VJEŽBE 10
29.5.2019.

BOZONI → čestice za koje vrijedi Bose-Einsteinova statistika

→ $\underbrace{\text{fotoni, gluoni, W i Z}}_{\text{baždarni}}$ bozoni, $\underbrace{\text{Higgsov bozon}}_{\text{skalarni}}$

mi ćemo promatrati: fotoni

fononi → pobuđenja kristalne rešetke, kvanti titranja u kristalu

BOSE-EINSTEIN RASPODJELA:

Za čestice čiji broj nije očuvan → $\boxed{\mu = 0}$ $(T \approx 0)$

broj ovisi o T

$\mu < 0 \quad T > 0$

DISPERZIJA ZA EM ZRAČENJE: (ovisnost energije o valnom vektoru)

$\boxed{\varepsilon_k = \hbar \omega_k}$ $\boxed{\omega_k = \underset{\text{brzina}}{v} k}$

Bose-Einstein raspodjela:

$$n(k) = \frac{1}{e^{\beta \varepsilon_k} - 1} = \frac{1}{e^{\beta \hbar \omega_k} - 1} = \frac{1}{e^{\beta \hbar v k} - 1}$$

$$n(\omega) = \frac{1}{e^{\frac{\hbar \omega}{kT}} - 1}$$

BOZONI OPĆENITO

Uvodimo gustoću stanja: (3D)

$$g(\varepsilon) = \sum_{k\sigma} \delta(\varepsilon - \varepsilon_k) \rightarrow \frac{(2s+1)V}{(2\pi)^3} \int_0^{\infty} 4\pi k^2 dk \, \delta(\varepsilon - \hbar v k)$$

nema ograničenja

bozoni: $s=1$

$\rightarrow (2s+1) = 3$

$$= \frac{3V \, 4\pi}{8\pi^3} \underbrace{\int_0^{\infty} dk \, k^2 \, \delta(\varepsilon - \hbar v k)}_{\frac{k_0^2}{|\hbar v|}} \quad ; \quad k_0 = \frac{\varepsilon}{\hbar v} > 0$$

$$\boxed{g(\varepsilon) = \frac{3}{2} \frac{V}{\pi^2} \frac{\varepsilon^2}{(\hbar v)^3}}$$

$$N(T) = \int_0^{\infty} g(\varepsilon) \, n(\varepsilon) \, d\varepsilon = \frac{3}{2} \frac{V}{\pi^2} \frac{1}{(\hbar v)^3} \int_0^{\infty} \frac{\varepsilon^2}{e^{\beta\varepsilon} - 1} d\varepsilon$$

uvodim: $x = \beta\varepsilon \rightarrow dx = \beta d\varepsilon$

$$= \frac{3}{2} \frac{V}{(\pi)^2} \frac{1}{(\hbar v)^3} \frac{1}{\beta^3} \underbrace{\int_0^{\infty} \frac{x^2}{e^x - 1} dx}_{\Gamma(3)\zeta(3) \rightarrow \sim 2.4} \rightarrow \boxed{N(T) \sim T^3}$$

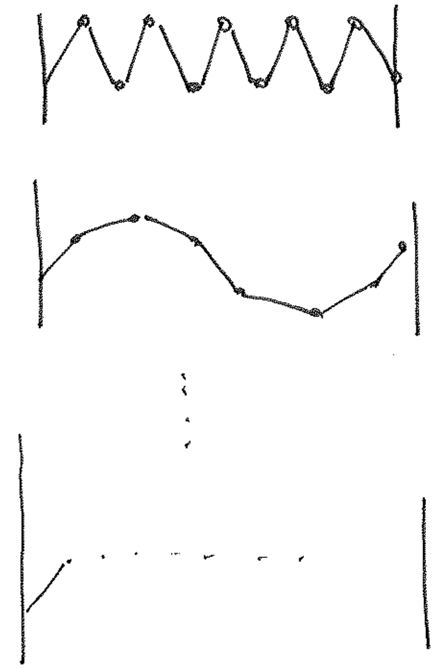
Zeta funkcija

Gama funkcija

Općenito:

$$\Gamma(s)\zeta(s) = \int_0^{\infty} \frac{x^{s-1}}{e^x - 1} dx$$

FONONI → kristal – titranje $\qquad N_0 = 3N$ jer p_x, p_y, p_z $\underbrace{\qquad}_{3\ \text{smjera}}$ x, y, z



Postoji maksimalni valni vektor → minimalna valna duljina do koje možemo ići:

$$N_0 = \sum_{\lambda, k < k_{max}} 1 \quad u \ 3D$$

$\lambda_{fotoni} = 2$
$\lambda_{fononi} = 3$

λ – polarizacija, to ćemo koristiti umjesto $(2s+1)$

$$\downarrow \quad \frac{V\lambda}{(2\pi)^3} \int_0^{k_{MAX}} 4\pi k^2 \, dk = \boxed{\frac{\lambda V}{6\pi^2} k_{max}^3 = N_0} = 3N$$

$$k_{max}^3 = \frac{3N \cdot 6\pi^2}{\cancel{\lambda} V} = \frac{N}{V} 6\pi^2 \quad \rightarrow \quad \boxed{k_{max}^3 = 6\pi^2 n}$$

(λ = 3)

PRETPOSTAVKA DEBYEVOG MODELA: $\boxed{\omega_k = vk}$

linearna disperzija → **zvuk!** $\quad \boxed{v = \frac{\partial \omega}{\partial k}}$

Koja je maksimalna frekvencija? $\qquad v$ → brzina zvuka

$$\boxed{\omega_{max} = v \cdot k_{max}}$$

$\boxed{k_{max} = k_D}$ Debyev valni vektor

$\boxed{\omega_{max} = \omega_D}$ Debyeva frekvencija

Definiramo maksimalnu energiju: $\boxed{\varepsilon_D = \hbar \omega_D}$

te Debyevu temperaturu: $k_B T_D = k_B \Theta = \varepsilon_D = \hbar \omega_D$ $\qquad$ Debyeva temperatura $\boxed{\Theta = \frac{\hbar \omega_D}{k_B}}$

$\varepsilon_D = \hbar v k_D$

Ovo je dovoljno za ići dalje:

$$g(\varepsilon) = \sum_{k\lambda} \delta(\varepsilon - \varepsilon_k) = \frac{\lambda V}{(2\pi)^3} \int_0^{k_D} 4\pi k^2 \delta(\varepsilon - \hbar v k)$$

$\lambda \rightarrow$ polarizacija = 3 za 3D

$$= \frac{\lambda V}{2\pi^2} \underbrace{\int_0^{k_D} k^2 \delta(\varepsilon - \hbar v k) dk}_{\frac{k_0^2}{|-\hbar v|}} \qquad k_0 = \frac{\varepsilon}{\hbar v}$$

$$\boxed{g(\varepsilon) = \frac{\lambda V}{2\pi^2} \frac{\varepsilon^2}{(\hbar v)^3}}$$

$$u = \int_0^{\varepsilon_D} g(\varepsilon)\, \varepsilon\, n_B(\varepsilon)\, d\varepsilon = \frac{\lambda V}{2\pi^2} \frac{1}{(\hbar v)^3} \int_0^{\varepsilon_D} \frac{\varepsilon^2}{e^{\beta\varepsilon} - 1} d\varepsilon$$

$$x = \beta\varepsilon \qquad \boxed{\varepsilon = \frac{x}{\beta}} \qquad \beta\varepsilon_D \rightarrow \frac{\varepsilon_D}{kT} = \frac{\theta}{T}$$

$$= \frac{\lambda V}{2\pi^2} \frac{1}{(\hbar v)^3} \frac{1}{\beta^4} \int_0^{\beta\varepsilon_D} \frac{x^3 dx}{e^x - 1}$$

$$\boxed{u(T) = \frac{\lambda V}{2\pi^2} \frac{1}{(\hbar v)^3} k_B^4 T^4 \int^{\theta/T} \frac{x^3 dx}{e^x - 1}}$$

$$U(T) = \frac{2V}{2\pi^2}\cdot\frac{3}{3}\,\overbrace{k_D^3}^{N_0}\,\underbrace{\frac{1}{k_D^3\,(\hbar v)^3}}_{\varepsilon_D^3}\,k_B^4 T^4 \int_0^{\theta/T} \frac{x^3\,dx}{e^x - 1}$$

$$U(T) = \frac{3N_0 (k_B T)^4}{\varepsilon_D^3} \int_0^{\theta/T} \frac{x^3\,dx}{e^x - 1}$$

REŽIMI:

$\boxed{T \to 0}$ $\qquad \theta/T \to \infty$

$$U(T) = \frac{3N_0 (k_B T)^4}{\varepsilon_D^3} \underbrace{\int_0^{\infty} \frac{x^3\,dx}{e^x - 1}}_{\Gamma(4)\zeta(4) = 6\cdot 1{,}08 = 6{,}48}$$

$$U(T) \simeq T^4$$

$$C_V = \frac{\partial U}{\partial T} \sim T^3$$

$\boxed{T \to \infty}$ $\qquad \theta/T \to 0$

$$U(T) = \frac{3N_0 (k_B T)^4}{\varepsilon_D^3} \int_0^{\theta/T} \frac{x^3}{\underbrace{e^x - 1}_{\text{razvij oko } 0}}\,dx \qquad \int \approx \frac{x^3}{\cancel{1} + x + \frac{x^2}{2} - \cancel{1}} \simeq x^2$$

$$\int_0^{\theta/T} x^2\,dx = \frac{1}{3}\left(\frac{\theta}{T}\right)^3$$

$$U(T) = \frac{\cancel{3}N_0}{\cancel{3}}\,\frac{1}{\varepsilon_D^3}\,k_B^4 T^4\,\frac{\theta^3}{\cancel{T^3}} = N_0\,\frac{1}{\cancel{\varepsilon_D^3}}\,k_B^4\,T\cdot\frac{\cancel{\varepsilon_D^3}}{\cancel{k_B^3}} \doteq N_0 k_B T$$

$$C_V = \frac{\partial U}{\partial T} = N_0 k_B = \underline{\underline{3Nk}} \longrightarrow \text{ekviparticijski teorem}$$

$$N_0 = 3N$$