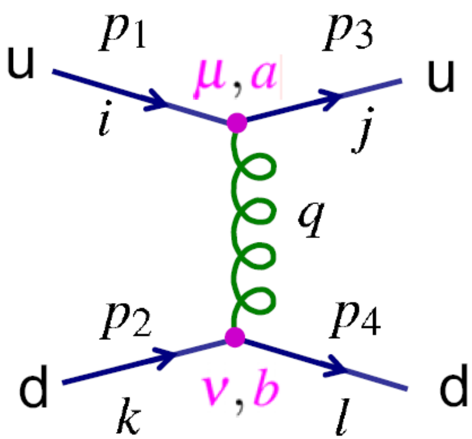

QCD vertex: $\overline{v}(p_2)c_k^{\dagger}\{-\tfrac{1}{2}ig_s\lambda^a\gamma^{\mu}\}c_i u(p_1)$

with $c_k^{\dagger}\lambda^a c_i = \lambda^a_{ki}$

$$\overline{v}(p_2)c_k^{\dagger}\{-\tfrac{1}{2}ig_s\lambda^a\gamma^{\mu}\}c_i u(p_1) \equiv \overline{v}(p_2)\{-\tfrac{1}{2}ig_s\lambda^a_{ki}\gamma^{\mu}\}u(p_1)$$

# RASPRŠENJE KVARKOVA U QCD-u

★ Consider QCD scattering of an up and a down quark




- The incoming and out-going quark colours are labelled by $i, j, k, l = \{1,2,3\}$ (or $\{r,g,b\}$)
- In terms of colour this scattering is $$ik \rightarrow jl$$
- The 8 different gluons are accounted for by the colour indices $a, b = 1, 2, ..., 8$
- NOTE: the $\delta$-function in the propagator ensures $a = b$, i.e. the gluon emitted at $a$ is the same as that absorbed at $b$

★ Applying the Feynman rules:

$$-iM = \left[\overline{u}_u(p_3)\{-\tfrac{1}{2}ig_s\lambda^a_{ji}\gamma^\mu\}u_u(p_1)\right]\frac{-ig_{\mu\nu}}{q^2}\delta^{ab}\left[\overline{u}_d(p_4)\{-\tfrac{1}{2}ig_s\lambda^b_{lk}\gamma^\nu\}u_d(p_2)\right]$$

where summation over $a$ and $b$ (and $\mu$ and $\nu$) is implied.

★ Summing over $a$ and $b$ using the $\delta$-function gives:

$$M = -\frac{g_s^2}{4}\lambda^a_{ji}\lambda^a_{lk}\frac{1}{q^2}g_{\mu\nu}[\overline{u}_u(p_3)\gamma^\mu u_u(p_1)][\overline{u}_d(p_4)\gamma^\nu u_d(p_2)]$$

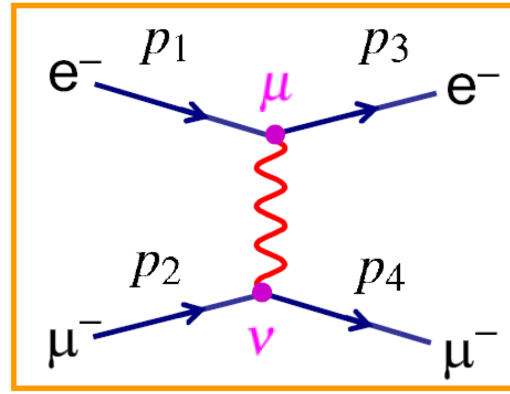
# USPOREDBA QCD i QED

## QCD vs QED

**QED**

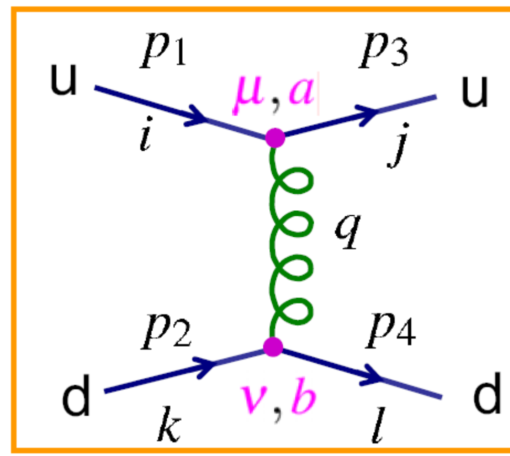
$$-iM = [\overline{u}(p_3) ie\gamma^\mu u(p_1)] \frac{-ig_{\mu\nu}}{q^2} [\overline{u}(p_4) ie\gamma^\nu u(p_2)]$$

$$M = -e^2 \frac{1}{q^2} g_{\mu\nu} [\overline{u}(p_3)\gamma^\mu u(p_1)][\overline{u}(p_4)\gamma^\nu u(p_2)]$$




**QCD**

$$M = -\frac{g_s^2}{4} \lambda^a_{ji} \lambda^a_{lk} \frac{1}{q^2} g_{\mu\nu} [\overline{u}_u(p_3)\gamma^\mu u_u(p_1)][\overline{u}_d(p_4)\gamma^\nu u_d(p_2)]$$




★ **QCD Matrix Element = QED Matrix Element with:**

- $e^2 \rightarrow g_s^2$ **or equivalently** $\alpha = \frac{e^2}{4\pi} \rightarrow \alpha_s = \frac{g_s^2}{4\pi}$

**+ QCD Matrix Element includes an additional "colour factor"**

$$C(ik \rightarrow jl) \equiv \frac{1}{4} \sum_{a=1}^{8} \lambda^a_{ji} \lambda^a_{lk}$$

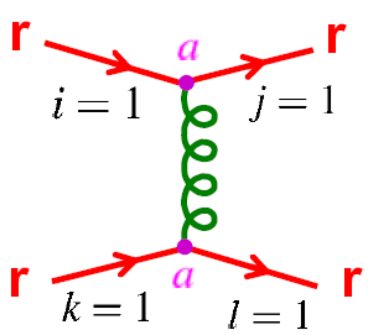
# RAČUN BOJNIH FAKTORA

QCD colour factors reflect the gluon states that are involved

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

## ❶ Configurations involving a single colour




•Only matrices with non-zero entries in **11** position are involved

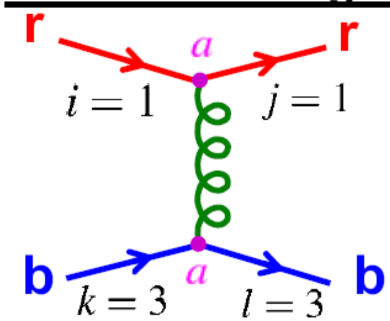
$$C(rr \to rr) = \frac{1}{4}\sum_{a=1}^{8} \lambda^a_{11}\lambda^a_{11} = \frac{1}{4}(\lambda^3_{11}\lambda^3_{11} + \lambda^8_{11}\lambda^8_{11})$$

$$= \frac{1}{4}\left(1 + \frac{1}{3}\right) = \frac{1}{3}$$

Similarly find

$$\boxed{C(rr \to rr) = C(gg \to gg) = C(bb \to bb) = \frac{1}{3}}$$

## ❷ Other configurations where quarks don't change colour e.g. $rb \rightarrow rb$



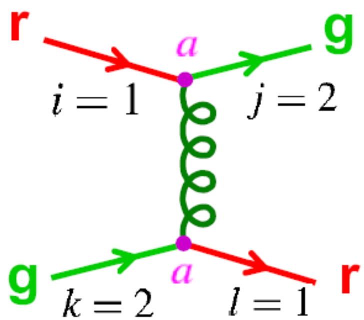

- Only matrices with non-zero entries in **11 and 33** position are involved

$$C(rb \rightarrow rb) = \frac{1}{4}\sum_{a=1}^{8} \lambda_{11}^a \lambda_{33}^a = \frac{1}{4}(\lambda_{11}^8 \lambda_{33}^8)$$

$$= \frac{1}{4}\left(1 . \frac{-2}{3}\right) = -\frac{1}{6}$$

Similarly $C(rb \rightarrow rb) = C(rg \rightarrow rg) = C(gr \rightarrow gr) = C(gb \rightarrow gb) = C(br \rightarrow br) = C(bg \rightarrow bg) = -\frac{1}{6}$

## ❸ Configurations where quarks swap colours e.g. $rg \rightarrow gr$



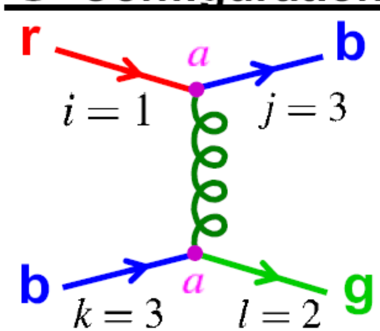

- Only matrices with non-zero entries in **12 and 21** position are involved

$$C(rg \rightarrow gr) = \frac{1}{4}\sum_{a=1}^{8} \lambda_{21}^a \lambda_{12}^a = \frac{1}{4}(\lambda_{21}^1 \lambda_{12}^1 + \lambda_{21}^2 \lambda_{12}^2)$$

$$= \frac{1}{4}(i(-i) + 1) = \frac{1}{2}$$

$$C(rb \rightarrow br) = C(rg \rightarrow gr) = C(gr \rightarrow rg) = C(gb \rightarrow bg) = C(br \rightarrow rb) = C(bg \rightarrow gb) = \frac{1}{2}$$

## ❹ Configurations involving 3 colours e.g. $rb \rightarrow bg$



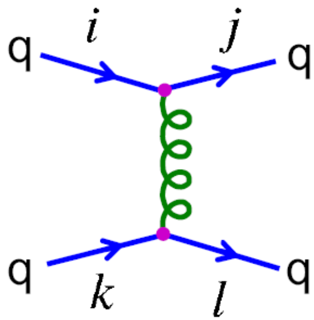

- Only matrices with non-zero entries in the **13 and 32** position
- But none of the $\lambda$ matrices have non-zero entries in the **13 and 32** positions. Hence the colour factor is zero

★ **colour is conserved**

# TRI TIPA BOJNIH FAKTORA

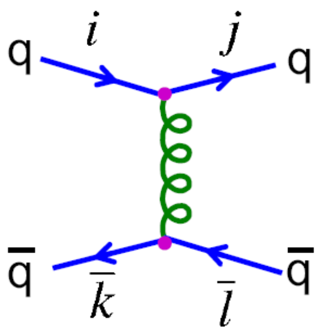
- Consequently the colour factors for the different diagrams are:

e.g.



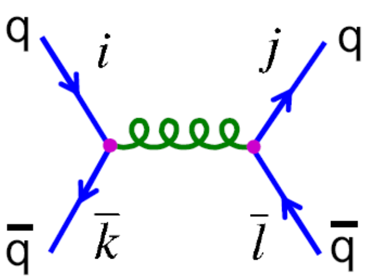

$$C(ik \rightarrow jl) \equiv \frac{1}{4}\sum_{a=1}^{8} \lambda^a_{ji}\lambda^a_{lk}$$

$$C(rr \rightarrow rr) = \frac{1}{3}$$
$$C(rg \rightarrow rg) = -\frac{1}{6}$$
$$C(rg \rightarrow gr) = \frac{1}{2}$$




$$C(i\bar{k} \rightarrow j\bar{l}) \equiv \frac{1}{4}\sum_{a=1}^{8} \lambda^a_{ji}\lambda^a_{kl}$$

$$C(r\bar{r} \rightarrow r\bar{r}) = \frac{1}{3}$$
$$C(r\bar{g} \rightarrow r\bar{g}) = -\frac{1}{6}$$
$$C(r\bar{r} \rightarrow g\bar{g}) = \frac{1}{2}$$



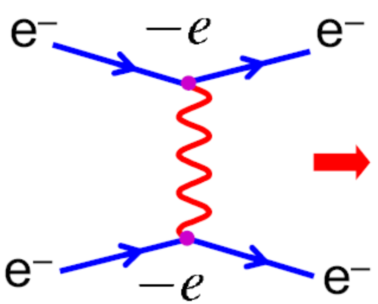

$$C(i\bar{k} \rightarrow j\bar{l}) \equiv \frac{1}{4}\sum_{a=1}^{8} \lambda^a_{ki}\lambda^a_{jl}$$

$$C(r\bar{r} \rightarrow r\bar{r}) = \frac{1}{3}$$
$$C(r\bar{g} \rightarrow r\bar{g}) = \frac{1}{2}$$
$$C(r\bar{r} \rightarrow g\bar{g}) = -\frac{1}{6}$$

Colour index of adjoint spinor comes first

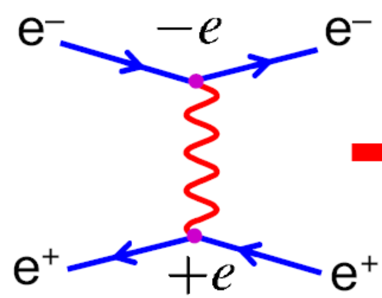
# TRAŽENJE PRIVLAČNIH KONFIGURACIJA

**QED**




$$V(r) = +\frac{\alpha}{r}$$

**Repulsive Potential**

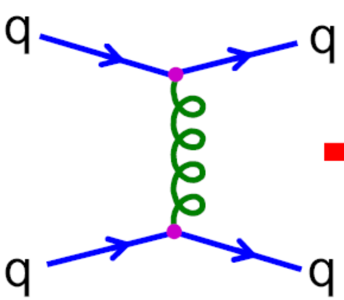



$$V(r) = -\frac{\alpha}{r}$$

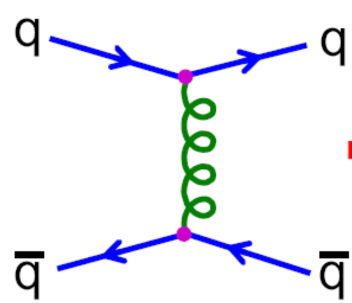
**Attractive Potential**

**QCD**

★ by analogy with QED expect potentials of form




$$V(r) = +C\frac{\alpha_S}{r}$$




$$V(r) = -C\frac{\alpha_S}{r}$$

★ Whether it is a attractive or repulsive potential depends on sign of colour factor

# PRIVLAČENJE U SINGLETU

★ Consider the colour factor for a qq system in the colour singlet state:

$$\psi = \frac{1}{\sqrt{3}}(r\bar{r} + g\bar{g} + b\bar{b})$$

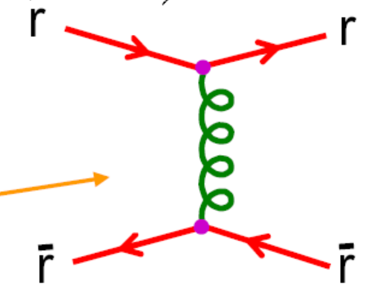
with colour potential $\langle V_{q\bar{q}} \rangle = \langle \psi | V_{\rm QCD} | \psi \rangle$

$$\Rightarrow \langle V_{q\bar{q}} \rangle = \frac{1}{3}\left(\langle r\bar{r}|V_{\rm QCD}|r\bar{r}\rangle + \ldots + \langle r\bar{r}|V_{\rm QCD}|b\bar{b}\rangle + \ldots\right)$$

•Following the QED analogy:

$$\langle r\bar{r}|V_{\rm QCD}|r\bar{r}\rangle = -C(r\bar{r} \to r\bar{r})\frac{\alpha_S}{r}$$

which is the term arising from $r\bar{r} \to r\bar{r}$




•Have 3 terms like $r\bar{r} \to r\bar{r}, b\bar{b} \to b\bar{b}, \ldots$ and 6 like $r\bar{r} \to g\bar{g}, r\bar{r} \to b\bar{b}, \ldots$

$$\langle V_{q\bar{q}} \rangle = -\frac{1}{3}\frac{\alpha_S}{r}[3 \times C(r\bar{r} \to r\bar{r}) + 6 \times C(r\bar{r} \to g\bar{g})] = -\frac{1}{3}\frac{\alpha_S}{r}\left[3 \times \frac{1}{3} + 6 \times \frac{1}{2}\right]$$

$$\Rightarrow \boxed{\langle V_{q\bar{q}} \rangle = -\frac{4}{3}\frac{\alpha_S}{r}}$$

**NEGATIVE ➡ ATTRACTIVE**

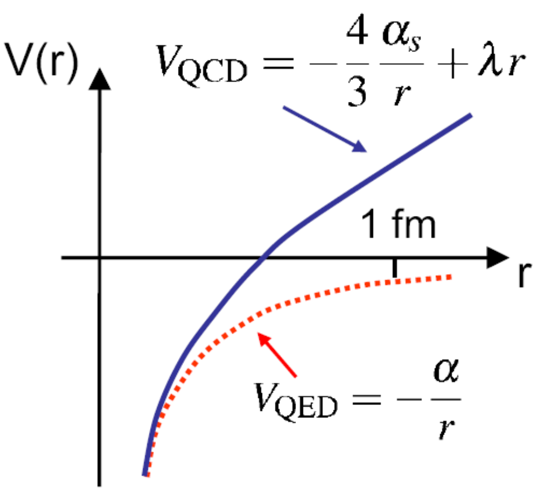
•The same calculation for a colour octet state, e.g. $r\bar{g}$ gives a positive repulsive potential: $C(r\bar{g} \to r\bar{g}) = -\frac{1}{6}$

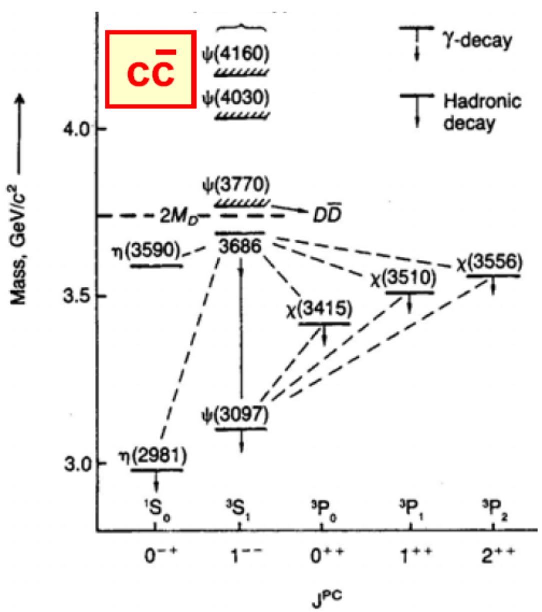
# KRATKO- & DUGO-DOSEŽNI DIO

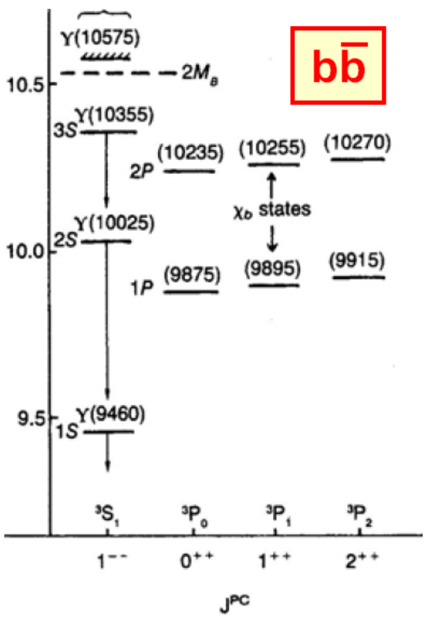
★ Combining the short-range QCD potential with the linear long-range term discussed previously:

$$V_{\text{QCD}} = -\frac{4}{3}\frac{\alpha_s}{r} + \lambda r$$

★ This potential is found to give a good description of the observed charmonium $(c\bar{c})$ and bottomonium $(b\bar{b})$ bound states.




**NOTE:**

- c, b are heavy quarks
- approx. non-relativistic
- orbit close together
- probe 1/r part of $V_{QCD}$

Agreement of data with prediction provides strong evidence that $V_{\text{QCD}}$ has the Expected form