Energija čestice (u ovisnosti o valnom vektoru) Vježbe 8 15.5.2019.

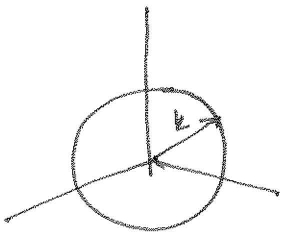
$$\boxed{\mathcal{E}(k) = \frac{\hbar^2 k^2}{2m}}$$ — Slobodni fermionski plin (elektroni)

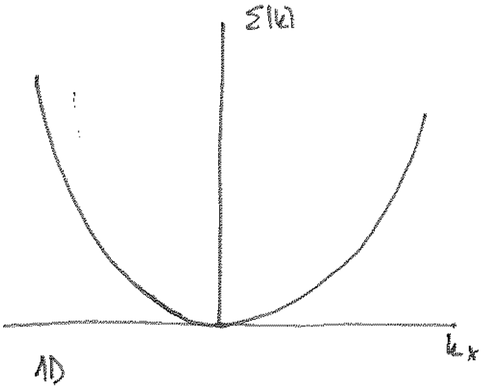
1D, 2D, 3D

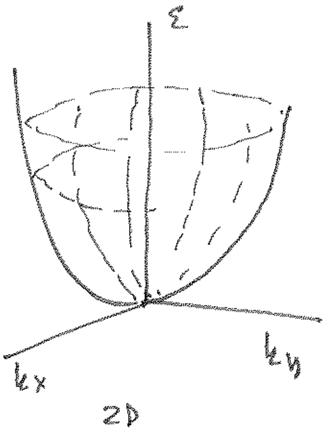
$\hbar k = p$ → impuls

$$\mathcal{E}(p) = \frac{p^2}{2m}$$

$$\boxed{k^2 = k_x^2 + k_y^2 + k_z^2}$$







slike za kontinuirani k

GUSTOĆA STANJA – broj stanja po jediničnoj energiji

$$\boxed{g(\mathcal{E}) = \sum_{k,s} \delta(\mathcal{E} - \mathcal{E}(k))}$$

spin

I $$\int f(x)\,\delta(z(x))\,dx = \sum_i \frac{f(x_i)}{\left|\frac{\partial z(x)}{\partial x}\Big|_{x=x_i}\right|}$$

x_i ... nultočke $z(x)$ funkcije

II $$\sum_k \to \frac{V}{(2\pi)^d}\int d^d k$$ tako da:

1D → $\frac{V}{2\pi}\int dk$

2D → $\frac{V}{(2\pi)^2}\int 2\pi k\,dk$

3D → $\frac{V}{(2\pi)^3}\int 4\pi k^2\,dk$

izotropan slučaj

Tako idemo proraći 3D gustoću stanja

$$g(\varepsilon) = \sum_{k,\sigma} \delta(\varepsilon - \varepsilon_k) = \frac{V}{(2\pi)^3} \cdot 2 \int_0^\infty 4\pi k^2 \, dk \, \delta(\varepsilon - \varepsilon_k)$$

$$\sum_\sigma \rightarrow (2s+1) = 2$$

$s = 1/2$ (fermioni)

$$= \frac{V}{(2\pi)^3} 2 \cdot 4\pi \int_0^\infty \underbrace{k^2}_{f(k)} \underbrace{\delta(\varepsilon - \varepsilon_k))}_{\delta(z(k))} dk$$

$$z(k) = \varepsilon - \varepsilon_k) = \varepsilon - \frac{\hbar^2 k^2}{2m} = \varepsilon - ck^2$$

I Nultočke od $z(k)$

$$z(k) = 0 = \varepsilon - ck^2 = 0$$

$$\varepsilon = ck^2 \rightarrow \boxed{k_0 = \pm\sqrt{\frac{\varepsilon}{c}}}$$

kako k ide od 0 do ∞ samo $k>0$ dolaze

II $\frac{\partial z(k)}{\partial k} = \frac{\partial}{\partial k}(\varepsilon - ck^2) = -2ck$

Konačno:

$$\frac{V}{\cancel{8\pi^3}} \cancel{8\pi} \cdot \frac{f(k_0)}{\left| \frac{\partial z(k)}{\partial k}\Big|_{k=k_0} \right|} = \frac{V}{\pi^2} \frac{k_0^2}{2ck_0} = \frac{V}{2c\pi^2} k_0$$

$$= \frac{V}{2c\pi^2}\sqrt{\frac{\varepsilon}{c}} = \frac{V}{2\pi^2} \frac{2m}{\hbar^2} \sqrt{\frac{2m\varepsilon}{\hbar^2}}$$

$$\boxed{g(\varepsilon) = \frac{Vm}{\pi^2 \hbar^3} \sqrt{2m\varepsilon}}$$

$$g(\varepsilon) \sim \sqrt{\varepsilon}$$

$\boxed{2D}$, Slobodni plin

$$\varepsilon_k = \frac{\hbar^2}{2m}k^2 = ck^2$$

$$g(\varepsilon) = \sum_{k\sigma} \delta(\varepsilon - \varepsilon_k) = \frac{V}{(2\pi)^2} \overset{\nearrow \sigma}{2} \cdot \int_0^\infty 2\pi\, k\, dk\, \delta(\varepsilon - \varepsilon(k))$$

$$= \frac{V}{4\pi^2} \cdot 4\pi \underbrace{\int_0^\infty k\, \delta(\varepsilon - \varepsilon_k)\, dk}_{\int_0^\infty k\, \delta(\varepsilon - ck^2)\, dk}$$

I $\quad z(k) = \varepsilon - ck^2 = 0 \quad \Longrightarrow \quad \boxed{k = \pm\sqrt{\frac{\varepsilon}{c}}}$

II $\quad \frac{\partial z}{\partial k} = -2ck$

Sve zajedno:

$$\frac{V}{\pi} \cdot \frac{f(k_0)}{\left| \frac{\partial z}{\partial k}\Big|_{k=k_0} \right|} = \frac{V}{\pi} \frac{k_0}{2ck_0} = \frac{V}{\pi 2c} = \frac{V}{\pi \cancel{2}} \frac{\cancel{2}m}{\hbar^2}$$

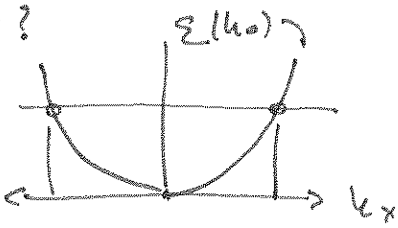
$$\longrightarrow \boxed{g(\varepsilon) = \frac{V}{\pi \hbar^2} m} = \text{Const.}$$

Konačno 1D: Uzimamo obje multočke

$$\varepsilon_k = \frac{\hbar^2 k^2}{2m} = ck^2$$

$$g(\varepsilon) = \sum_{k\sigma} \delta(\varepsilon - \varepsilon_k) \Longrightarrow \frac{N}{2\pi} \cdot 2^{\sigma} \int_{-\infty}^{\infty} 1\, dk\, \delta(\varepsilon - \varepsilon_k)$$

?



$$= \frac{V}{\pi} \int_{-\infty}^{\infty} 1\, \delta(\varepsilon - ck^2)\, dk$$

I multočke $\quad z(k) = \varepsilon - ck^2 \longrightarrow \boxed{k_0 = \pm\sqrt{\frac{\varepsilon}{c}}}$

2 multočke!

II $\frac{\partial z(k)}{\partial k} = -2ck$

$$\frac{V}{2\pi} \sum_i \frac{f(k_i)}{\left|\frac{dz}{\partial k}\Big|_{k=k_i}\right|} \quad f(k_i) \rightarrow 1$$

$$\frac{V}{2\pi}\left(\frac{1}{|-2ck_1|} + \frac{1}{|-2ck_2|}\right) = \frac{2}{2\pi} V \frac{1}{2c|k_0|} \cdot 2^{\sigma} \qquad c = \frac{\hbar^2}{2m}$$

$$= \frac{V}{\pi} \frac{1}{2c\sqrt{\frac{\varepsilon}{c}}} = \frac{V}{\pi} \frac{1}{\sqrt{\varepsilon c}} = \frac{V}{\pi} \frac{\sqrt{2m}}{\sqrt{\hbar^2 \varepsilon}} = \frac{V}{\pi} \frac{1}{\hbar} \frac{2m}{\sqrt{2m\varepsilon}}$$

$$g(\varepsilon) \sim \frac{1}{\sqrt{\varepsilon}}$$

Zadana je energija elektrona $E = E_0 + \frac{\hbar^2 k^2}{2m}$

Kolika je gustoća stanja?

$$g(E) = \frac{\lambda V}{(2\pi)^3} \int_0^\infty \delta(E - E(k))\, 4\pi k^2 \, dk$$

$$= 4\pi \frac{2V}{8\pi^3} \int_0^\infty \delta(\underbrace{E - E_0 - \frac{\hbar^2 k^2}{2m}}_{z}) k^2 \, dk$$

$z = 0$

$$E - E_0 = \frac{\hbar^2 k^2}{2m} \qquad \left| \frac{\partial z}{\partial k} \right| = \left| -\frac{2\hbar^2 k}{2m} \right| = \frac{\hbar^2 k}{m}$$

$$k_0 = \frac{1}{\hbar} \sqrt{2m(E - E_0)}$$

$$g(E) = \frac{V}{\pi^2} \frac{k_0^{\cancel{2}}}{\frac{\hbar^2 \cancel{k_0}}{m}} = \frac{V}{\pi^2} \frac{m}{\hbar^2} \frac{\sqrt{2m(E - E_0)}}{\hbar}$$

$$\boxed{g(E) = \frac{Vm}{\pi^2 \hbar^3} \sqrt{2m(E - E_0)}}$$