VIII. PREDAVANJE

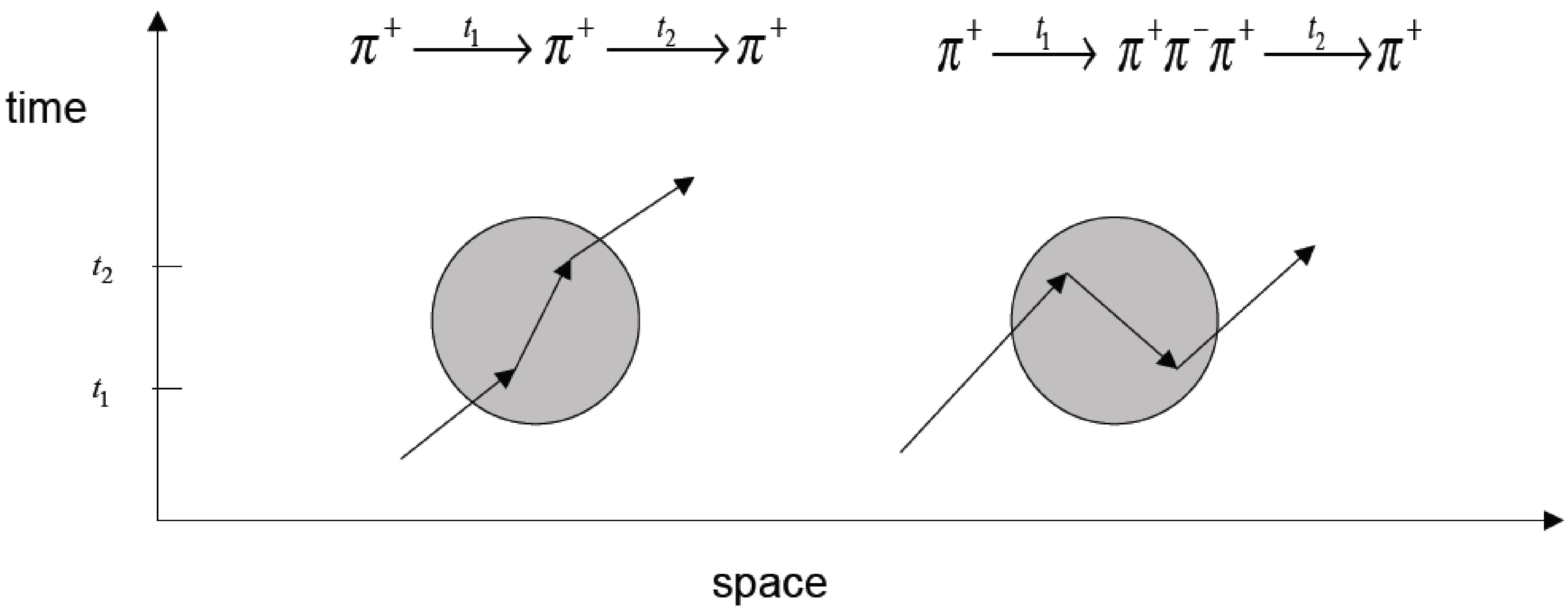
LAGRANGEOV OPIS & DVIJE POSLJEDICE VARIJACIJSKOG NAČELA:

PRIJELAZ NA RELATIVIST. RAČUN SMETNJE:
cilj - Fejnmanova pravila;
međukorak – virtualna st.

- JEDNADŽBE GIBANJA i NOETHERIČIN TEOREM:
  - za prost-vrem simetrije
  - za interne simetrije

DVA RAZLIČITA VREMENSKA UREĐENJA KOJA VODE NA ISTI OPAŽENI DOGAĐAJ

Two different time orderings giving same observable event :



ITERATIVNO DOBIVENA AMPLITUDA QM PRIJELAZA

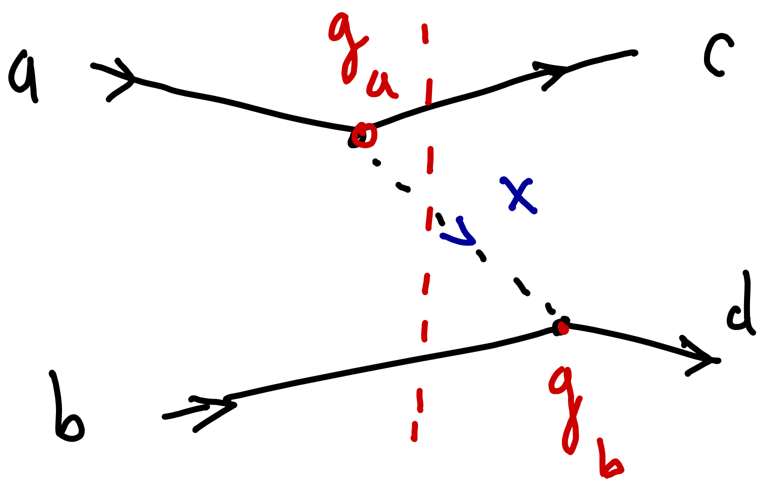
$$T_{fi} = \langle f | V | i \rangle + \sum_{n \neq i} \frac{\langle f | V | n \rangle \langle n | V | i \rangle}{E_i - E_n}$$

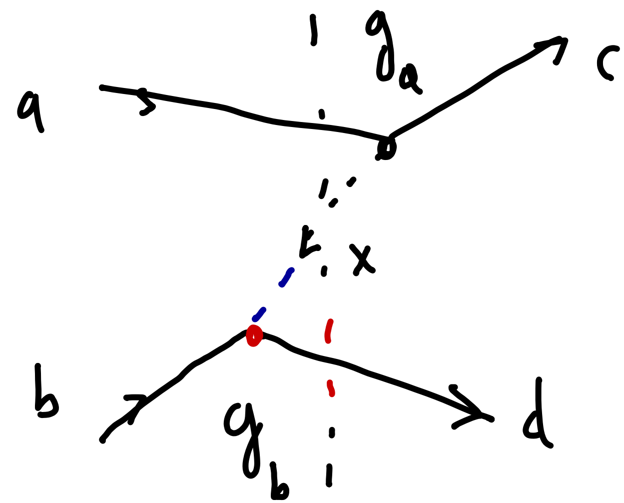
$$\langle f | V | i \rangle = \int d^4x \, \phi_f^*(x) V(x) \phi_i(x) \quad ; \quad \int \phi^* \phi \, dV = 1$$

Prijelaz na relativističko normiranje $\quad \int \psi^* \psi \, dV = 2E$

$$\phi \rightarrow \psi = \sqrt{2E}\,\phi \Rightarrow M_{fi} = \langle \psi_1 \psi_2 .. | V | .. \psi_{n-1} \psi_n \rangle = (2E_1 \cdots 2E_n)^{1/2} T_{fi}$$

PRIMJER a+b->c+d



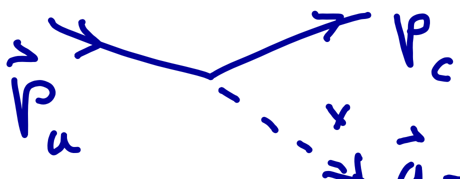


$$\sim \frac{g_a}{\sqrt{2E_x}} \qquad \frac{\langle d|V|x+b\rangle\langle c+x|V|a\rangle}{(E_a+E_b)-(E_c+E_x+E_b)} \; ; \; \frac{\langle c|V|a+x\rangle\langle d+x|V|b\rangle}{(E_a+E_b)-(E_a+E_d+E_x)}$$

$$\sim \frac{g_b}{\sqrt{2E_x}}$$

Uvedeno virtualno međustanje

$$M_{fi} = M_{fi}^{ab} + M_{fi}^{ba} = \frac{g_a g_b}{2E_x}\left(\frac{1}{E_a - E_c - E_x} + \frac{1}{E_b - E_c + E_x}\right)$$

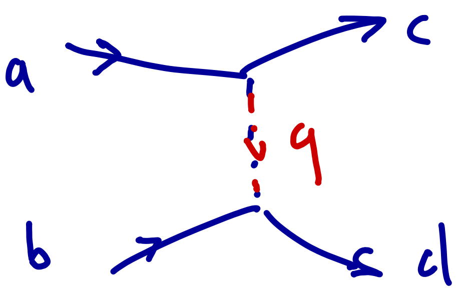


$\vec q = \vec p_a - \vec p_c$

$$= \frac{g_a g_b}{(E_a - E_c)^2 - E_x^2}$$

$E_x^2 = (\vec p_a - \vec p_c)^2 + m_x^2$

$q = p_a - p_c$



$q^2 \neq m_x^2$

$$= \frac{g_a g_b}{q^2 - m_x^2}$$

Primjeri virtualnosti prijenosa impulsa:

- Prostorna virtualnost za elastično raspršenje;
- Negativna virtualnost za anihilacijski proces.

EKSTREMALNOST (MINIMUM) FUNKCIJE DJELOVANJA

Formulacija
principa invarijantnosti u 3

- KLASIČNOJ FIZICI

  Lagrange-ov formalizam

  $$L(q, \dot{q}, t)$$

  - princip najmanjeg djelovanja ($S = \int_{t_1}^{t_2} L(q, \dot{q}, t)\, dt$)

    $$\delta S = 0 \Rightarrow \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0$$

    Euler-Lagrange j-dž.

  - princip simetrije

    $\delta L = 0$ za danu transformaciju simetrije

PRINCIP INVARIJANTNOSTI u TEORIJI POLJA i jedn. gibanja

POSLJEDICE VARIJACIJSKOG NAČELA

$$\delta S = 0 \quad ; \quad S = \int d^4x\, \mathcal{L}(\phi, \partial_\mu \phi)$$

◊ **Euler-Lagrange**-ove jedn. gib.

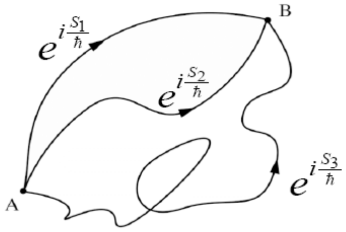
$$\phi(x) \to \phi'(x) = \phi(x) + \delta_0 \phi(x) \qquad (x^\mu \to x'^\mu = x^\mu + \delta x^\mu)$$

$$\frac{\partial \mathcal{L}}{\partial \phi} - \frac{\partial}{\partial x^\mu}\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\right) = 0$$

I. Jednadžbe gibanja

Euler Lagrange equs

$$S=\int_{t_1}^{t_2} L\,dt=\int \mathfrak{L}(\phi,\partial_\mu\phi)\,d^4x$$



Principle of least action :

0 (surface integral)

$$0=\delta S=\int d^4x\left\{\frac{\partial\mathfrak{L}}{\partial\phi}\delta\phi+\frac{\partial\mathfrak{L}}{\partial(\partial^\mu\phi)}\delta(\partial^\mu\phi)\right\}$$

$$=\int d^4x\left\{\frac{\partial\mathfrak{L}}{\partial\phi}\delta\phi-\partial_\mu\left(\frac{\partial\mathfrak{L}}{\partial(\partial^\mu\phi)}\right)\delta\phi+\partial_\mu\left(\frac{\partial\mathfrak{L}}{\partial(\partial^\mu\phi)}\delta\phi\right)\right\}$$

$$\Rightarrow\quad \boxed{\frac{\partial\mathfrak{L}}{\partial\phi}-\partial^\mu\frac{\partial\mathfrak{L}}{\partial(\partial^\mu\phi)}=0}$$

Euler Lagrange equation

II. Očuvana struja za unutrašnju simetriju

Noether current

$$\mathfrak{L} = \left(\partial_\mu \phi(x)\right)^\dagger \partial^\mu \phi(x) - m^2 \phi(x)^\dagger \phi(x)$$

Is invariant under $\phi(x) \rightarrow e^{i\alpha}\phi(x)$ …an Abelian (U(1)) gauge symmetry

$$0 = \delta\mathfrak{L} = \frac{\partial\mathfrak{L}}{\partial\phi}\underbrace{\delta\phi}_{i\alpha\phi} + \frac{\partial\mathfrak{L}}{\partial(\partial^\mu\phi)}\underbrace{\delta(\partial^\mu\phi)}_{i\alpha\partial_\mu\phi} + (\phi \leftrightarrow \phi^\dagger)$$

0 (Euler lagrange eqs.)

$$= i\alpha\left[\frac{\partial\mathfrak{L}}{\partial\phi} - \partial^\mu\left(\frac{\partial\mathfrak{L}}{\partial(\partial^\mu\phi)}\right)\right]\phi + i\alpha\partial^\mu\left(\frac{\partial\mathfrak{L}}{\partial(\partial^\mu\phi)}\phi\right) - (\phi \leftrightarrow \phi^\dagger)$$

$$\Rightarrow \quad \boxed{\partial^\mu j_\mu = 0, \quad j_\mu = \frac{ie}{2}\left(\frac{\partial\mathfrak{L}}{\partial(\partial^\mu\phi)}\phi - \frac{\partial\mathfrak{L}}{\partial(\partial^\mu\phi^\dagger)}\phi^\dagger\right)}$$

Noether current