Analysis of the Vacuum Cannon

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We develop a theoretical model for the velocity of a projectile in a vacuum cannon. The ideal maximum velocity is independent of vacuum cannon diameter and projectile mass, and is significantly lower than the speed of sound. Experimental measurements support the theory as an upper limit.

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I. INTRODUCTION

The vacuum cannon, or vacuum bazooka as it is sometimes called, is a spectacular classroom demonstration of the very real nature of air pressure.[1] A 1–2 m length of PVC pipe, with a 'T' fitting to facilitate attachment to a vacuum pump, is loaded with one or more ping-pong balls. (Fig. 1) The pipe is then sealed with caps of aluminum foil or plastic tape.[2] The apparatus is evacuated, and the foil cap nearest the ping-pong balls is popped. The ping-pong balls are driven out of the other end of the pipe by atmospheric pressure at a startlingly high velocity.

In the course of discussion about this apparatus with fellow physics professors, it became evident that there was no clear understanding of the maximum theoretical velocity such an apparatus should produce. The "zeroth order" approximation is that the acceleration of the projectile would be a constant:

$$a_o = \frac{F}{m} = \frac{P_o A}{m} \tag{1}$$

where P_o and A are atmospheric pressure and projectile cross-sectional area, respectively. This predicts a maximum velocity dependent on tube length[3] L:

$$v(L) = \sqrt{\frac{2P_o A L}{m}} \tag{2}$$

This estimate runs into problems for longer barrels, where it predicts velocity greater than the speed of sound. The ensuing "common-sense correction" is that the projectile asymptotically approaches the speed of sound.

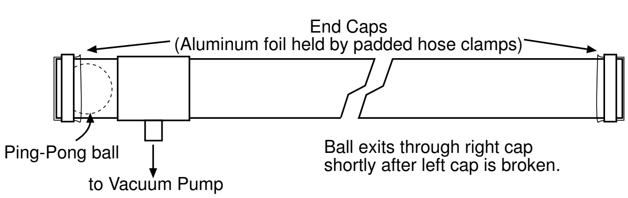

FIG. 1: Vacuum Cannon

II. ANALYSIS

The actual situation is somewhat more complicated, because the air pressure must accelerate not only the projectile but also the air column behind the projectile. A closed-form solution exists, however. Consider a projectile of mass m and cross section A in a horizontal vacuum cannon. The projectile begins at position $x = 0$. We will assume for this "first-order" calculation that the projectile fits —and seals— the barrel perfectly, and that effects due to friction, viscosity, and compressibility are negligible. We also assume the pressure at $x = 0$ remains constant at P_o.

Beginning with $F = \frac{dP}{dt}$, we obtain

$$P_o A = \frac{d}{dt}\left\{(m + \rho x A)v\right\} \tag{3}$$

which can be integrated directly to obtain

$$P_o A t = m\left(1 + \frac{\rho x A}{m}\right) v \tag{4}$$

where ρ is the air density and x is the position of the projectile. (The integration constant is zero if the initial velocity is taken to be zero.)

If we define for convenience a characteristic length[4] $\lambda \equiv \frac{m}{\rho A}$, this may be rewritten as

$$P_o A t = m\left(1 + \frac{x}{\lambda}\right) v. \tag{5}$$

Multiplying by dt and integrating, noting that $v\,dt = dx$, gives

$$\frac{1}{2} P_o A t^2 = m\left(x + \frac{x^2}{2\lambda}\right). \tag{6}$$

It is convenient to substitute a_o for the collection of constants:

$$t^2 = \frac{2x}{a_o}\left(1 + \frac{x}{2\lambda}\right). \tag{7}$$

Solve this for x to obtain

$$x(t) = \lambda\left[\sqrt{1 + \frac{a_o t^2}{\lambda}} - 1\right]. \tag{8}$$

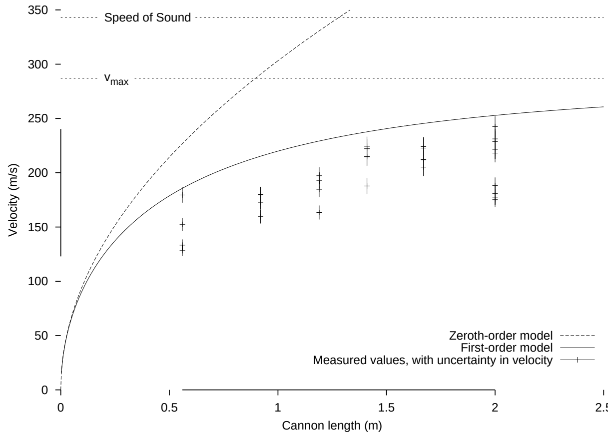

FIG. 2: Comparison of experimental measurements with various models for vacuum cannon velocity.

We are most interested in the velocity, of course:

$$v(t) = \frac{dx}{dt} = \frac{a_o t}{\sqrt{1 + \frac{a_o t^2}{\lambda}}}. \tag{9}$$

The initial acceleration of the projectile is

$$\left.\frac{dv}{dt}\right|_{t\to 0} = a_o = \frac{P_o A}{m} \tag{10}$$

which is as one would expect from the zeroth-order approximation (Eq.1). Just as an aside, it is interesting to note that the magnitude of this acceleration, for a ping-pong ball, is approximately 4,700 g's!

By factoring $\sqrt{\frac{a_o t^2}{\lambda}}$ out of the denominator of Eq. 9, we may express the velocity in an asymptotic form:

$$v(t) = v_{max}\left[\frac{1}{\sqrt{1 + \frac{\lambda}{a_o t^2}}}\right], \tag{11}$$

where

$$v_{max} \equiv \sqrt{a_o \lambda} = \sqrt{\frac{P_o}{\rho}}. \tag{12}$$

For comparison, the speed of sound is[5] $v_s = \sqrt{\frac{\gamma P_o}{\rho}}$, with $\gamma \equiv \frac{C_p}{C_v}$.

Substituting Eq. 7 into Eq. 11 allows us to determine, after some algebra, the velocity as a function of length:

$$v(x) = v_{max}\left[\frac{x}{x+\lambda}\sqrt{1 + 2\frac{\lambda}{x}}\right]. \tag{13}$$

This equation is plotted as "First-order model" in Fig. 2, using as numeric parameters the mass and cross-sectional area of a ping-pong ball. For comparison, the "Zeroth-order model" (Eq. 2) and various limits are also shown.

The energy of the projectile depends, of course, on projectile mass and on the length of the cannon. For a given length L,

$$\begin{aligned} E &= \frac{1}{2}mv^2(L) \\ &= \frac{1}{2}mv_{max}^2\left(\frac{L}{L+\lambda}\right)^2\left(1 + 2\frac{\lambda}{L}\right). \end{aligned} \tag{14}$$

Substituting for λ and v_{max} we obtain

$$E = \frac{1}{2}m\frac{P_o}{\rho}\left[1 - \left(1 + \frac{\rho A L}{m}\right)^{-2}\right]. \tag{15}$$

If we take the limit of this energy we obtain, to nobody's surprise,

$$\lim_{m\to\infty}[E] = P_o A L \tag{16}$$

which is the energy required to evacuate the vacuum cannon in the first place. The momentum of the projectile, $p = mv(L)$, is not limited but increases (in the limit of large masses) as the square root of mass.

III. EXPERIMENTAL VALIDATION

The experimental values shown in Fig. 2 were obtained by firing the ping-pong ball through two PASCO[6] photogates at a fixed spacing. The photogate signals were sent to an HP 5600B digital oscilloscope and to an HP 5300A timer. The oscilloscope was used to determine whether the shot was "good": i.e. whether there were two distinct photogate events at about the right time interval, and the timer was used to obtain a more precise value for the time of flight between the gates.

There is considerable scatter in the data, due to unavoidable random factors in the firing and measurement process. The end-cap does not collapse instantaneously, and sometimes a significant area of foil around the edge may remain and interfere with the incoming airflow. This lowers the ball velocity by an unpredictable amount. The ball itself comes out with wildly varying amounts of spin, judging by the unpredictable curve of its subsequent trajectory. If this spin arises from friction with the tube wall during transit, the amount of energy lost to that friction also must vary considerably from shot to shot, and this contributes to the scatter in the measured data. Finally, there is the inherent uncertainty in measurement of position of a spherical object by a photogate. Depending on the exact path of the ball between the two gates, the effective gate position could vary by more than a centimeter at each gate.

There are several weaknesses to this approximation. It does not take into account any "blowby" of air past the ball. The ping-pong ball is not a perfect fit in the PVC pipe: there is a 1.2 mm gap around the edge. Any such blowby would result in a buildup of air pressure in front of the ball which would slow the ball down, although this effect has not been experimentally quantified. This approximation does not take into account any drop in the air pressure at $x = 0$ during the firing process. Such a drop would lower the final velocity of the ball. Finally, it assumes that ρ remains at its constant equilibrium value in the tube during the firing process. It is not immediately apparent what effect the non-equilibrium value of ρ in the tube would have, but all the other sources of error — including those deliberately not included in the model, such as the impact of the ball with the exit endcap — would lower the measured ball velocity. This is consistent with the experimental data shown in figure 2.

IV. CONCLUSIONS

This model predicts that the maximum velocity available to a vacuum cannon is not the speed of sound v_s, but $\frac{v_s}{\sqrt{\gamma}}$. This asymptotic limit is independent of the vacuum cannon length, diameter, or projectile mass. The actual projectile velocity, as measured experimentally, is well below this limit. The kinetic energy of the projectile is limited, for a given cannon length, although the momentum is not.

V. CAUTIONARY NOTE

A ping-pong ball slows down very rapidly in air, so if you were to get hit with one of these from across a room it would not be a problem. However, at the muzzle of the vacuum cannon the ping-pong ball could be moving as fast as $\sqrt{\frac{P_o}{\rho}} = 287$ m/s. The kinetic energy of a ping-pong ball at this velocity is higher than that of a bullet from most .22 caliber handguns,[7] so be careful where you point this thing. It is best, when doing this as a demonstration, to load the cannon with 3–6 balls instead of one. This is just as impressive as a single ball, and the increased mass lowers the exit velocity (for a 1–2 m cannon) to a much safer level.

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[1] Physics Instructional Resource Association (PIRA) Demonstration Classification Scheme 2B30.70.

[2] J. Cockman, The Physics Teacher **41**, 246 (2003).

[3] N. A. Downie, *Vacuum Bazookas, Electric Rainbow Jelly, and 27 Other Saturday Science Projects* (Princeton University Press, 2001).

[4] This definition was chosen purely to simplify the analysis. Interestingly enough, it is also the length of an air column with a mass equal to the mass of the projectile.

[5] R. Feynman, R. B. Leighton, and M. L. Sands, *The Feynman Lectures on Physics*, vol. 1 (Addison-Wesley, 1989).

[6] PASCO Scientific, (800) 772-8700, part #ME-9498A.

[7] `http://www.volny.cz/buchtik/Revo/Ballistic_Info_komplet.htm`