Posredno mjerene veličine:

izravno mjerimo X i Y, a zanima nas $h(X,Y)$

Ako su rezultati neposrednih veličina $x = (\bar{x} \pm M_X)$ i $y = (\bar{y} \pm M_Y)$,

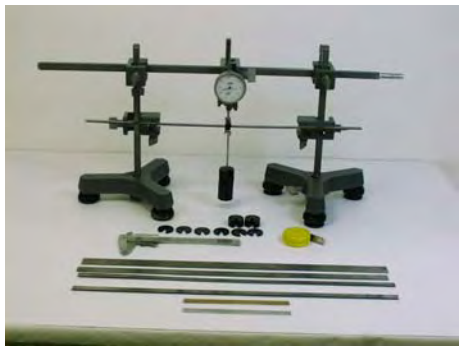
onda je:

$$\bar{h} = h(\bar{x}, \bar{y}) \qquad M_h = \sqrt{\left(\left.\frac{\partial h}{\partial X}\right|_{\substack{X=\bar{x} \\ Y=\bar{y}}} \cdot M_X \right)^2 + \left(\left.\frac{\partial h}{\partial Y}\right|_{\substack{X=\bar{x} \\ Y=\bar{y}}} \cdot M_Y \right)^2}$$

Relativna pogreška?

Linearna regresija

Mjerimo ovisnost jedne veličine o drugoj - parovi varijabli.

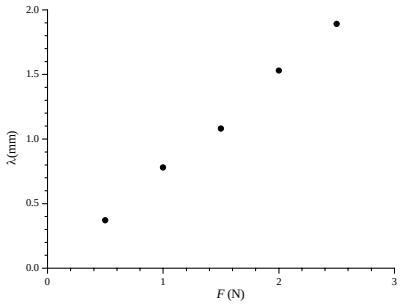


Primjer mjerenja:

F(N)	λ(mm)
0,5	0,37
1	0,78
1,5	1,08
2	1,53
2,5	1,89

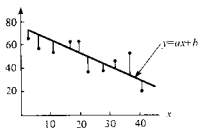
Odabir nezavisne i zavisne varijable.

Crtanje grafa!



Za izmjerene (opažene) parove vrijedi:

$$y_i = ax_i + b + \varepsilon_i \ ,$$



Princip najmanjih kvadrata:

Od svih pravaca $y = ax + b$, najvjerojatniji pravac regresije jest onaj za koji je suma kvadrata odstupanja

$$f(a,b) = \sum_{i=1}^{n} [y_i - (ax_i + b)]^2 = \sum_{i=1}^{n} \varepsilon_i^2$$

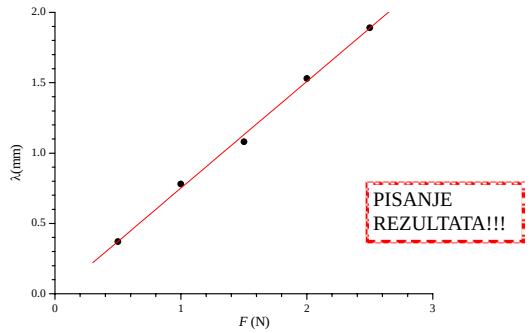
minimalna.

Konačni rezultati:

$$a = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2} \qquad b = \frac{\sum_{i=1}^{n} x_i^2 \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} x_i y_i}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}$$

$$M_a = \sqrt{\frac{1}{(n-2)} \left[\frac{n\sum_{i=1}^{n} y_i^2 - \left(\sum_{i=1}^{n} y_i\right)^2}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2} - a^2 \right]} \qquad M_b = M_a \sqrt{\frac{\sum_{i=1}^{n} x_i^2}{n}}$$

$a = 0{,}758$ mm/N $b = -0{,}007$ mm $M_a = 0{,}02248$ mm/N $M_b = 0{,}03728$ mm



PISANJE REZULTATA!!!

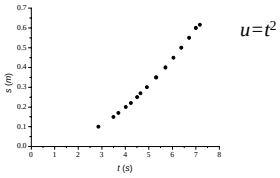
Nelinearne regresije

Regresija s transformiranim varijablama

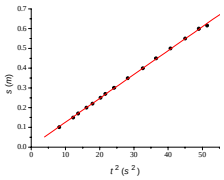
nelinearnu ovisnost prikazati u linearnom obliku:

- Mogu se primijeniti jednadžbe za linearnu regresiju.
- Takav grafički prikaz zorno potvrđuje (ili odbacuje) ispravnost primijenjene teorije.

Primjer iz praktikuma:
Dubina poniranja Maxwellova diska $s = \frac{1}{2}\frac{mg}{m + \frac{I_z}{r^2}}t^2$



$u = t^2$



Logaritamski grafovi

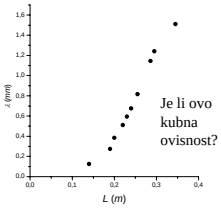
$Y = X^{\alpha}$ α ne znamo ili želimo provjeriti

Primjer: modul elastičnosti
Savijenost šipke je:

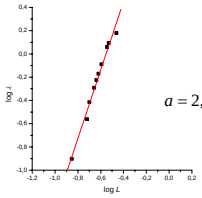
$$\lambda = \frac{1}{4E}\frac{L^3}{ab^3}F$$

$$\log\lambda = \log\frac{1}{4E}\frac{F}{ab^3} + 3\log L$$

$x = \log L$ $y = \log\lambda$



Je li ovo kubna ovisnost?



$a = 2{,}9 \pm 0{,}1$

Nelinearne regresije

Zavisna varijabla nelinearno ovisi o nezavisnoj

Npr.: tjerani prigušeni harmonički oscilator

$$y = \frac{A}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + (\omega/\tau)^2}}$$

parametri A, ω_0 i τ

Suma kvadrata odstupanja minimalna:

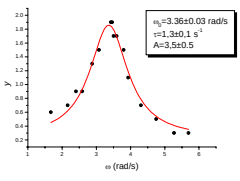
$$f(A,\omega_0,\tau) = \sum_i \left(y_i - \frac{A}{\sqrt{\left(\omega_0^2 - \omega_i^2\right)^2 + (\omega/\tau)^2}} \right)^2$$

Tri jednadžbe:

$$\frac{\partial f(A,\omega_0,\tau)}{\partial A} = 0 \quad ; \quad \frac{\partial f(A,\omega_0,\tau)}{\partial \omega_0} = 0 \quad ; \quad \frac{\partial f(A,\omega_0,\tau)}{\partial \tau} = 0$$

⇒ računalom!

Primjer s naprednog praktikuma:



$$y = \frac{A}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + (\omega/\tau)^2}}$$