$$\boxed{S(U,V,N) = a(UVN)^{1/3}}$$

N = konst.

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3. VJEŽBE STATISTIČKA

① jednadžba stanja ? $\quad dU = TdS - pdV$

$$\rightarrow dS = \underbrace{\left(\frac{\partial S}{\partial U}\right)}_{\frac{1}{T}} dU + \underbrace{\left(\frac{\partial S}{\partial V}\right)}_{p/T} dV$$

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right) = \frac{a(VN)^{1/3}}{3} U^{-2/3} \cdot \frac{U}{U} \rightarrow \boxed{\frac{S}{3U} = \frac{1}{T}}$$

isto tako

$$\boxed{\frac{p}{T} = \left(\frac{\partial S}{\partial V}\right) = \frac{1}{3}\frac{S}{V}}$$

iskombiniramo

$$p \cdot \frac{S}{3U} = \frac{1}{3}\frac{S}{V} \rightarrow \boxed{U = pV}$$

$$\frac{1}{T} = \frac{a}{3}\frac{(VN)^{1/3}}{U^{2/3}} = \frac{a}{3}\frac{(VN)^{1/3}}{(pV)^{2/3}} \Big/^3$$

$$\boxed{\frac{1}{T^3} = \frac{a^3}{27}\frac{VN}{p^2V^2}}$$ Jednadžba stanja!

$$\boxed{p^2V = \frac{T^3a^3}{27}N} = mT^3 \qquad m = \frac{a^3N}{27}$$

Tražim $U(V,T) \longrightarrow$

$$\frac{1}{T^3} = \frac{a^3}{27}\frac{VN}{U^2} \qquad U^2 \rightarrow (pV)^2$$

$$U(V,T) = \sqrt{\frac{VNT^3a^3}{27}} \longrightarrow U(V,T) = \sqrt{mVT^3}$$

$S(V,T)$?

$$S(V,T) = \frac{3U}{T} = \sqrt{\frac{\cancel{9}}{\cancel{T^2}}\frac{VNT^{\cancel{3}}a^3}{\cancel{27}\cdot 3}} = \sqrt{\frac{a^3VNT}{3}} \rightarrow S(V,T) = 3\sqrt{mVT}$$

Jedn. adijabate i izoterme?

$$dS = \left(\frac{\partial S}{\partial V}\right)_T dV + \left(\frac{\partial S}{\partial T}\right)_V dT = 0 \rightarrow$$ za adijabatski proces

$$\frac{1}{2}\sqrt{\frac{a^3NT}{3V}} \qquad \frac{1}{2}\sqrt{\frac{a^3VN}{3T}}$$

2 i 3 isto makneno

$$\frac{1}{2}\sqrt{\frac{a^3NT}{3V}}\,dV + \frac{1}{2}\sqrt{\frac{a^3VN}{3T}}\,dT = 0 \qquad \Big/ \cdot \frac{1}{\sqrt{VT}} \cdot 2\sqrt{3} : a^3N$$

$$\frac{dV}{V} + \frac{dT}{T} = 0 \qquad \Big/ \int$$

$$\ln V + \ln T = \text{const}$$

$$\ln VT = \text{Const} \quad / e^{\wedge} \longrightarrow \boxed{VT = \text{Const}}$$

$\boxed{\text{p-V dijagram}}$

$$T^3 = \frac{27}{a^3}\frac{p^2V}{N} \longrightarrow V \cdot \frac{3}{a}\frac{p^{2/3}V^{1/3}}{N^{1/3}} = \text{Const}$$

$$p^{2/3}V^{4/3} = \text{const} \quad \Big/^{3/2} \longrightarrow \underline{pV^2 = \text{const}} \quad \text{ADIJABATA}$$

dok je $\underline{p^2V = \text{const}}$ izoterma

$$p^2V = nT^3 \qquad T = \text{const}$$

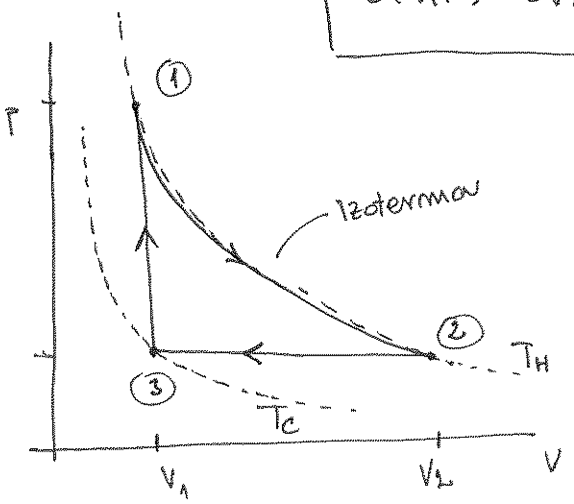
$$p^2V = \text{Const}.$$

(2) $S(U,V,N) = a(U,V,N)^{1/3}$

$m = \frac{a^3 N}{27}$

$$\boxed{\begin{aligned} U(V,T) &= \sqrt{mVT^3} \\ S(V,T) &= 3\sqrt{mVT} \end{aligned}}$$

$$\boxed{p^2 V = mT^3}$$



$$\boxed{dU = \delta W + \delta Q}$$

Promjena energije duž svakog koraka:

$$dU = \left(\frac{\partial U}{\partial V}\right)_T dV + \left(\frac{\partial U}{\partial T}\right)_V dT$$

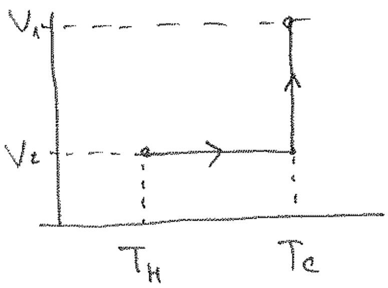
$$\left(\frac{\partial U}{\partial V}\right)_T = \frac{1}{2}\sqrt{\frac{mT^3}{V}}$$

$$\left(\frac{\partial U}{\partial T}\right)_V = \frac{3}{2}\sqrt{mVT}$$

$$dU_{1\to2} = \left(\frac{\partial U}{\partial V}\right)_T dV + \cancel{dT} \quad \text{izoterma}$$

$$\Delta U_{1\to2} = \int_{V_1}^{V_2} \left(\frac{\partial U}{\partial V}\right)_T dV = \underline{\underline{\sqrt{mT_H^3}\left(\sqrt{V_2} - \sqrt{V_1}\right)}}$$

$dU_{2\to3}$ → tu idemo iz T_H & V_2 ⟶ T_C & V_1



$$\Delta U_{2\to 3} = \int_{T_H}^{T_c} \cancel{\left(\frac{\partial U}{\partial V}\right)_T dV} \Bigg|_{V=V_2=\text{Const}} + \int_{T_H}^{T_c} \left(\frac{\partial U}{\partial T}\right)_V dT \Bigg|_{V=V_2=\text{Const}}$$

$$+ \int_{V_2}^{V_1} \left(\frac{\partial U}{\partial V}\right)_T dV \Bigg|_{T=T_c=\text{Const}} + \int_{V_2}^{V_1} \cancel{\left(\frac{\partial U}{\partial T}\right)_V dT} \Bigg|_{T=T_c=\text{Const}}$$

$$= \int_{T_H}^{T_c} \frac{3}{2}\sqrt{mVT}\, dT \Bigg|_{V=V_2} + \int_{V_2}^{V_1} \frac{1}{2}\sqrt{\frac{mT^3}{V}}\, dV \Bigg|_{T=T_c}$$

$$= \underline{\sqrt{mV_2}\left(T_c^{3/2} - T_H^{3/2}\right) + \sqrt{mT_c^3}\left(\sqrt{V_1} - \sqrt{V_2}\right)}$$

$$\Delta U_{3\to 1} = \int_{T_c}^{T_H} \left(\frac{\partial U}{\partial T}\right)_V dT \qquad V = \text{Const} = V_1$$

$$= \underline{\sqrt{mV_1}\left(T_H^{3/2} - T_c^{3/2}\right)}$$

$$\sum \Delta U = \sqrt{mT_H^3}\left(\sqrt{V_2} - \sqrt{V_1}\right) + \sqrt{mV_2}\left(T_c^{3/2} - T_H^{3/2}\right)$$

$$+ \sqrt{mT_c^3}\left(\sqrt{V_1} - \sqrt{V_2}\right) + \sqrt{mV_1}\left(T_H^{3/2} - T_c^{3/2}\right)$$

$$= \sqrt{m}\left\{\left(\sqrt{V_2} - \sqrt{V_1}\right)\left(T_H^{3/2} - T_c^{3/2}\right) + \left(T_H^{3/2} - T_c^{3/2}\right)\left(\sqrt{V_1} - \sqrt{V_2}\right)\right\}$$

$$= 0 \,/\!/$$

Rad po komadima $\qquad p^2 V = mT^3$

$$\delta W_{1\to2} = -p\,dV = -\sqrt{\frac{mT^3}{V}}\,dV$$

$$\Delta W_{1\to2} = -\int_{V_1}^{V_2}\sqrt{\frac{mT^3}{V}}\,dV\,\Bigg|_{T=T_H} = \underline{-2\sqrt{mT_H^3}\left(\sqrt{V_2}-\sqrt{V_1}\right)}.$$

$$\delta W_{2\to3} = -p\,dV = -\sqrt{\frac{mT^3}{V}}\,dV \overset{\Delta W_{2\to3}=}{=} \underline{-2\sqrt{mT_c^3}\left(\sqrt{V_1}-\sqrt{V_2}\right)}$$

$\delta W_{3\to1} = 0$ (jer je volumen konstantan)

<u>δQ po komadima:</u>

$$\delta Q_{1\to2} = \delta U_{1\to2} - \delta W_{1\to2} = \sqrt{mT_H^3}\left(\sqrt{V_2}-\sqrt{V_1}\right) + 2\sqrt{mT_H^3}\left(\sqrt{V_2}-\sqrt{V_1}\right)$$

$$= 3\sqrt{mT_H^3}\left(\sqrt{V_2}-\sqrt{V_1}\right)$$

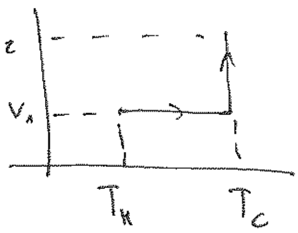
ovo je isto

$$\delta Q = T\,dS = T\left(\frac{\partial S}{\partial V}\Big|_T dV + \cancel{\frac{\partial S}{\partial T}\Big|_V dT}\right) = \underbrace{3\sqrt{mT_H^3}\left(\sqrt{V_2}-\sqrt{V_1}\right)}_{\text{za } 1\to 2}$$

od $1\to2$ $T=\text{const}$

$$\delta Q_{2\to3} = \frac{3}{2}\sqrt{mVT}\,dT + \frac{3}{2}\sqrt{\frac{mT^3}{V}}\,dV$$

$$= \sqrt{mV_2}\left(T_c^{3/2} - T_H^{3/2}\right) + 3\sqrt{mT_c^3}\left(\sqrt{V_1}-\sqrt{V_2}\right)$$



$$\delta Q_{31} = T\,dS = T\left(\frac{\partial S}{\partial T}\right)_{V=V_1} dT = \frac{3}{2}\sqrt{mVT}\,dT \quad \Big/ \int_{T_c}^{T_H}$$

$$\Delta Q_{3\to1} = \sqrt{mV_1}\left(T_H^{3/2} - T_c^{3/2}\right)$$

<u>Iskoristivost:</u>

$$\eta = \left|\frac{\overset{\Delta W_{12}+\Delta W_{23}+\Delta W_{31}}{\sum W}}{Q_{1\to2}}\right| = \frac{2}{3}\left(1-\left(\frac{T_c}{T_H}\right)^{3/2}\right)$$