IX. Dinamika okusa I

FIZIKA OKUSA kao OKUS FIZIKE

- KVARKOVSKI OKUSI - CKM MIJEŠANJE
- LEPTONSKI OKUSI - PMNS MIJEŠANJE

Konvencija predznaka za KVARKOVSKE OKUSE

◊ of a QUARK I_3, S, C, B, T has the same sign as its charge

$Q = -1/3$		$Q = 2/3$	
d	$I_3 = -1/2$	u	$I_3 = +1/2$
s	$S = -1$	c	$C = +1$
b	$b = -1$	t	$T = +1$

& konvencija za predznak OKUSA MEZONA

$\Rightarrow$ flavour carried by a charged meson

$\Diamond$ of a MESON – same sign as it's charge

examples

$S=1$ $K^0(d\bar{s})$ $K^+(u\bar{s})$ $D^0(c\bar{u})$ $D^+(c\bar{d})$ $C=1$

$B=1$ $B^0(d\bar{b})$ $B_c^+(c\bar{b})$ $D_s^+(c\bar{s})$

KVARKOVSKI OKUSI & CKM MIJEŠANJE

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

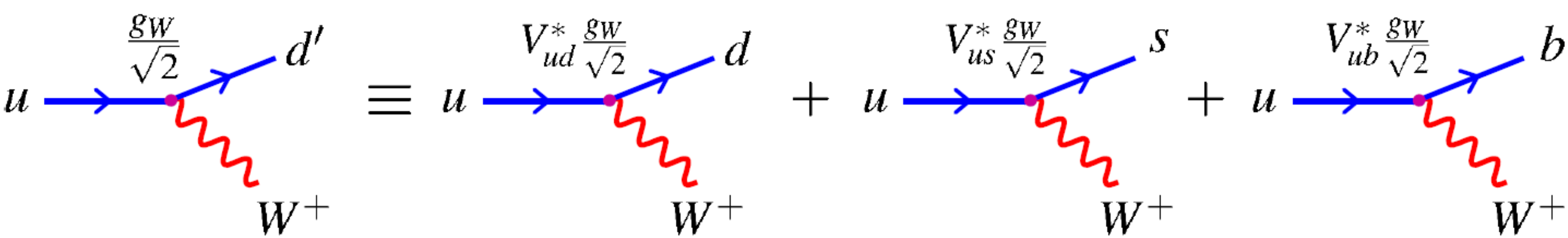
By convention CKM matri
defined as acting on
quarks with charge $-\frac{1}{3}e$

Weak eigenstates

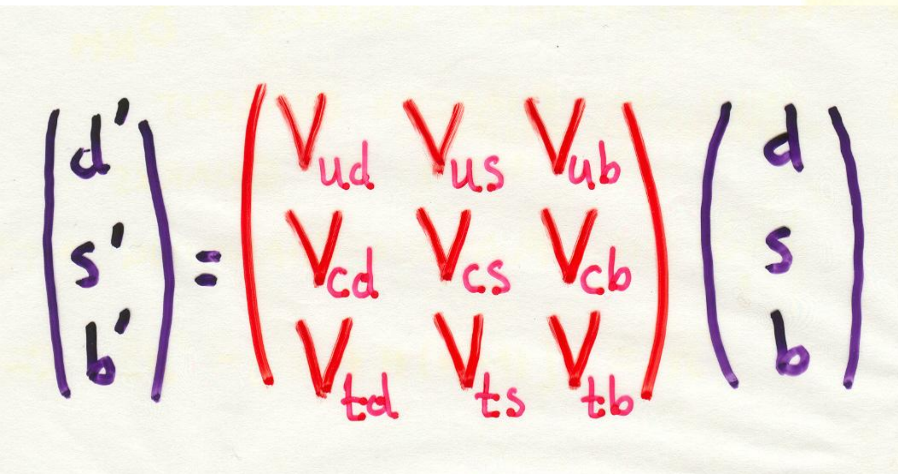
$\underbrace{\text{CKM Matrix}}_{\text{(Cabibbo, Kobayashi, Maskawa)}}$

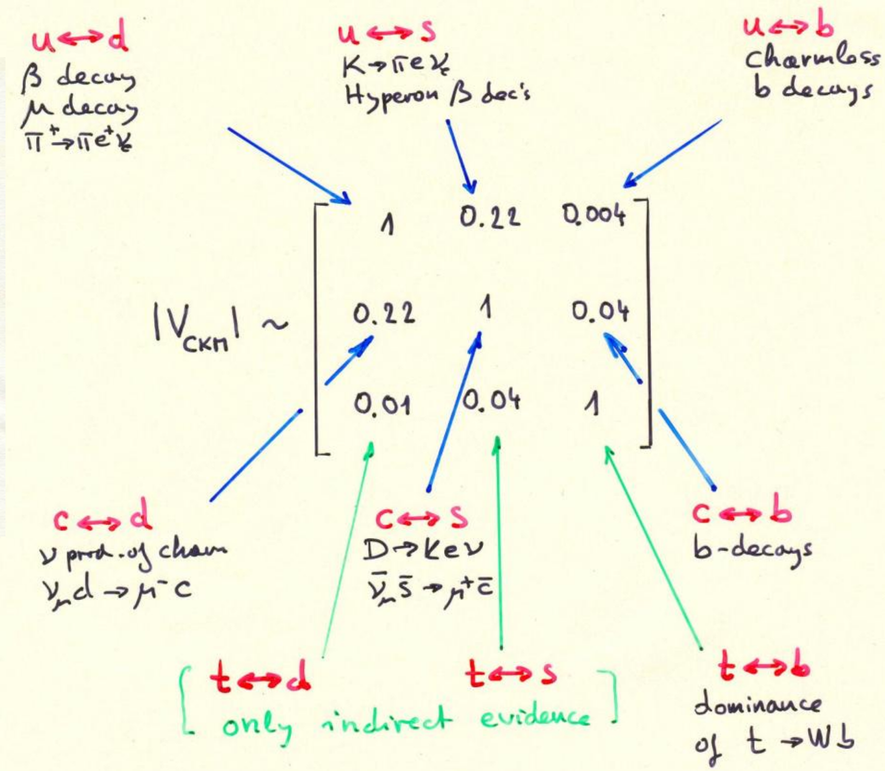
Mass Eigenstates

★ e.g. Weak eigenstate d' is produced in weak decay of an up quark:



Kvarkovsko miješanje i CKM-matrica (Cabibbo-Kobayashi-Maskawa)





$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.974 & 0.226 & 0.004 \\ 0.23 & 0.96 & 0.04 \\ ? & ? & ? \end{pmatrix}$$

Currently little direct experimental information on V_{td}, V_{ts}, V_{tb}

Assuming unitarity of CKM matrix, e.g. $|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1$ gives:

Cabibbo matrix

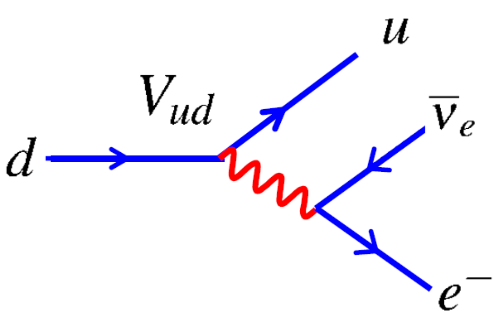
$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.974 & 0.226 & 0.004 \\ 0.23 & 0.96 & 0.04 \\ 0.01 & 0.04 & 0.999 \end{pmatrix}$$

Near diagonal – very different from PMNS

Slabi raspadi temeljnih fermiona -inačice raspada miona (FEČ §6.3)

❶ $|V_{ud}|$ from nuclear beta decay

$$\begin{pmatrix} \times & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$



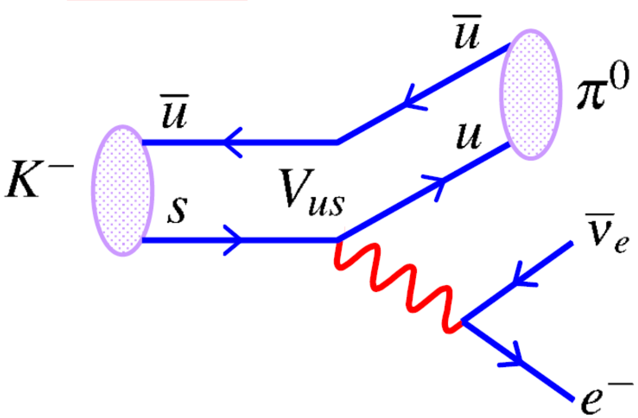
Super-allowed $0^+ \to 0^+$ beta decays are relatively free from theoretical uncertainties

$$\Gamma \propto |V_{ud}|^2$$

$$\boxed{|V_{ud}| = 0.97377 \pm 0.00027} \qquad (\approx \cos\theta_c)$$

❷ $|V_{us}|$ from semi-leptonic kaon decays

$$\begin{pmatrix} \cdot & \times & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$



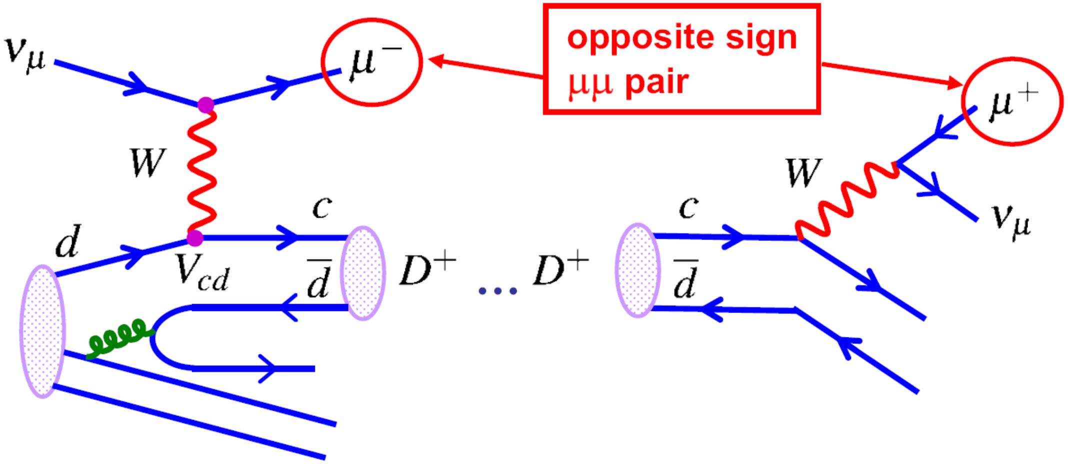
$$\Gamma \propto |V_{us}|^2$$

$$\boxed{|V_{us}| = 0.2257 \pm 0.0021} \qquad (\approx \sin\theta_c)$$

❸ $|V_{cd}|$ from neutrino scattering $\nu_\mu + N \to \mu^+\mu^- X$

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \times & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

Look for opposite charge di-muon events in ν_μ scattering from production and decay of a $D^+(c\bar{d})$ meson



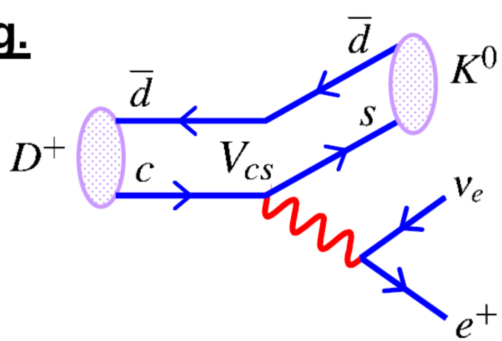
$$\text{Rate} \propto |V_{cd}|^2 \underbrace{\text{Br}(D^+ \to X\mu^+\nu_\mu)}_{\text{Measured in various collider experiments}}$$

⇨ $\boxed{|V_{cd}| = 0.230 \pm 0.011}$

❹ $|V_{cs}|$ from semi-leptonic charmed meson decays

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \times & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

e.g.



$$\Gamma \propto |V_{cs}|^2$$

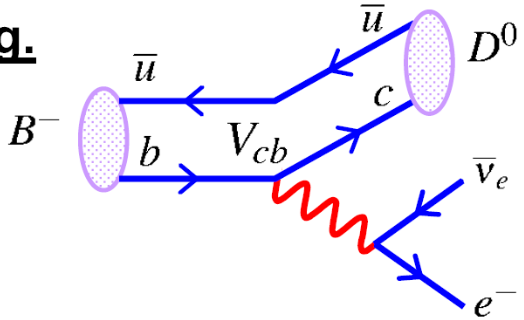
• Precision limited by theoretical uncertainties

$$|V_{cs}| = 0.957 \underbrace{\pm 0.017}_{\text{experimental error}} \underbrace{\pm 0.093}_{\text{theory uncertainty}}$$

❺ $|V_{cb}|$ **from semi-leptonic B hadron decays**

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \end{pmatrix}$$

e.g.



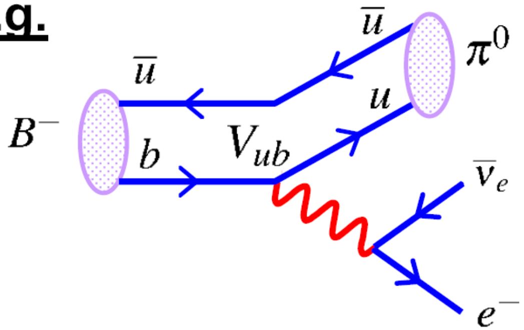
$$\Gamma \propto |V_{cb}|^2$$

$$\boxed{|V_{cb}| = 0.0416 \pm 0.0006}$$

❻ $|V_{ub}|$ **from semi-leptonic B hadron decays**

$$\begin{pmatrix} \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

e.g.



$$\Gamma \propto |V_{ub}|^2$$

$$\boxed{|V_{ub}| = 0.0043 \pm 0.0003}$$

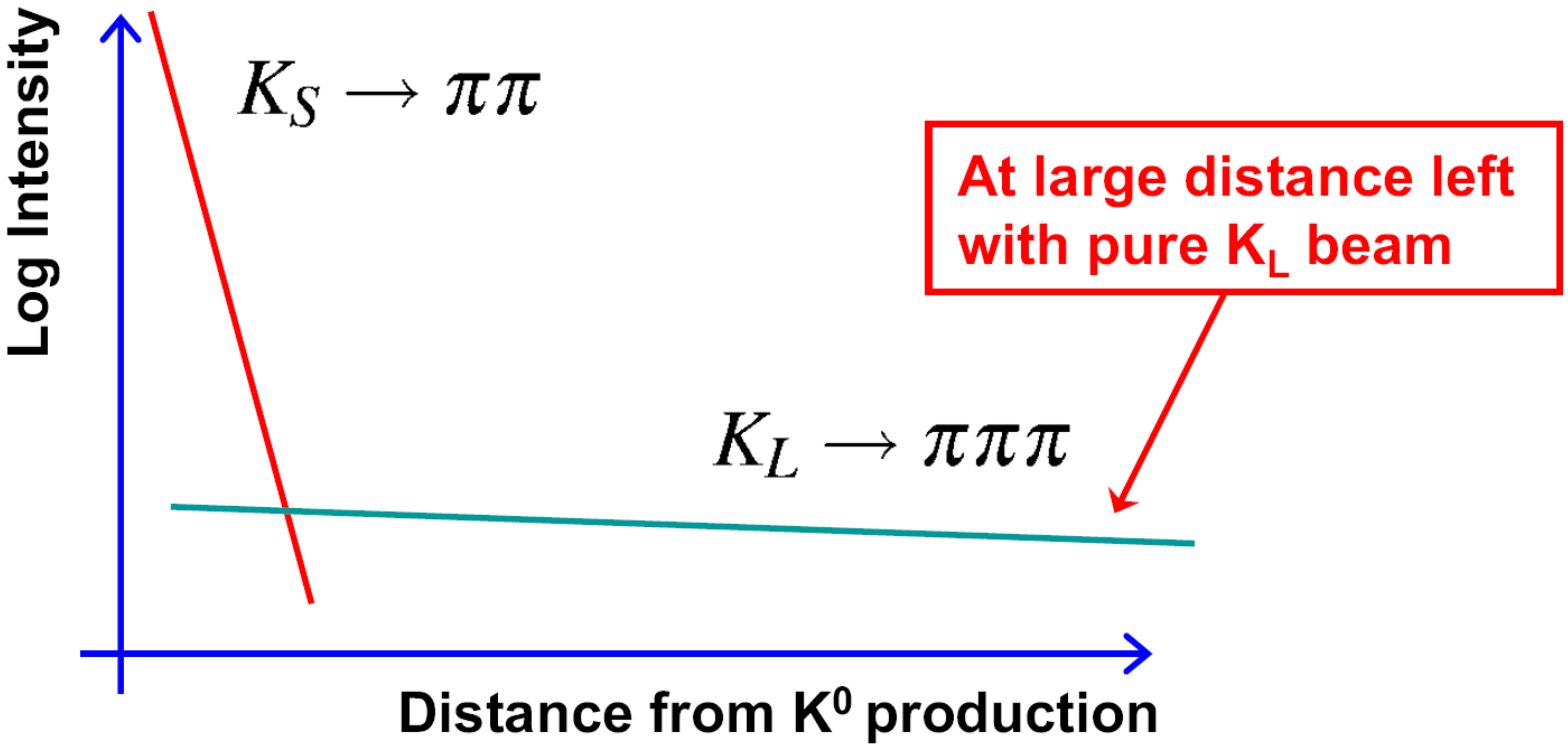
Uočavamo hijerarhijsku strukturu – dominanta je interakcija između kvarkova iste generacije

FIZIKALNA STANJA NEUTRALNIH KAONA

$$|K_S\rangle \simeq |K_1\rangle \equiv \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle)$$

$$|K_L\rangle \simeq |K_2\rangle \equiv \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle)$$

RASPAD KAONA U PIONE



CP NARUŠENJE U SUSTAVU NEUTRALNIH KAONA

$$
\begin{array}{llll}
K_L \rightarrow & \pi^+\pi^-\pi^0 & BR = 12.6\% & CP = -1 \\
\rightarrow & \pi^0\pi^0\pi^0 & BR = 19.6\% & CP = -1 \\
\rightarrow & \pi^+\pi^- & BR = 0.20\% & \boxed{CP = +1} \\
\rightarrow & \pi^0\pi^0 & BR = 0.08\% & \boxed{CP = +1}
\end{array}
$$

NEIZRAVNO I IZRAVNO (DIREKTNO) CP NARUŠENJE

$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[|K_2\rangle + \varepsilon|K_1\rangle\right]$$

$|K_1\rangle \rightarrow \pi\pi$ CP = +1

$|K_2\rangle \rightarrow \pi\pi\pi$ CP = -1

$|K_L\rangle = |K_2\rangle$ CP = -1

$\rightarrow \pi\pi\pi$ CP = -1

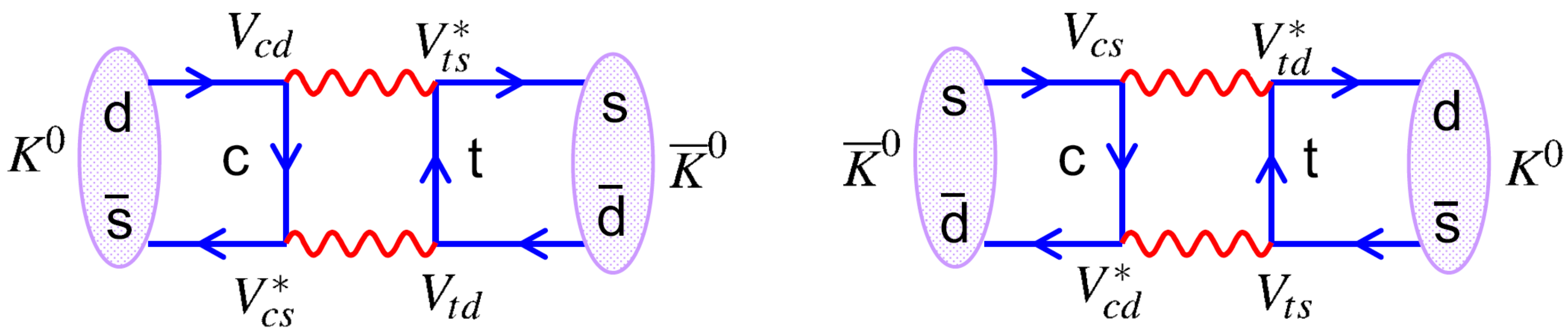
$\rightarrow \pi\pi$ CP = +1

Parameterised by ε'

$$\varepsilon'/\varepsilon = (1.7 \pm 0.3) \times 10^{-3}$$

NA48 (CERN)
KTeV (FermiLab)

CP PARAMETAR MIJEŠANJA



$$M_{fi} \propto A_{ct} V_{cd} V_{cs}^* V_{td} V_{ts}^* \qquad M'_{fi} \propto A_{ct} V_{cd}^* V_{cs} V_{td}^* V_{ts} = M_{fi}^*$$

Therefore difference in rates

$$\Gamma(K^0 \to \overline{K}^0) - \Gamma(\overline{K}^0 \to K^0) \propto M_{fi} - M_{fi}^* = 2\Im\{M_{fi}\}$$

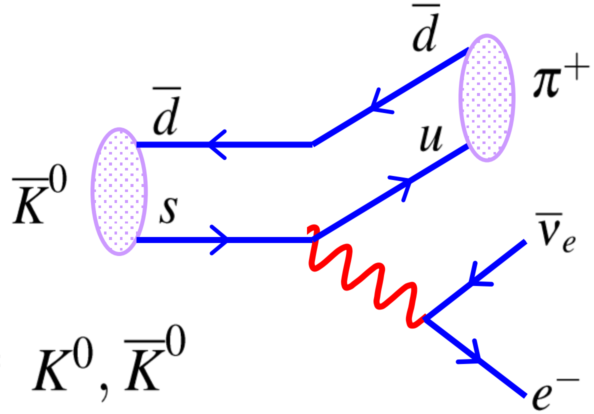
Hence the rates can only be different if the CKM matrix is imaginary

$$|\varepsilon| \propto \Im\{M_{fi}\}$$

$$|\varepsilon| \propto A_{ut}.\Im\{V_{ud}V_{us}^*V_{td}V_{ts}^*\} + A_{ct}.\Im\{V_{cd}V_{cs}^*V_{td}V_{ts}^*\} + A_{tt}.\Im\{V_{td}V_{ts}^*V_{td}V_{ts}^*\}$$

Shows that CP violation is related to the imaginary parts of the CKM matrix

Neutral Kaon Decays to Leptons



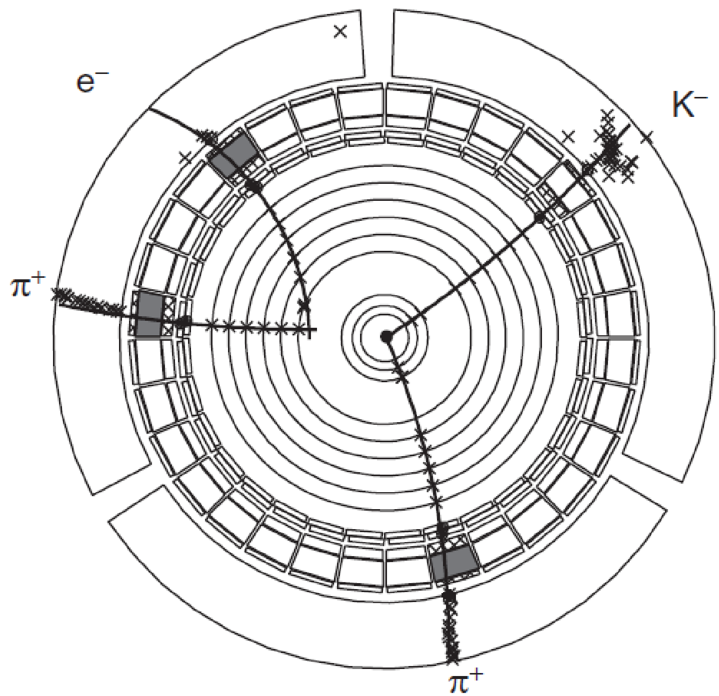
- Neutral kaons can also decay to leptons

$$\overline{K}^0 \to \pi^+ e^- \overline{\nu}_e \qquad \overline{K}^0 \to \pi^+ \mu^- \overline{\nu}_\mu$$
$$K^0 \to \pi^- e^+ \nu_e \qquad K^0 \to \pi^- \mu^+ \nu_\mu$$

- Note: the final states are not CP eigenstates which is why we express these decays in terms of $K^0, \overline{K}^0$
- Neutral kaons propagate as combined eigenstates of weak + strong interaction i.e. the K_S, K_L. The main decay modes/branching fractions are:

$$\begin{array}{llll} K_S & \to & \pi^+\pi^- & BR = 69.2\% \\ & \to & \pi^0\pi^0 & BR = 30.7\% \\ & \to & \pi^- e^+ \nu_e & BR = 0.03\% \\ & \to & \pi^+ e^- \overline{\nu}_e & BR = 0.03\% \\ & \to & \pi^- \mu^+ \nu_\mu & BR = 0.02\% \\ & \to & \pi^+ \mu^- \overline{\nu}_\mu & BR = 0.02\% \end{array} \qquad \begin{array}{llll} K_L & \to & \pi^+\pi^-\pi^0 & BR = 12.6\% \\ & \to & \pi^0\pi^0\pi^0 & BR = 19.6\% \\ & \to & \pi^- e^+ \nu_e & BR = 20.2\% \\ & \to & \pi^+ e^- \overline{\nu}_e & BR = 20.2\% \\ & \to & \pi^- \mu^+ \nu_\mu & BR = 13.5\% \\ & \to & \pi^+ \mu^- \overline{\nu}_\mu & BR = 13.5\% \end{array}$$

- Leptonic decays are more likely for the K-long because the three pion decay modes have a lower decay rate than the two pion modes of the K-short



An event in the CPLEAR detector where a K^0 is produced in $\bar{p}p \rightarrow K^-\pi^+K^0$ and decays as $\bar{K}^0 \rightarrow \pi^+e^-\bar{\nu}_e$. The grey boxes indicate signals from relativistic particles in the Čerenkov detectors. Courtesy of the CPLEAR Collaboration.

Strangeness Oscillations (neglecting CP violation)

- The "semi-leptonic" decay rate to $\pi^- e^+ \nu_e$ occurs from the K^0 state. Hence to calculate the expected decay rate, need to know the K^0 component of the wave-function. For example, for a beam which was initially K^0 we have (1)

$$|\psi(t)\rangle \;=\; \frac{1}{\sqrt{2}}(\theta_S(t)|K_S\rangle + \theta_L(t)|K_L\rangle)$$

- Writing K_S, K_L in terms of $K^0, \overline{K}^0$

$$\begin{aligned} |\psi(t)\rangle &= \tfrac{1}{2}\left[\theta_S(t)(|K^0\rangle - |\overline{K}^0\rangle) + \theta_L(t)(|K^0\rangle + |\overline{K}^0\rangle)\right] \\ &= \tfrac{1}{2}(\theta_S + \theta_L)|K^0\rangle + \tfrac{1}{2}(\theta_L - \theta_S)|\overline{K}^0\rangle \end{aligned}$$

- Because $\theta_S(t) \neq \theta_L(t)$ a state that was initially a K^0 evolves with time into a mixture of K^0 and $\overline{K}^0$ - "strangeness oscillations"

- The K^0 intensity (i.e. K^0 fraction):

$$\Gamma(K^0_{t=0} \to K^0) = |\langle K^0|\psi(t)\rangle|^2 = \tfrac{1}{4}|\theta_S + \theta_L|^2 \qquad (2)$$

- Similarly $$\Gamma(K^0_{t=0} \to \overline{K}^0) = |\langle \overline{K}^0|\psi(t)\rangle|^2 = \tfrac{1}{4}|\theta_S - \theta_L|^2 \qquad (3)$$

- Using the identity $|z_1 \pm z_2|^2 = |z_1|^2 + |z_2|^2 \pm 2\Re(z_1 z_2^*)$

$$\begin{aligned}
|\theta_S \pm \theta_L|^2 &= |e^{-(im_S+\frac{1}{2}\Gamma_S)t} \pm e^{-(im_L+\frac{1}{2}\Gamma_L)t}|^2 \\
&= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2\Re\{e^{-im_S t}e^{-\frac{1}{2}\Gamma_S t}.e^{+im_L t}e^{-\frac{1}{2}\Gamma_L t}\} \\
&= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\Re\{e^{-i(m_S-m_L)t}\} \\
&= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\cos(m_S-m_L)t \\
&= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\cos\Delta m t
\end{aligned}$$

- Oscillations between neutral kaon states with frequency given by the mass splitting $\Delta m = m(K_L) - m(K_S)$
- Reminiscent of neutrino oscillations ! Only this time we have decaying states.
- Using equations (2) and (3):

$$\Gamma(K^0_{t=0} \to K^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right] \quad (4)$$

$$\Gamma(K^0_{t=0} \to \overline{K}^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right] \quad (5)$$

- Experimentally we find: $\boxed{\tau(K_S) = 0.9 \times 10^{-10}\,\mathrm{s}}$ $\boxed{\tau(K_L) = 0.5 \times 10^{-7}\,\mathrm{s}}$

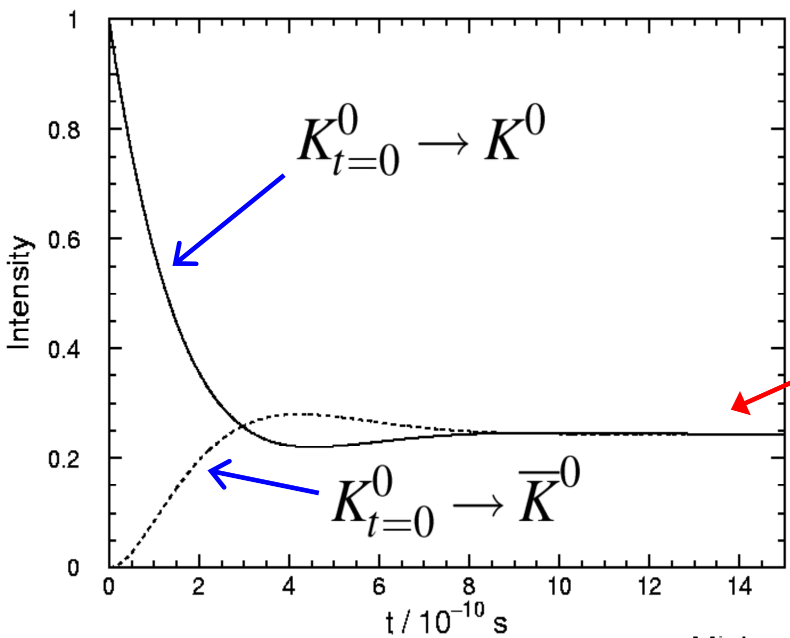
and $\boxed{\Delta} m = (3.506 \pm 0.006) \times 10^{-15}\,\mathrm{GeV}$

i.e. the K-long mass is greater than the K-short by 1 part in 10^{16}

- The mass difference corresponds to an oscillation period of

$$T_{osc} = \frac{2\pi\hbar}{\Delta m} \approx 1.2 \times 10^{-9}\,\mathrm{s}$$

- The oscillation period is relatively long compared to the K_S lifetime and consequently, do not observe very pronounced oscillations



$$\Gamma(K^0_{t=0} \to K^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

$$\Gamma(K^0_{t=0} \to \overline{K}^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

After a few K_S lifetimes, left with a pure K_L beam which is half K^0 and half $\overline{K}^0$

Narušenje CP pariteta, VREMENSKE MIKROOBRATIVOSTI

$\cancel{CP}$ 1964. Cronin & Fitch

na raspadima dugoživućih neutralnih kaona

- $K_L \not\rightarrow 2\pi$ ako je CP očuvano

  u pokusu se pojavljuje s grananjem $2 \cdot 10^{-3}$

- $K_L \rightarrow \pi^- \underline{e^+} \nu_e$ češći od CP-konjugiranog $K_L \rightarrow \pi^+ e^- \bar{\nu}_e$

  omogućuje apsolutnu definiciju pozitivnog naboja

  – razlikovanje materije i antimaterije !

CPT = I teorem potvrđen na točnost

$$\frac{m_{\bar{K}^0} - m_{K^0}}{m_{K^0}} \begin{array}{l} < 3.5 \cdot 10^{-18} \\ < 9 \cdot 10^{-19} \end{array}$$

FCNC u rijetkim raspadima arXiv:1501.04838

$$\mathcal{B}(B_s^0 \to \mu^+\mu^-)_{\mathrm{SM}} = (3.65 \pm 0.23) \times 10^{-9}$$
$$\mathcal{B}(B^0 \to \mu^+\mu^-)_{\mathrm{SM}} = (1.06 \pm 0.09) \times 10^{-10}$$

- **Prvi rezultati CMS i LHCb:**

$$\mathcal{B}(B_s^0 \to \mu^+\mu^-) = (2.8\,^{+0.7}_{-0.6}) \times 10^{-9}$$
$$\mathcal{B}(B^0 \to \mu^+\mu^-) = (3.9\,^{+1.6}_{-1.4}) \times 10^{-10}$$

Osjetljivost na “NP”

$$\mathcal{B}(K^+ \to \pi^+ \nu\overline{\nu})_{\mathrm{SM}} = (7.81 \pm 0.80) \times 10^{-11}$$
$$\mathcal{B}(K^0_{\mathrm{L}} \to \pi^0 \nu\overline{\nu})_{\mathrm{SM}} = (2.43 \pm 0.39) \times 10^{-11}$$

- **BNL E787/E949 mjere omjer grananja**

$$\mathcal{B}(K^+ \to \pi^+ \nu\overline{\nu}) = 1.73^{+1.15}_{-1.05} \times 10^{-10}$$

KVARKOVSKI OKUSI & CKM MIJEŠANJE

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

By convention CKM matri
defined as acting on
quarks with charge $-\frac{1}{3}e$

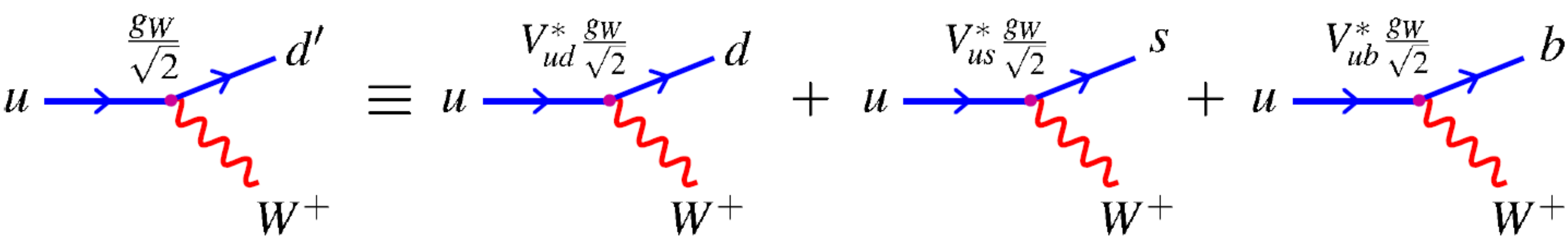
Weak eigenstates

CKM Matrix

Mass Eigenstates

(Cabibbo, Kobayashi, Maskawa)

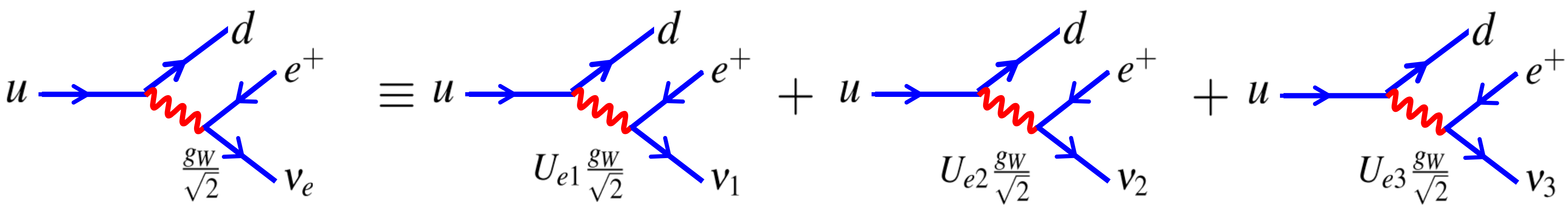
★ e.g. Weak eigenstate d' is produced in weak decay of an up quark:



Neutrinske oscilacije i mase neutrina

★ It is simple to extend this treatment to three generations of neutrinos.
★ In this case we have:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



★ The 3x3 Unitary matrix U is known as the Pontecorvo-Maki-Nakagawa-Sakata matrix, usually abbreviated PMNS

★ Note : has to be unitary to conserve probability

• Using $U^\dagger U = I \quad \Rightarrow \quad U^{-1} = U^\dagger = (U^*)^T$

gives
$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} U^*_{e1} & U^*_{\mu 1} & U^*_{\tau 1} \\ U^*_{e2} & U^*_{\mu 2} & U^*_{\tau 2} \\ U^*_{e3} & U^*_{\mu 3} & U^*_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$