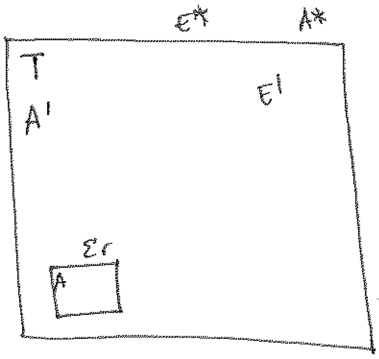
KANONSKA RASPODJELA

4. VJEŽBE STATISTIČKA
27.3.2019.



Ogroman sustav na temperaturi T
↑
toplinski
rezervoar / spremnik

• ako je sustav ocean, umočimo li noge, one će se ohladiti, ali ocean to neće baš osjetiti.

Vjerojatnost da se mali podsustav A nađe u stanju r je proporcionalna br. stanja dostupnih A* za znani A u stanju r.

$E' = E^* - \varepsilon_r$

$\Omega'(E^* - \varepsilon_r)$ → br. stanja dostupnih A' za A s energijom ε_r

$\Rightarrow P_r \propto \Omega'(E^* - \varepsilon_r)$, $\varepsilon_r \ll E^*$

...

$$P_r \sim e^{-\beta \varepsilon_r}$$ KANONSKA RASPODJELA

$P_r = C e^{-\beta \varepsilon_r}$
↑
konst. (ne ovisi o r)

Suma svih vjerojatnosti mora biti 1

$\sum_r P_r = 1$ (u nekom stanju MORA biti)

$C \sum_r e^{-\beta \varepsilon_r} = 1 \Rightarrow C = \frac{1}{\left(\sum_r e^{-\beta \varepsilon_r}\right)}$, $\sum_r e^{-\beta \varepsilon_r} = Z$ → PARTICIJSKA FUNKCIJA

$$P_r = \frac{e^{-\beta \varepsilon_r}}{Z}$$ $\beta = \frac{1}{kT}$

SREDNJA VRIJEDNOST NEKE VELIČINE:

y → neka veličina koja poprima vrijednost y_r u stanju r sustava A

$$\bar{y} \equiv \sum_r P_r y_r = \frac{\sum e^{-\beta \varepsilon_r} y_r}{\sum_r e^{-\beta \varepsilon_r}}$$ sumacija po svim stanjima sustava.

① N atoma s magnetskim momentom stavimo u vanjsko polje.

Svaki atom ima spin $\frac{1}{2}\hbar$, te pripadni magnetski moment μ_0.

$E = -\vec{\mu} \cdot \vec{B} = -g\mu_B B z m$, $m \in [-1/2, 1/2]$ $\quad$ $g = 2$ žiromag. faktor

$\hookrightarrow = \pm \mu_0 B$

μ_B - Bohrov magneton

$\mu_B = \frac{e\hbar}{2m_e}$ elektronski intrinzični spinski magnetni moment

$B \uparrow \quad \uparrow \quad E_\uparrow = -\mu_0 B$,

$B \uparrow \quad \downarrow \quad E_\downarrow = \mu_0 B$

$$P_\uparrow = C e^{(-)\beta E_\uparrow} = C e^{\beta \mu_0 B}$$

$$P_\downarrow = C e^{-\beta E_\downarrow} = C e^{-\beta \mu_0 B}$$

imamo samo 2 stanja

$$P_\uparrow + P_\downarrow = 1 = C\left(e^{\beta\mu_0 B} + e^{-\beta\mu_0 B}\right)$$

$$C = \frac{1}{e^{\beta\mu_0 B} + e^{-\beta\mu_0 B}}$$

stanje $E_\uparrow \rightarrow \mu \uparrow\uparrow B$, $E_\downarrow \rightarrow \mu \uparrow\downarrow B$

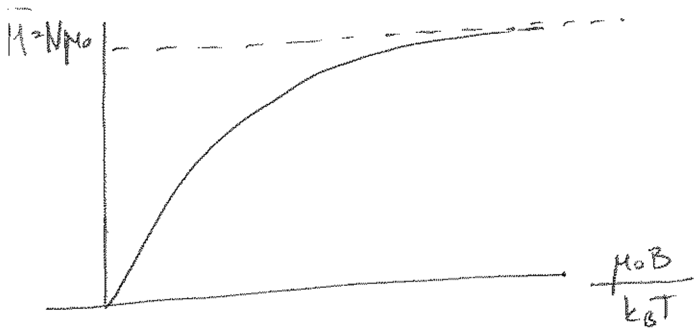
$$\bar{\mu} = \frac{\sum_r \mu_r e^{-\beta E_r}}{\sum_r e^{-\beta E_r}} = \frac{\mu_0 e^{\beta\mu_0 B} - \mu_0 e^{-\beta\mu_0 B}}{e^{\beta\mu_0 B} + e^{-\beta\mu_0 B}} \quad /\cdot 2$$

$$\boxed{\bar{\mu} = \mu_0 \tanh \beta\mu_0 B}$$

atoma u vanjskom polju

Magnetski moment po jedinićnom volumenu:

$$\overline{M} = N\overline{\mu} = N\mu_0 \, tgh \, \beta\mu_0 B$$



Imamo 2 limesa

I $\quad T \to 0 \quad \to \quad \frac{\mu_0 B}{k_B T} \to \infty = X$

$$tgh \, X = \frac{e^X - e^{-X}}{e^X + e^{-X}} \qquad e^{-X} \to 0 \qquad \lim_{X \to \infty} tgh \, X = 1$$

$$\overline{M} = N\mu_0$$

II $\quad T \to \infty \qquad X = \frac{\mu_0 B}{k_B T} \to 0$

$$tgh \, x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{1 + x + \frac{x^2}{2} \ldots - 1 + x - \frac{x^2}{2} \ldots}{1 + x + \frac{x^2}{2} + \ldots + 1 - x + \frac{x^2}{2} \ldots} = \frac{2x}{2 + \frac{x^2}{2}} \simeq x$$

$$\overline{M} = N\mu_0 \frac{\mu_0 B}{k_B T} = \frac{N\mu_0^2 B}{k_B} \frac{1}{T} \longrightarrow \overline{M} = \chi B$$

$$\chi(T) = \frac{N\mu_0^2}{k_B} \frac{1}{T} = \frac{C}{T}$$

$\to$ Curiev zakon za paramagnetike

TOPLINSKI KAPACITET :

$$E_\uparrow = -\mu_0 B = -E_0 \quad , \quad E_\downarrow = \mu_0 B = E_0$$

$$\bar{E} = \frac{-E_0 e^{\beta E_0} + E_0 e^{-\beta E_0}}{e^{+\beta E_0} + e^{-\beta E_0}} = \boxed{-E_0 \operatorname{tgh} \beta E_0}$$

$$C_V = \frac{\partial \bar{E}}{\partial T} = \frac{\partial \beta}{\partial T} \frac{\partial \bar{E}}{\partial \beta} \quad , \quad \frac{\partial \beta}{\partial T} = \frac{\partial}{\partial T} \frac{1}{k_B T} = -\frac{1}{k_B T^2} \frac{\cdot k_B}{\cdot k_B} = -k_B \beta^2$$

$$= -k_B \beta^2 \frac{\partial \bar{E}}{\partial \beta}$$

$$= k_B E_0 \beta^2 \underbrace{\frac{\partial}{\partial \beta} \operatorname{tgh} \beta E_0}_{\frac{E_0}{\operatorname{ch}^2 \beta E_0}}$$

$$\boxed{C_V = \frac{k_B E_0^2 \beta^2}{\operatorname{ch}^2 \beta E_0}}$$

I $\quad T \to 0 \qquad \beta E_0 \to \infty \equiv x$

$$\frac{1}{\operatorname{ch}^2 x} = \frac{4}{(e^x + e^{-x})^2} = 4 e^{-2x} \qquad (e^{-x} \to 0)$$

$$C_V = k_B E_0^2 \beta^2 4 e^{-2\beta E_0} = \frac{4 k_B E_0^2}{k_B^2 T^2} e^{-\frac{2E_0}{k_B T}}$$

II $\quad T \to \infty \qquad \beta E_0 \to 0 \equiv x \to 0$

$$\frac{1}{\operatorname{ch}^2 x} = \frac{4}{\left(1 + x + \frac{x^2}{2} + 1 - x + \frac{x^2}{2}\right)^2} = 1 \qquad \left(\frac{x^2}{2} \to 0\right)$$

$$C_V = \frac{k_B E_0^2}{k_B^2 T^2} \sim \frac{1}{T}$$