2. VJEŽBE 6.3.2019.

Dakle: $\boxed{dS = \frac{\delta Q}{T}}$ $\longrightarrow$ $\oint dS = 0$

Clausius: količina topline reverzibilno izmjenjena na T

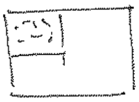
Izolirani sustav → $\delta Q = 0$ → $\underline{dS = 0}$ → ekstrem!

Izolirani sustav u ravnoteži $\boxed{S = S_{max}}$ iskustvo eksperiment

<u>II</u> zakon termodinamike

$\boxed{dS = 0}$ $\boxed{S = S_{max}}$ → za izolirani sustav

U ireverzibilnom procesu kad sustav ide prema ravnoteži entropija <u>raste</u>.

Pr:  plin, pregrada

$S \rightarrow$ ekstenzivna veličina

I. zakon

$$dU = \delta Q_{rev} + \delta W_{rev} = TdS - pdV + \mu dN \Leftarrow dU(S,V)$$

dodamo li $\mu dN \rightarrow$ $U \rightarrow U(S,V,N)$

$U(S,V,N)$ .... funkcija prirodnih varijabli

iz I:

$$T = \frac{\partial U}{\partial S}\Big|_{V,N} \; ; \; -p = \frac{\partial U}{\partial V}\Big|_{S,N} \; ; \; \mu = \frac{\partial U}{\partial N}\Big|_{S,V}$$

kompletno znanje o sustavu

intenzivne veličine

I zakon

$dU = TdS - pdV + \mu dN$ HdM, EdP

↳ ext funkcija ext. veličina

$U(\alpha S, \alpha V, \alpha N) = \alpha U(S,V,N)$ → homogeno 1. reda

pr. $f(x) = x$

$f(\alpha x) = \alpha x = \alpha f(x)$

$f(x^2)$ nije homogena

Intenzivne varijable su homogene 0. reda

$T(\alpha S, \alpha V, \alpha N) = T(S,V,N)$

$f(x) = C$

$f(\alpha x) = C$

$\alpha = 1 + \varepsilon \quad , \varepsilon \to 0$

$$U\left[(1+\varepsilon)S, (1+\varepsilon)V, (1+\varepsilon)N\right] = (1+\varepsilon)U$$

$$\approx U(S,V,N) + \underbrace{\frac{\partial U}{\partial S}\bigg|}_{T} \varepsilon S + \underbrace{\frac{\partial U}{\partial V}\bigg|}_{-p} \varepsilon V + \underbrace{\frac{\partial U}{\partial N}\bigg|_{SV}}_{\mu} \varepsilon N$$

$$\to \cancel{U} + \varepsilon U = \cancel{U} + \varepsilon TS - \varepsilon pV + \varepsilon \mu N$$

$$\to \boxed{U = TS - pV + \mu N}$$ Eulerova relacija

$dU = SdT + TdS - pdV - Vdp + \mu dN + Nd\mu$

$dU = SdT - pdV + \mu dN$

$$\boxed{SdT - Vdp + Nd\mu = 0}$$ Gibbs-Duheim relacija

Intenzivne varijable nisu međusobno neovisne!

ZADATAK: Entropija idealnog plina! $N = \text{const}$

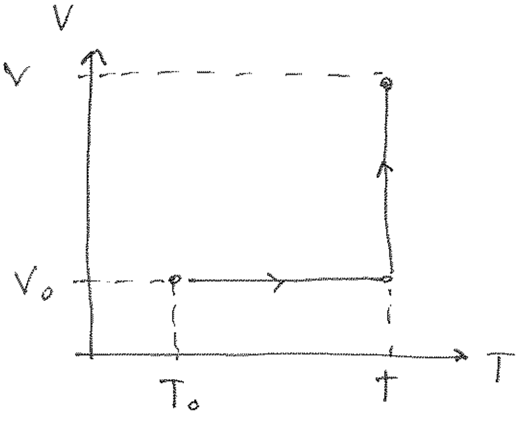
$S = S(T,V)$

I. zakon $dU = TdS - pdV \rightarrow \boxed{dS = \frac{1}{T}dU + \frac{p}{T}dV}$

Iz $\boxed{U = \frac{3}{2}NkT}$ $\boxed{pV = NkT}$

$dU = \frac{3}{2}Nk\,dT \qquad p = \frac{NkT}{V}$

$dS = \frac{3}{2}Nk\frac{dT}{T} + Nk\frac{dV}{V}$




$V_0, T_0 \rightarrow V, T$

$$S = \int_{T_0}^{T} dS\Big|_{V=V_0=\text{const}} + \int_{V_0}^{V} dS\Big|_{T=T=\text{const}}$$

$$S = \frac{3}{2}Nk\int_{T_0}^{T}\frac{dT}{T}\Big|_{V=V_0=\text{const}} + \cancel{Nk\int_{T_0}^{T}\frac{dV}{V}}\Big|_{V=V_0=\text{const}}$$

$$+\cancel{\frac{3Nk}{2}\int_{V_0}^{V}\frac{dT}{T}}\Big|_{T=T=\text{const}} + Nk\int_{V_0}^{V}\frac{dV}{V}\Big|_{T=T=\text{const}}$$

$$= \frac{3}{2}Nk\ln\frac{T}{T_0} + Nk\ln\frac{V}{V_0} + S_0(T_0, V_0)$$

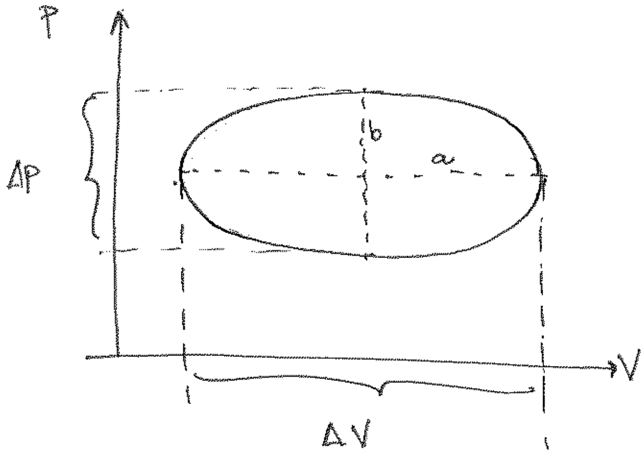
$$S(T,V) - S_0(T_0,V_0) = \frac{3}{2}Nk\ln\frac{T}{T_0} + Nk\ln\frac{V}{V_0} = Nk\ln\left[\left(\frac{T}{T_0}\right)^{3/2}\frac{V}{V_0}\right]$$

$$S(T,P) - S_0(T_0,P_0) = Nk\ln\left[\left(\frac{T}{T_0}\right)^{5/2}\frac{P_0}{P}\right]$$

$\boxed{V \sim \frac{T}{P}}$

ZAD PV dijagram kružnog procesa, u kojem sustav prima toplinu od okoline, ima oblik elipse.

Koliko puta moramo ponoviti ciklus da bi sveukupna primljena toplina bila $\Delta Q = 2,827\ MJ$ ako je razlika između maksimalnog i minimalnog tlaka $\Delta P = 48 kPa$, a max i min volumena $\Delta V = 1.5 m^3$?




• nakon svakog ciklusa $\Delta U = 0$

→ po ciklusu onda imamo da je primljena toplina = radu

rad 1. ciklusa?

$W = A_0$ (površina elipse)

$a = \Delta V / 2$

$\boxed{A = ab\pi}$

$b = \Delta P / 2$

$$A_0 = \frac{\Delta P \Delta V}{4}\pi$$

$A_0 = Q_{1 ciklus}$

u n ciklusa primi n puta $A_0$ topline

$\Delta Q = n A_0$

$$n = \frac{\Delta Q}{A_0} = \frac{4\Delta Q}{\pi \Delta P \Delta V} = 50$$

ZAD (Sa ispitnog roka).

$$S(U,V,N) = \frac{a}{b}(UVN)^{1/3}$$ a,b pozitivne konst

a) $T = ?$ b) $p = ?$

c) Mogu li, i ako da, koje vrijednosti moraju imati konstante a i b da bismo dobili jednadžbe stanja za idealni plin?

a) $T = ?$

$$dU = TdS - pdV$$

$$\frac{1}{T} = \frac{\partial S}{\partial U}$$

$$\frac{\partial S}{\partial U} = \frac{a}{b}(VN)^{1/3}\frac{1}{3}U^{-2/3} = \frac{1}{T}$$

$$\rightarrow \quad T = \frac{b}{a}\frac{3U^{2/3}}{(VN)^{1/3}}$$

b) $p = ?$

$$\frac{p}{T} = \frac{\partial S}{\partial V}$$

$$\frac{\partial S}{\partial V} = \frac{a}{b}(UN)^{1/3}\frac{1}{3}V^{-2/3} = \frac{p}{T}$$

$$p = T\frac{a}{b}(UN)^{1/3}\frac{1}{3}V^{-2/3}$$

$$p = \frac{b}{a}\frac{3U^{2/3}}{(VN)^{1/3}}\cdot\frac{a}{b}(UN)^{1/3}\frac{1}{3}V^{-2/3}$$

$$\rightarrow \quad p = \frac{U}{V}$$

c) $pV = U$ → Nikako ne možemo namjestiti a i b.

(da bi bila jedn. idealnog plina mora biti $3/2\,U$)