# VELOCITY DISPERSIONS IN A CLUSTER OF STARS: HOW FAST COULD USAIN BOLT HAVE RUN?

H. K. Eriksen<sup>1,5,6</sup>, J. R. Kristiansen<sup>2,5</sup>, Ø. Langangen<sup>3,5</sup> and I. K. Wehus<sup>4,7</sup> (Dated: Received - / Accepted -) Draft version September 2, 2008

### ABSTRACT

Since that very memorable day at the Beijing 2008 Olympics, a big question on every sports commentator's mind has been "What would the 100 meter dash world record have been, had Usain Bolt not celebrated at the end of his race?" Glen Mills, Bolt's coach suggested at a recent press conference that the time could have been 9.52 seconds or better. We revisit this question by measuring Bolt's position as a function of time using footage of the run, and then extrapolate into the last two seconds based on two different assumptions. First, we conservatively assume that Bolt could have maintained Richard Thompson's, the runner-up, acceleration during the end of the race. Second, based on the race development prior to the celebration, we assume that he could also have kept an acceleration of  $0.5~\text{m/s}^2$  higher than Thompson. In these two cases, we find that the new world record would have been  $9.61\pm0.04$  and  $9.55\pm0.04$  seconds, respectively, where the uncertainties denote 95% statistical errors.

Subject headings: popular science — image analysis — Beijing 2008

### 1. INTRODUCTION

On Saturday, August 16th 2008, Usain Bolt shattered the world record of 100 meter dash in the Bird's Nest at the Beijing Olympics 2008. In a spectacular run dubbed "the greatest 100 meter performance in the history of the event" by Michael Johnson, Bolt finished at 9.69 seconds, improving his own previous world record from earlier this year by 0.03 seconds. However, the most impressive fact about this run was the way in which he did it: After accelerating away from the rest of the field, he looked to his sides when two seconds and 20 meters remained, and when that noting he was completely alone, he started celebrating! He extended his arms, and appeared to almost dance along the track.

Despite this, he broke the world record by 0.03 seconds. But, needless to say, this celebration left spectators and commentators all over the world wondering about one big question: What would the world record have been if he had *not* celebrated the last 20 meters? Bolt's coach, Glen Mills, recently suggested at a press conference of the Golden League tournament in Zürich, that the record could have been 9.52 seconds, or even better.

We wanted to check this for ourselves, by attempting to measure Bolt's position as a function of time, and extrapolate from the dynamics before the celebration began, into the last two seconds of the race. Based on (hopefully) reasonable assumptions, we could then obtain an estimate of the new world record.

In this paper we analyze footage of the run obtained from various web sites and the Norwegian Broadcasting Corporation (NRK), with the goal of estimating this "hy-

<sup>1</sup> email: h.k.k.eriksen@astro.uio.no

pothetical" world record. The main technical difficulty in performing this analysis lies in obtaining accurate distance measurements as a function of time for each runner. Fortunately, this task is made considerably easier by the presence of a moving camera mounted to a rail along the track. This rail is bolted to the ground at regular intervals, and thereby provides the required standard ruler. Using the methods detailed in the following sections, and properly taking into account all major sources of statistical uncertainty, we believe that our measurements are sufficiently accurate and robust to support interesting conclusions.

# 2. METHOD

Our analysis is based on the following simple steps:

- 1. We first obtained several different videos of the race from the Internet (NBC and BBC) and the Norwegian Broadcasting Company (NRK), and printed out  $\sim 30$  screen shots at different times from these.
- 2. We then constructed a standard ruler by counting the total number of bolts (called "ticks" in the following) on the rail of the moving camera along the 100 meter track (see Figure 1). We assumed the distance between these to be constant.
- 3. Next, we drew lines orthogonal to the track, using whatever means most accurate for a given screen shot. For early and late frames, lines in the actual track itself (e.g., starting and finishing lines) were most useful, while for intermediate frames, the lower right edge of the camera mount was utilized (Figure 1).
- 4. For a given frame, we then read off the positions of Usain Bolt and Richard Thompson, the runner-up, with the ruler, and recorded these together with the time from the screen clock.
- 5. Next, we assigned an uncertainty to each distance measurement, by estimating how many ticks we

<sup>&</sup>lt;sup>2</sup> email: j.r.kristiansen@astro.uio.no

<sup>&</sup>lt;sup>3</sup> email: oystein.langangen@astro.uio.no

<sup>&</sup>lt;sup>4</sup> email: i.k.wehus@fys.uio.no

<sup>&</sup>lt;sup>5</sup> Institute of Theoretical Astrophysics, University of Oslo, P.O. Box 1029 Blindern, N-0315 Oslo, Norway

<sup>&</sup>lt;sup>6</sup> Centre of Mathematics for Applications, University of Oslo, P.O. Box 1053 Blindern, N-0316 Oslo

<sup>&</sup>lt;sup>7</sup> Department of Physics, University of Oslo, P.O. Box 1048 Blindern, N-0316 Oslo, Norway



Fig. 1.— Example screen shot used to estimate the runners' position as a function of time.

believed we were off in a given frame. For later frames, when the camera angle is almost orthogonal to the track, this uncertainty is smaller than in the beginning of the race because of the camera perspective.

- 6. Based on these uncertainties, we fitted a smooth spline with inverse variance weights to the data. This provided us with a smooth approximation to the runners' positions as a function of time, and also with the first and second derivatives, i.e., their speeds and accelerations.
- 7. To make the projections, we consider two cases: First, we conservatively assume that Bolt would have been able to keep up with Thompson's acceleration profile in the end race after 8 seconds of elapsed time, and project a new finishing time. Second, given his clearly stronger acceleration around 6 seconds, we also consider the case in which he is able to maintain a  $\Delta a = 0.5 \text{ m/s}^2$  higher acceleration than Thompson through to the end.

The final goal is the new projected world record, which is found by extrapolating the resulting motion profile to 100 meters. We also estimate the uncertainty in this number by repeating the above analysis 10 000 times, each time adding a random fluctuation with specified uncertainties to each time and tick count.

### 3. DATA AND OBSERVATIONS

## 3.1. Data sets

The data used for this analysis consist of three clips filmed by three cameras located along the finishing line at slightly different positions. Specifically, the clips were obtained from NRK, NBC, and BBC.

Unfortunately, the NRK and BBC clips were filmed with cameras positioned fairly close to the track, and the rail of the moving camera therefore disappears outside the field-of-view after about 6 seconds. This is not the

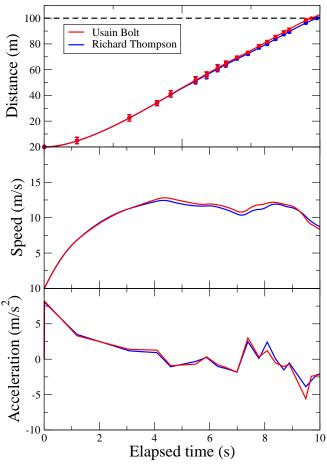


Fig. 2.— Estimated position (top), speed (middle) and acceleration (bottom) for Bolt (red curves) and Thompson (blue curves) as a function of time. Actual distance measurements are indicated in the top panel with  $5\sigma$  error bars.

TABLE 1								
POSITION AS A FUNCTION OF TIME FOR BOLT AND THOMPSO	ΟN							

Uncalibrated	Usain Bolt		Richard Thompson			
elapsed time (s)	Ticks (#)	Distance (m)	Ticks (#)	Distance (m)	$\begin{array}{c} \text{Uncertainty} \\ \text{(m)} \end{array}$	Data set
0.0*	-7.0	0.0	-7.0	0.0	0.0	None
(0.01	-7.0	0.0	-7.0	0.0	0.0	$None)^{\dagger}$
1.1	-2.0	5.0	-2.1	4.9	0.5	$NRK^{'}$
3.0	15.5	22.5	15.6	22.6	0.5	NRK
4.0	27.0	34.0	27.0	34.0	0.4	NRK
4.5	34.3	41.3	34.1	41.1	0.5	NRK
5.4	45.1	52.1	44.3	51.3	0.5	NBC
5.8	48.9	55.9	48.3	55.3	0.5	BBC
6.2	54.5	61.5	53.8	60.8	0.5	NBC
6.5	57.8	64.8	56.9	63.9	0.4	BBC
6.9	62.6	69.6	61.5	68.5	0.2	NBC
7.3	66.3	73.3	65.1	72.1	0.2	NBC
7.7	71.5	78.5	70.1	77.1	0.2	NBC
8.0	74.7	81.7	72.9	79.9	0.2	NBC
8.3	78.6	85.6	76.8	83.8	0.2	NBC
8.6	82.2	89.2	80.5	87.5	0.2	NBC
8.8	84.3	91.3	82.4	89.4	0.2	NBC
9.4	91.6	98.6	89.4	96.4	0.2	NBC
$9.69^{*}$	93.0	100.			0.0	NRK
$9.89^{*}$		• • •	93.0	100.	0.0	NRK
(13	105	112.	105.	112	5.0	NRK) <sup>†</sup>

Note. — Compilation of distance-vs-time observations for Usain Bolt and Richard Thompson in the 100 meter dash in Beijing 2008, obtained from screen shot prints of the race.

case for the NBC clip, which was filmed from further away. Even though the quality of this version is rather poor, it is possible to count the number of ticks to the end.

Using these data sets, we measured the position of Usain Bolt and Richard Thompson at 16 different times in units of ticks. These are all listed in Table 1.

### 3.2. Calibration of measurements

There are three issues that must be addressed before the tick counts listed in Table 1 can be translated into proper distance measurements. First, the camera rail is not visible entirely to the starting line, as the very first part is obscured by a camera man. The tick counts in Table 1 are therefore counted relative to the first visible tick. Fortunately, it is not very problematic to extrapolate into the obscured region by using the distance between the visible ticks, and knowing that the distance between the starting lines for the 100 meter dash and 110 meter hurdles is precisely 10 meters. We estimate the number of obscured ticks to be  $7\pm1$ .

Second, the precision of the screen clock is only a tenth of a second, and the clock also appears to truncate the time, not round off. We therefore add 0.05 seconds to each time measurement, and define our uncertainty in time to uniform between -0.05 and 0.05 seconds.

Finally, the screen clock is not calibrated perfectly with the stadium clock. (See Figure 1 for an example frame.) A little more than half of all frames appear to be synchronized, while in the rest the screen clock is lagging behind by 0.1 seconds. We assume that the stadium clock is the correct one, and re-calibrate the screen clock by adding an additional 0.04 seconds to each time measurement.

With these assumptions, it is straightforward to cal-

ibrate both the clock and distance measurements, and this is done in the corresponding columns in Table 1.

### 4. ESTIMATION OF MOTION PROFILES

With calibrated distance information ready at hand, it is straightforward to make the desired predictions. First, we compute a smooth spline (Green & Silverman 1994), s(t), through each of the two runners' measured positions. A nice bonus of using splines is that we automatically obtain the second derivatives of s (ie., acceleration) at each time step, and also the first derivatives (ie., speed),

$$v(t) = \frac{ds}{dt}; \quad a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}.$$
 (1)

To obtain a well-behaved spline, we impose three constraints. First, we add two auxiliary data points at t=0.01 and t=13.0 seconds. These are not measurements, but included only in order to guarantee sensible boundary conditions at each end: The first one implies that the starting velocity is zero, while the last one leads to a smooth acceleration at the finishing line. Thirdly, we adopt a smooth spline stiffness parameter of  $\alpha=0.5$  (Green & Silverman 1994) to minimize unphysical fluctuations. The results are fairly insensitive to the specific value of this parameter.

The resulting functions are plotted in Figure 2. Some interesting points to notice are the following:

- Bolt and Thompson are virtually neck by neck up to four seconds, corresponding to a distance of 35 meters.
- Bolt's Olympic gold medal is essentially won between 4 and 8 seconds.

<sup>\*</sup> The first point is taken from the known starting position, and the last is taken from a high-resolution picture of the finishing line. The times for these are not read from the screen clock, but are adopted from official sources. Zero uncertainties are assigned to these points.† These two points are not real observations, but auxiliary points to ensure sensible boundary conditions for the smooth spline; the first ensures zero starting velocity, and the last gives a smooth acceleration at the finishing line.

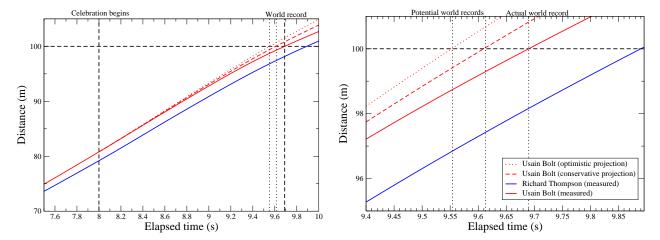


Fig. 3.— Comparison of real and projected distance profiles at the end of the race. The point where the profiles cross the horizontal 100 meter line is the new world record for a given scenario. The right panel is only a zoomed version of the left panel.

 At 8 seconds Bolt decelerates noticeably, and Thompson equalizes and surpasses Bolt's speed. Note however that Thompson is also not able to maintain his speed to the very end, but runs out of power after about 8.5 seconds. Still, his acceleration is consistently higher than Bolt's after 8 seconds.

### 5. WORLD RECORD PROJECTIONS

We are now in the position to quantitatively answer the original question: How fast would Bolt really have run, if he hadn't celebrated the last 2 seconds? To make this projection, we consider the following two scenarios:

- Bolt matches Thompson's acceleration profile after 8 seconds.
- 2. Bolt maintains a  $0.5 \text{ m/s}^2$  higher acceleration than Thompson after 8 seconds.

The justification of scenario 1 is obvious, as Bolt outran Thompson between 4 and 8 seconds. The justification of scenario 2 is more speculative, as it is difficult to quantify exactly *how much* stronger Bolt was. Still, looking at the acceleration profiles in Figure 2, and noting that Bolt traditionally was considered a 200 meter specialist, a value of  $0.5~\rm m/s^2$  seems fairly realistic.

Then, for each scenario we compute a new trajectory for Bolt by choosing initial conditions,  $s_0 = s(8 \text{ sec})$  and  $v_0 = v(8 \text{ sec})$ , and an acceleration profile as described above. The computation of these trajectories are performed by simply integrating Equation 1 with respect to time,

$$\hat{s}(t) = s_0 + \int_{t_0}^t \hat{v}(t)dt$$
 (2)

$$\hat{v}(t) = v_0 + \int_{t_0}^t \hat{a}(t)dt$$
 (3)

$$\hat{a}(t) = a_{\text{Thompson}}(t).$$
 (4)

In Figure 3 we compare the projected trajectories,  $\hat{s}(t)$  (dashed red line shows scenario 1, dotted red line shows scenario 2), with the actual trajectory, s(t) (solid red line). For comparison, Thompson's trajectory is indicated by a solid blue line.

The projected new world record is the time for which  $\hat{s}(t)$  equals 100 meter. Including 95% statistical errors estimated by Monte Carlo simulations as described in Section 2, we find that the new world record would be  $9.61 \pm 0.04$  seconds in scenario 1, and  $9.55 \pm 0.04$  in scenario 2.

### 6. CONCLUSIONS

Glen Mills, Usain Bolt's coach, suggested that the world record could have been 9.52 seconds if Bolt had not danced along the track in Beijing for the last 20 meters. According to our calculations, that seems like an good, but perhaps slightly optimistic, estimate: Depending on assumptions about Bolt's acceleration at the end of the race, we find that his time would have been somewhere between 9.55 and 9.61 seconds, with a 95% statistical error of  $\pm 0.04$  seconds. Clearly, the uncertainties due to the assumptions about the acceleration are comparable to or larger than the statistical uncertainties. Therefore, 9.52 seconds does by no means seem to be out of reach.

In Figure 4 we show an illustration of how such a record would compare to the actual world record of 9.69 seconds, relative to the rest of the field: The left version of Bolt shows his actual position at  $\sim 9.5$  seconds, while the right version indicates his position in the new scenarios.

Of course, there are potential several systematics errors involved in these calculations. For instance, it is impossible to know for sure whether Usain might have been tired at the end, which of course would increase the world record beyond our estimates. On the other hand, judging from his facial expressions as he crossed the finishing line, this doesn't immediately strike us as a very plausible hypothesis.

Another issue to consider is the wind. It is generally agreed that a tail wind speed of 1 m/s improves a 100 meter time by 0.05 seconds (Mureika 2000). Further, for IAAF (International Association of Athletics Federations) to acknowledge a given run as a record attempt, the wind speed must be less than +2 m/s. When Bolt ran in Beijing, there was no measurable wind speed at all, and one can therefore safely assume that the world record could have been further decreased, perhaps by as much as 0.1 seconds, under more favorable wind conditions.

A corollary of this study is that a new world record of



 $\rm Fig.~4.--$  Photo montage showing Bolt's position relative to his competitors for real (left Bolt) and projected (right Bolt) world records.

less than 9.5 seconds is within reach for Usain Bolt in the near future.

First and foremost, we would like to thank Christian Nitschke Smith at NRK Sporten for providing very useful high-resolution footage of the Beijing 100 dash run, and also BBC and NBC for making their videos available on their web pages. Second, we thank the pizza guy from Peppe's who provided us with a very good half-n-half "Thai Chicken" and "Heavy Heaven" pizza on a late Friday night. This article has been submitted to the American Journal of Physics. After it is published, it will be found at http://scitation.aip.org/ajp.

#### REFERENCES

BBC's web page, http://www.bbc.co.uk/blogs/thereporters/mihirbose/2008/08/bolt\_reignites\_100m\_interest.html
Green, P. J., & Silverman, B. W. 1994, Non-Parametric Regression and Generalized Linear Models, Chapman and Hall, 1994
Mureika, J. R. 2000, New Studies in Athletics (15) 3/4, 53-60 (physics/0102039)