$B \to K\eta'$ decay induced by the singlet-digluon $b \to s\eta'$ transition

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Collaboration with:
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7. $B \rightarrow K\eta'$ amplitude
8. Conclusions

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$$\mathrm{Br}(B^+ \to K^+\eta') = (72 \pm 5) \cdot 10^{-6}$$
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- What makes η''s special?
- Experience with η' mass (*U(1) problem*: $m_{\eta'} \gg m_\pi$) suggests: axial anomaly

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2. Other approaches

- Halperin and Zhitnitsky (1997) — $b \to s\bar{c}c$, intrinsic *charm* of η' — too large $\mathrm{Br}(B \to K^* \eta')$
- Hou and Tseng (1998), Kagan and Petrov (1997) — *new physics*
- Beneke and Neubert (2002) — *QCD factorization* approach — problems, large errors

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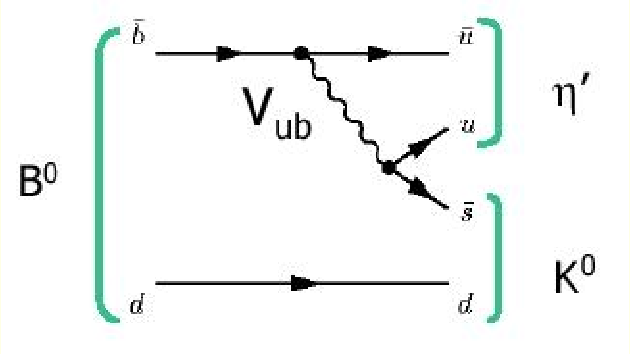
- Tree $T =$ 

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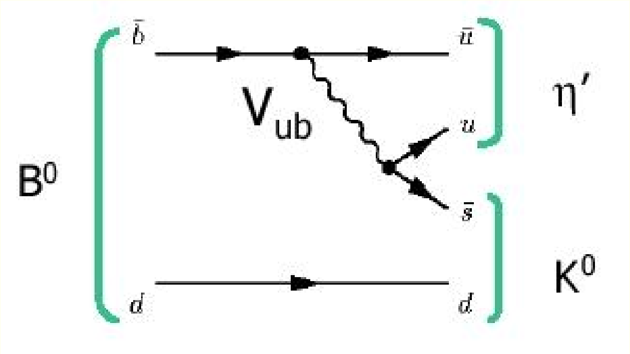
- Tree $T =$ 

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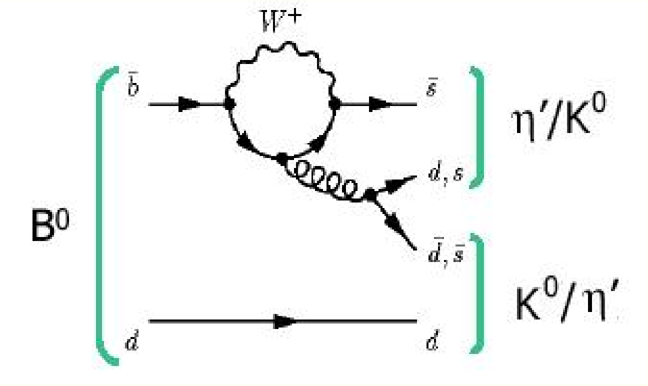
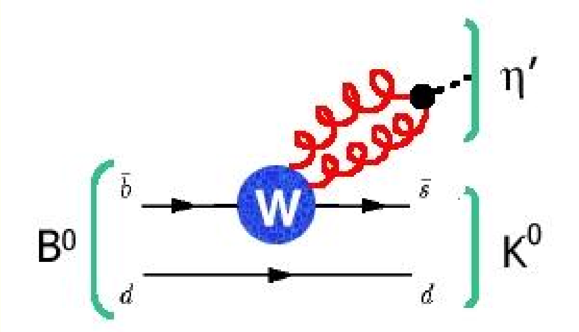
- Penguin $P =$ 

$\longrightarrow$ probably the most important contribution

- Experimental data call also for the *singlet penguin* S:

$$\mathcal{A}(B \to K\eta') = \frac{3}{\sqrt{6}} P + \frac{4}{\sqrt{3}} S$$

- $S =$ 

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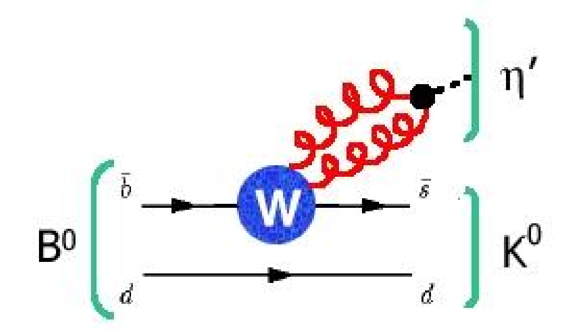
- $S =$ 

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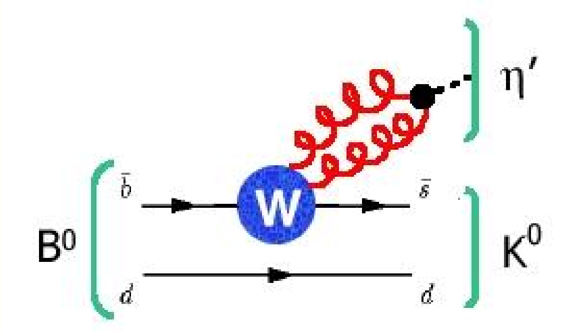
- $S =$ 

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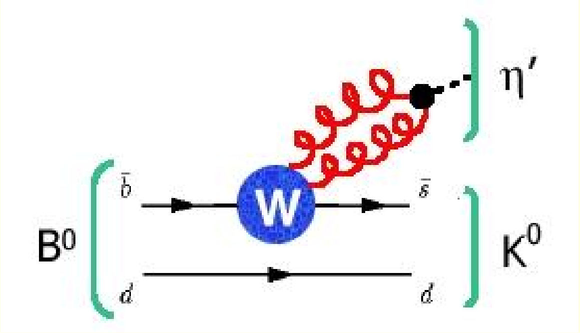
- $S =$ 

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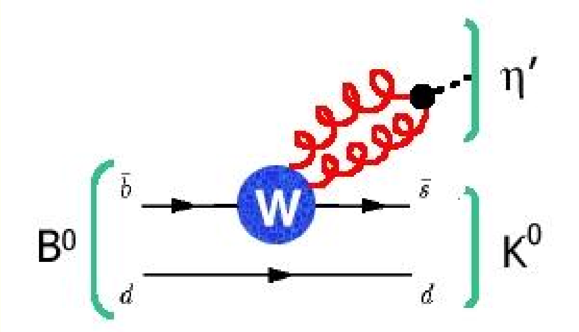
- $S =$ 

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[Back to results]

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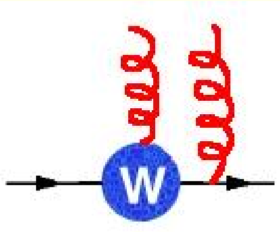

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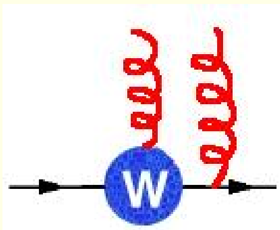

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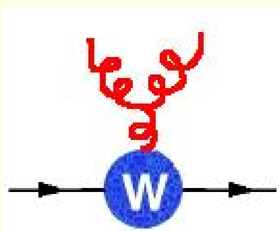

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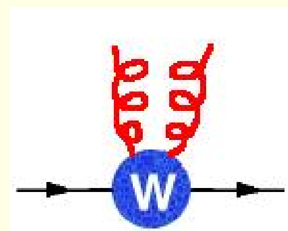

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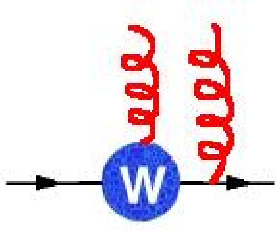

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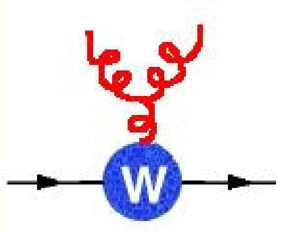

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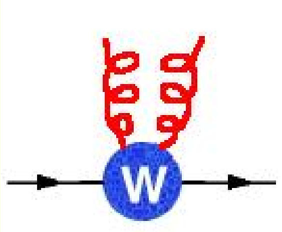

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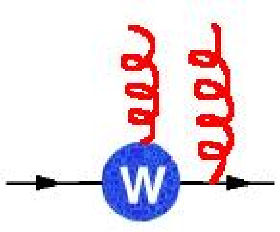

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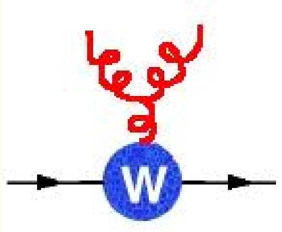

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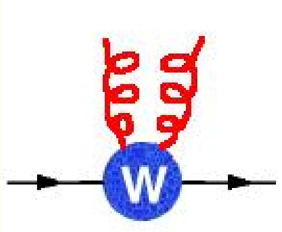

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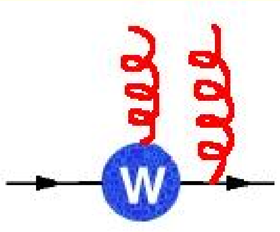

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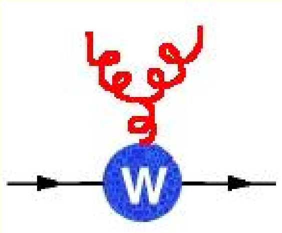

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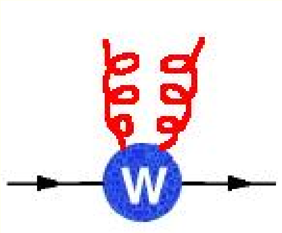

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- but not for hard off-shell gluons (Witten (1977))!

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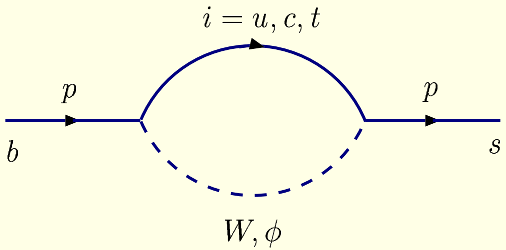
Building blocks

- Self-energy



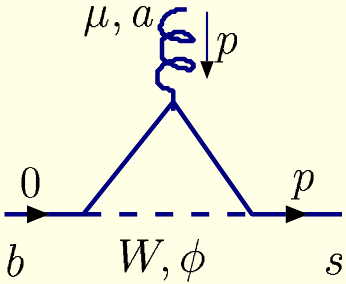
$$= \frac{i}{4\pi^2}\frac{G_F}{\sqrt{2}}\,\Sigma(p)$$

$$\Sigma(p) = -M_W^2 \not{p} L - 2M_W^2 \left(1 + \frac{m_i^2}{2M_W^2}\right) \not{p} L \int_0^1 \mathrm{d}x (1-x) \ln \frac{D}{\mu_*^2}$$

$$- \int_0^1 \mathrm{d}x \left[(1-x) m_b m_s \not{p} R - m_i^2 (m_b R + m_s L)\right] \ln \frac{D}{\mu_*^2}$$

$$\ln \mu_*^2 = \frac{1}{\epsilon} - \gamma_E + \ln 4\pi\mu^2$$

- Triangle



$$= \frac{i}{4\pi^2}\frac{G_F}{\sqrt{2}} g_s t^a \, \Gamma^\mu(0,p,-p)$$

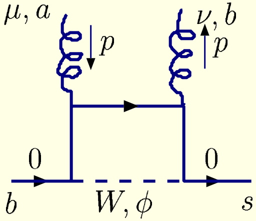
$$\Gamma^\mu(0,p,-p) = \frac{4M_W^2}{m_i^2 - M_W^2}\left(1+\frac{m_i^2}{2M_W^2}\right)(p^2 g^{\mu\nu} - p^\mu p^\nu)\gamma_\nu L \int_0^1 \mathrm{d}x\, x(1-x)\ln\frac{D}{C}$$

$$+ M_W^2 \gamma^\mu L + 2M_W^2\left(1+\frac{m_i^2}{2M_W^2}\right)\gamma^\mu L \int_0^1 \mathrm{d}x (1-x)\ln\frac{D}{\mu_*^2}$$

$$\begin{aligned} D &= x m_i^2 + (1-x)M_W^2 - x(1-x)p^2 \\ C &= m_i^2 - x(1-x)p^2 \end{aligned}$$

- Divergent parts of Γ^μ and Σ cancel among themselves in the final amplitude

♦ Box



$$= \frac{i}{4\pi^2}\frac{G_F}{\sqrt{2}} g_s^2 t^b t^a \, I^{\mu\nu}(0,0,-p,p)$$

$$I^{\mu\nu}(0,0,-p,p) = \frac{2M_W^2}{m_i^2 - M_W^2}\left(1 - \frac{m_i^2}{2M_W^2}\right)(-i\epsilon^{\mu\nu\rho\sigma} p_\sigma \gamma_\rho L) \times$$

$$\times \int_0^1 \mathrm{d}x (1-x) \left\{ (3x-1)\mathbb{Y}_1 + \left[x^2(1-x)p^2 + (x+1)m_i^2\right]\mathbb{Y}_2 \right\}$$

$$+ \frac{2M_W^2}{m_i^2 - M_W^2}\left(1 + \frac{m_i^2}{2M_W^2}\right) \int_0^1 \mathrm{d}x(1-x) \left\{ [-(x+1)\not{p} g^{\mu\nu} - (x-1)(p^\mu\gamma^\nu + p^\nu\gamma^\mu)]\mathbb{Y}_1 \right.$$

$$+ \left(x^2(1-x)\left[-(p^\mu\gamma^\nu + p^\nu\gamma^\mu)p^2 + \not{p}(4p^\mu p^\nu - g^{\mu\nu}p^2)\right] \right.$$

$$\left. \left. + [-(x+1)\not{p} g^{\mu\nu} - (x-1)(p^\mu\gamma^\nu + p^\nu\gamma^\mu)]m_i^2 \right) \mathbb{Y}_2 \right\} L$$

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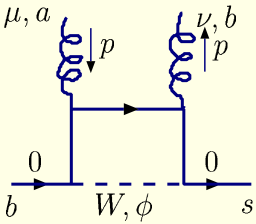

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♦ $\mathbb{Y}_{1,2} =$ complicated functions of x, m_i^2, M_W^2, p^2

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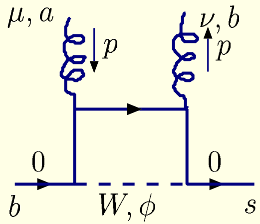

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$$I^{\mu\nu}(0,0,-p,p) = \frac{2M_W^2}{m_i^2-M_W^2}\left(1-\frac{m_i^2}{2M_W^2}\right)(-i\epsilon^{\mu\nu\rho\sigma}p_\sigma\gamma_\rho L)\times$$
$$\times\int_0^1 \mathrm{d}x(1-x)\left\{(3x-1)\mathbb{Y}_1+\left[x^2(1-x)p^2+(x+1)m_i^2\right]\mathbb{Y}_2\right\}$$
$$+\frac{2M_W^2}{m_i^2-M_W^2}\left(1+\frac{m_i^2}{2M_W^2}\right)\int_0^1 \mathrm{d}x(1-x)\left\{[-(x+1)\not{p}g^{\mu\nu}-(x-1)(p^\mu\gamma^\nu+p^\nu\gamma^\mu)]\mathbb{Y}_1\right.$$
$$+\left(x^2(1-x)\left[-(p^\mu\gamma^\nu+p^\nu\gamma^\mu)p^2+\not{p}(4p^\mu p^\nu-g^{\mu\nu}p^2)\right]\right.$$
$$\left.\left.+[-(x+1)\not{p}g^{\mu\nu}-(x-1)(p^\mu\gamma^\nu+p^\nu\gamma^\mu)]m_i^2\right)\mathbb{Y}_2\right\}L$$

- $\mathbb{Y}_{1,2} =$ complicated functions of x, m_i^2, M_W^2, p^2
- We agree with Simma and Wyler (1990) in appropriate regions of parameter space.

Complete amplitude for $b \to s g^* g^*$

$$\mathcal{A} = \mathrm{i}\frac{\alpha_s}{\pi}\frac{G_F}{\sqrt{2}}\bar{s}(0)t^b t^a \sum_i \lambda_i T_{i\mu\nu} b(0)\epsilon_a^\mu(-p)\epsilon_b^\nu(p) \; + \; \text{(crossed)} \; ,$$

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$$A_i = -\frac{8M_W^2}{m_i^2 - M_W^2}\left(1 + \frac{m_i^2}{2M_W^2}\right)\int_0^1 \mathrm{d}x\, x(1-x)\ln\frac{D}{C}$$

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- Expanding this one sees, as expected, that there is no power suppression of large logs.

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- General colour-singlet $\eta' \to g^*(k_1) g^*(k_2)$ amplitude:

$$N^{ab}_{\mu\nu}(k_1^2, k_2^2) = -i \, F_{\eta' g^* g^*}(k_1^2, k_2^2) \, \epsilon_{\mu\nu\rho\sigma} k_1^\rho k_2^\sigma \delta^{ab} \; .$$

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- Perturbative QCD, hard scattering approach Ali and Parkhomenko (2002), Kroll and Passek-Kumerički (2002) $\Rightarrow\ 1/Q^2$ suppression ($Q^2 \equiv |k_1|^2 = |k_2|^2$)

$$F_{\eta' g^* g^*}(Q^2)\Big|_{Q^2 > m_b^2} \longrightarrow 4\pi\alpha_s(Q^2) \frac{f^1_{\eta'}}{\sqrt{3} Q^2} \ ,$$

- $f^1_{\eta'} \approx 1.15\sqrt{2} f_\pi$ known from $\eta_1 - \eta_8$ mixing theory (Feldman and Kroll (1998))

Gluing two pieces together

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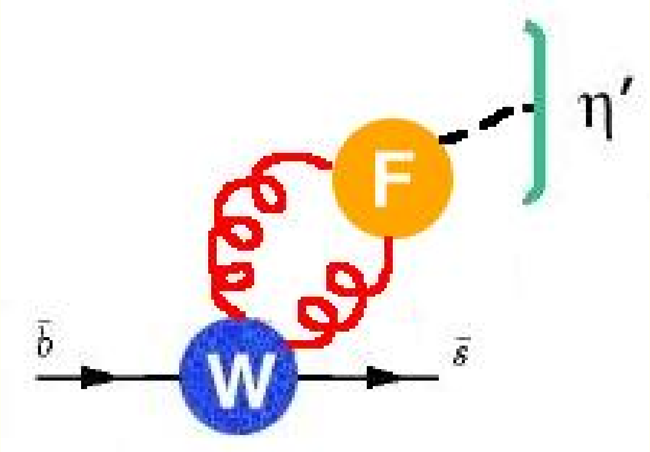

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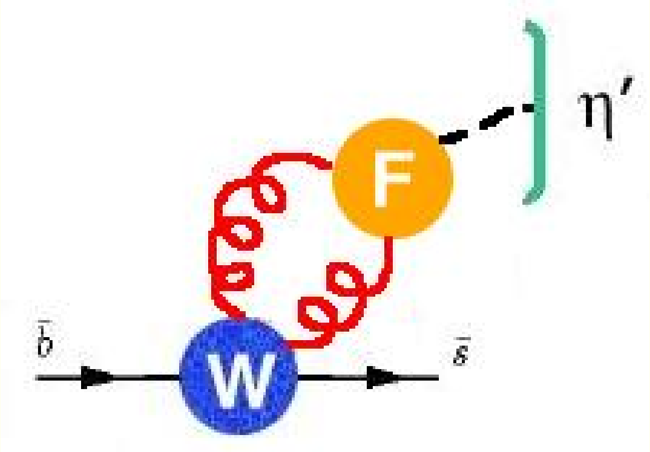
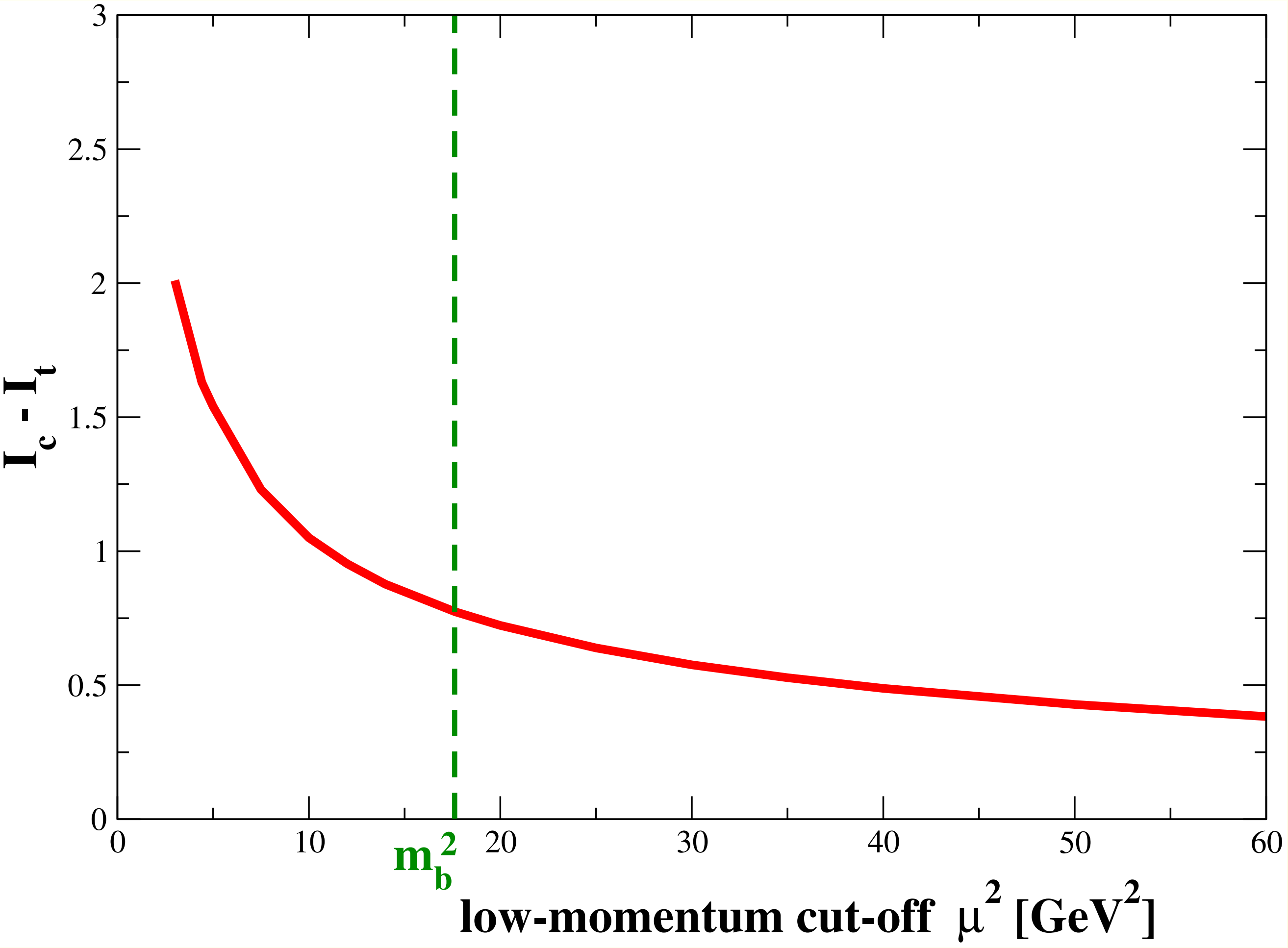


- to leading orders in $m^2_{\eta'}/Q^2$ and $m^2_{b,s}/Q^2$ we get

$$\mathcal{A}(b \to s\eta') = \frac{G_F f_\pi}{3\pi^2} (\phi_{\eta'} \bar{s} \not{P}_{\eta'} L b) \sum_{i=u,c,t} \lambda_i \int_{\mu^2 \sim m_b^2}^{M_W^2} dQ^2 \, \frac{\alpha_s^2(Q^2)}{Q^2} \, A_i(-Q^2) \ ,$$

♦ Check that the dependence on the infra-red cut-off μ^2 is mild:



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i.e. we have the desired $S \sim 0.5P$.

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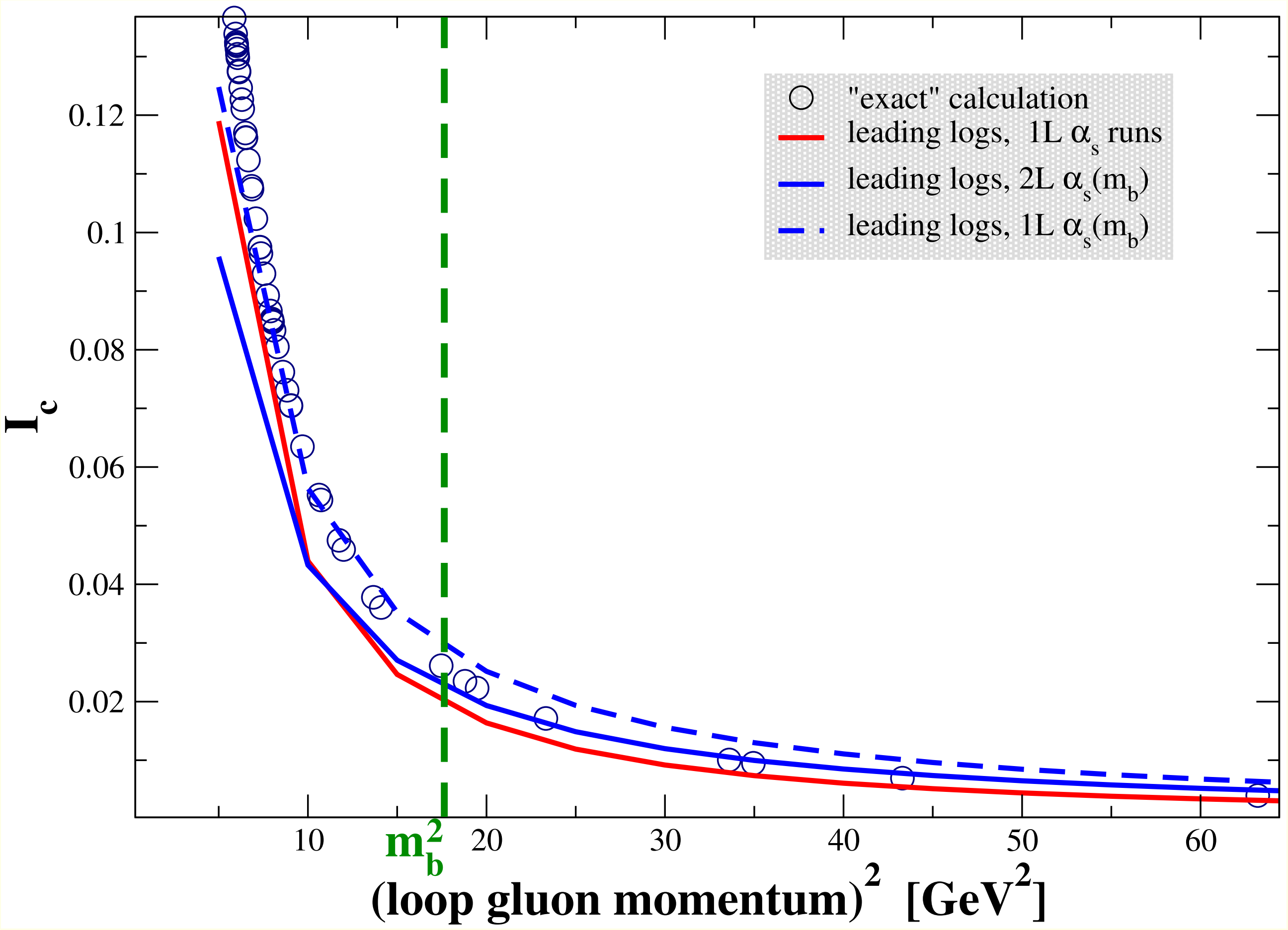
QCD corrections, leading logs



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