Applying Heavy-Light Chiral Quark Model to Rare B Decays

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Collaboration with:

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Ivica Picek (University of Zagreb)

Overview

- Introduction to B decays

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- Conclusions

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- ▶ parameters involving 3rd quark family still poorly known

[to exp.]

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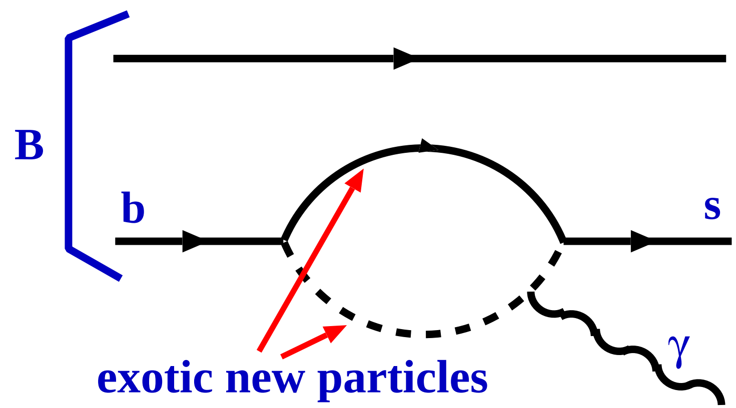

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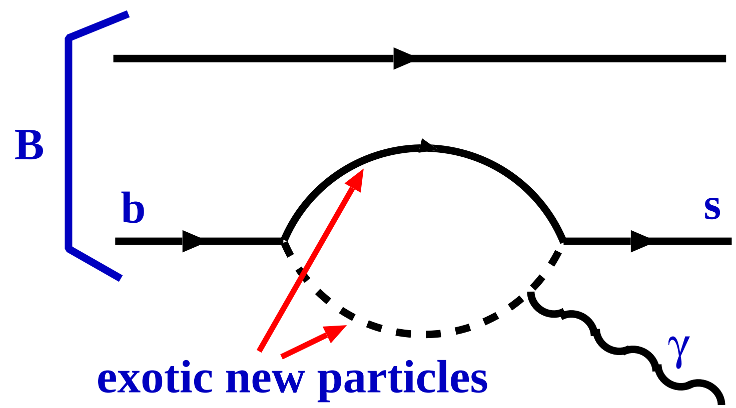

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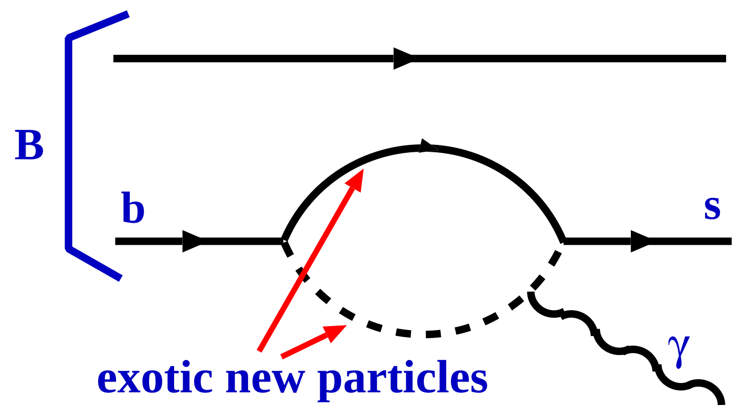

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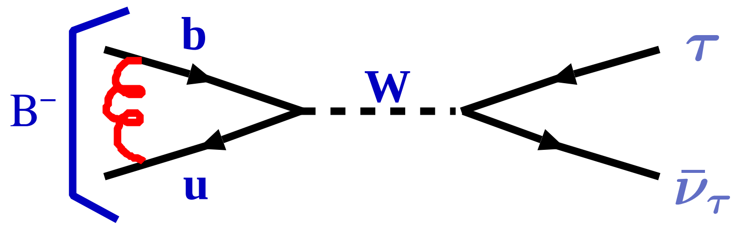
► large energy scale $\sim m_b$ $\Rightarrow$ less QCD-pollution

Types of B decays

- leptonic



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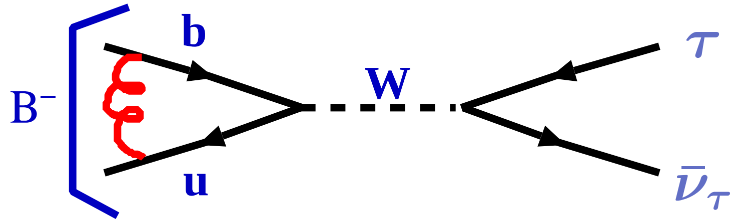

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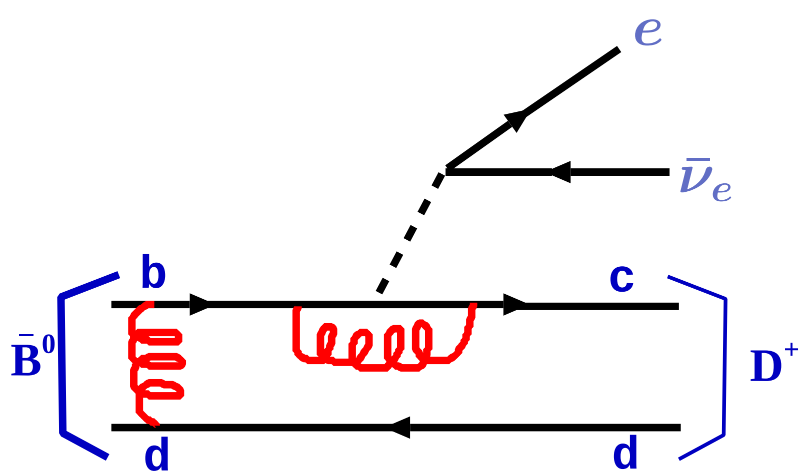
- semi-leptonic



$$\langle D | J^{\text{weak}}_{\text{hadr.}} | B \rangle \propto F_0(q^2), F_1(q^2)$$

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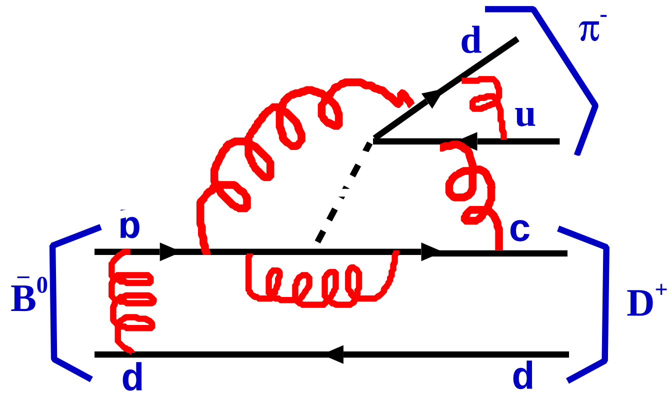

$\langle \boldsymbol{D\pi} | \boldsymbol{J}^{\text{weak}}_{\text{hadr.}\mathbf{1}} \boldsymbol{J}^{\text{weak}}_{\text{hadr.}\mathbf{2}} | \boldsymbol{B} \rangle \longrightarrow$ very complicated

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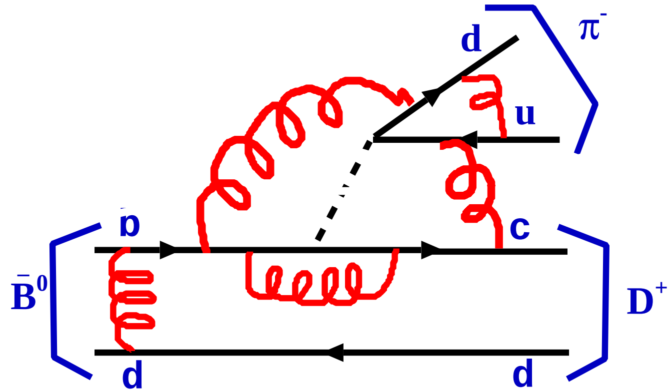

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$$\approx \langle \pi | J^{\text{weak}}_{\text{hadr.}1} | 0 \rangle \langle D | J^{\text{weak}}_{\text{hadr.}2} | B \rangle \propto f_\pi, F_0(q^2), F_1(q^2)$$

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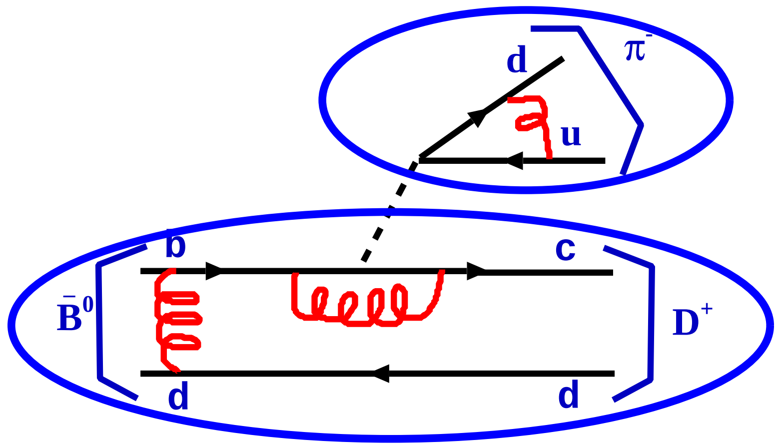

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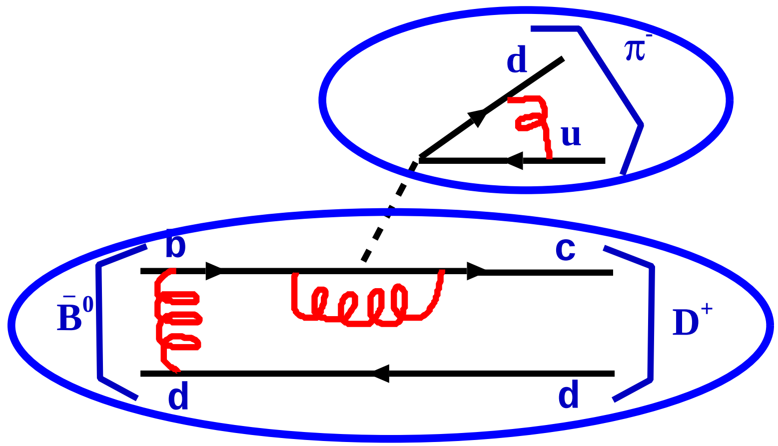

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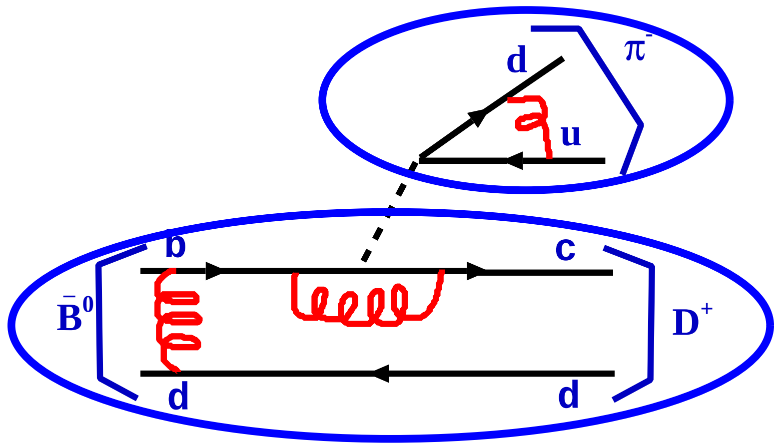

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- explanation: fast pion → "color transparency"
- good as a 1$^{\text{st}}$ approximation but needs improvement

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- Expand the $\langle D\pi | J^{\text{weak}}_{\text{hadr.}1} J^{\text{weak}}_{\text{hadr.}2} | B \rangle$ in both

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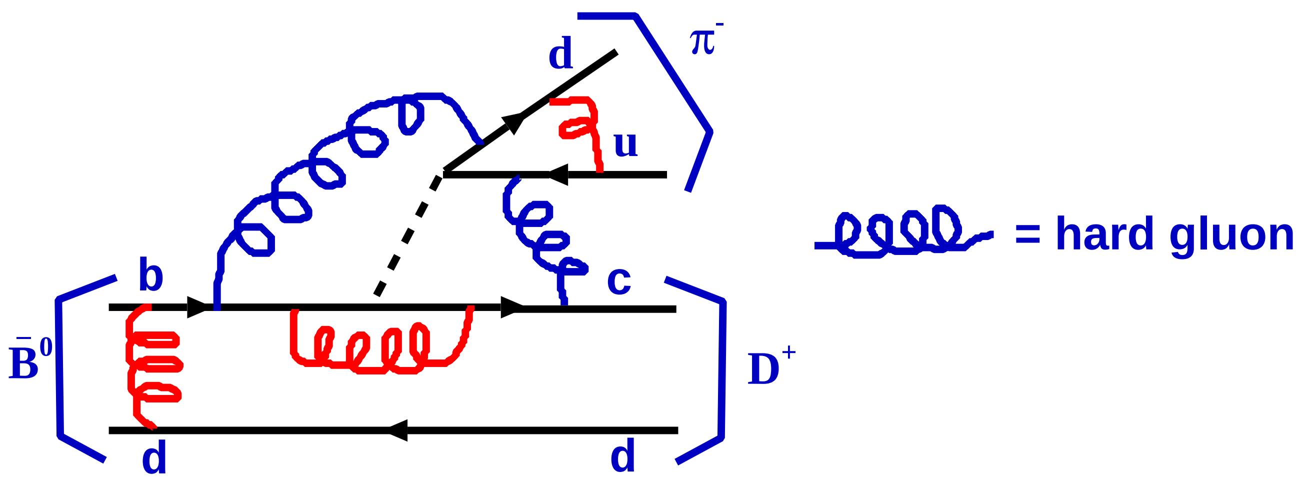
- To lowest order “non-factorizable gluons” are hard $\Rightarrow$ can be treated perturbatively [Beneke, Buchalla, Neubert, Sachrajda, (1999,2000)]



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- More "primitive" but has some advantages

At the meson level

- Implementing HQ and chiral symmetries at the meson level [Burdman and Donoghue (1992), Wise (1992)]

$$\mathcal{L} = \frac{f^2}{4}\partial_\mu \Sigma_{ab}\partial^\mu \Sigma^\dagger_{ba} - \mathrm{Tr}\left[\bar{H}_a\left(iv\cdot\mathcal{D}_{ba}\right)H_b\right]$$

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$$H_a = \frac{1 + \not{v}}{2}\left(\bar{B}^*_{a\mu}\gamma^\mu - i\bar{B}_a\gamma_5\right) \; ; \qquad \bar{B}_a = \left(B^-, \bar{B}^0, \bar{B}_s\right)$$

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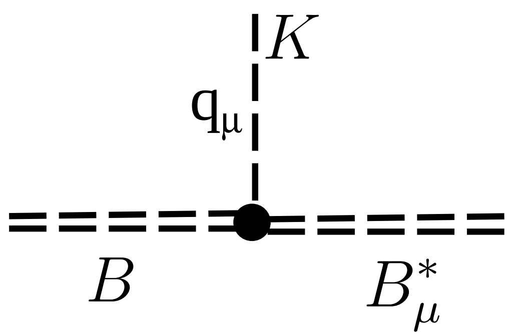
- 

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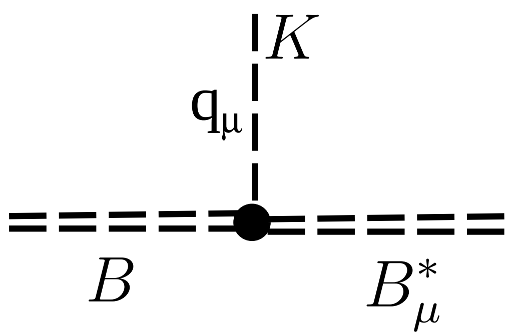
- 

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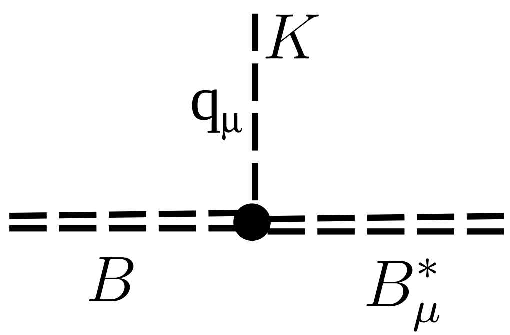
- 

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- Calculate chiral corrections to $f_B / f_{B_s}, \ldots$

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$$\mathcal{L}_{\chi QM} = \bar{\chi}\Big[\gamma^\mu(i\partial_\mu + \mathcal{V}_\mu + \gamma_5 \mathcal{A}_\mu) - m\Big]\chi$$

$\chi(x) = e^{i\gamma_5 \Pi/f}\, q(x)$ quarks with G. bosons “removed”

$m \approx 250$ MeV constituent mass

Heavy-Light Chiral Quark Model (2)

- The usual heavy quark effective theory

$$b \rightarrow e^{-im_b v\cdot x}\left(Q_v^{(+)} + \frac{1}{2m_b}\frac{1-\not{v}}{2} i\not{D} Q_v^{(+)}\right)$$

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- Heavy-light meson-quark interaction

$$\mathcal{L}_{Int} = -G_H\left[\bar{\chi}_a\,\overline{H_{va}^{(\pm)}}\,Q_v^{(\pm)} + \overline{Q_v^{(\pm)}}\,H_{va}^{(\pm)}\,\chi_a\right]$$

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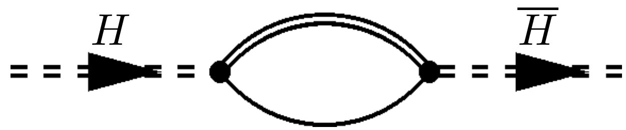
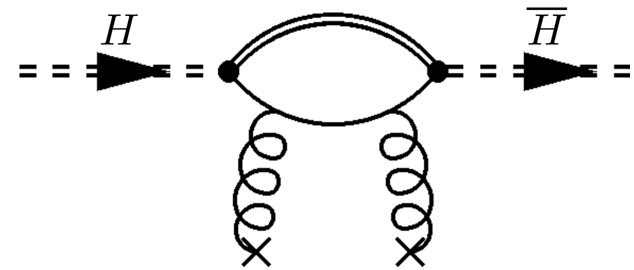
- To determine parameters of the theory (such as quark-meson coupling constant G_H):

  ► integrate out quarks ("bosonise theory") and create constraints on parameters

Bosonisation of HLχQM

- E.g. calculating self-energy of a heavy meson





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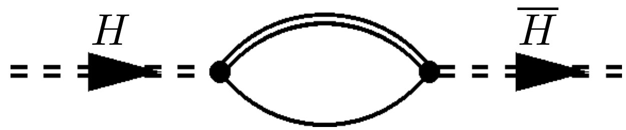

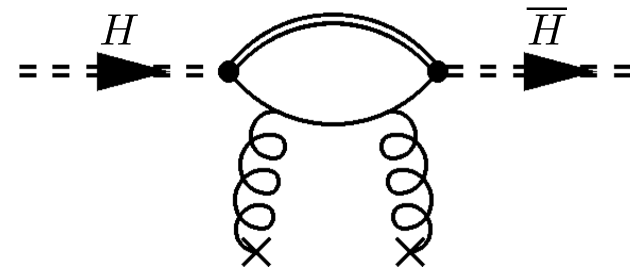

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kinetic Lagrangian for heavy boson leads to constraint

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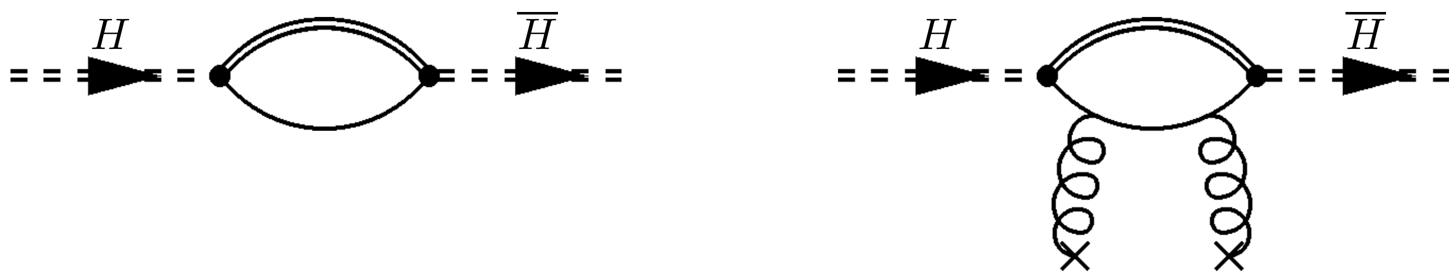

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- This completes definition of the model

Application: Isgur-Wise function

- Isgur-Wise function $\xi(\omega)$ defined by

$$\langle D|\overline{Q_{cv'}}\gamma^\mu Q_{bv}|B\rangle = \sqrt{M_B M_D}\,\xi(\omega)(v+v')^\mu$$

$$\omega \equiv v \cdot v'$$

Application: Isgur-Wise function

- Isgur-Wise function $\xi(\omega)$ defined by

$$\langle D|\overline{Q_{cv'}}\gamma^{\mu}Q_{bv}|B\rangle = \sqrt{M_B M_D}\,\xi(\omega)(v+v')^{\mu}$$

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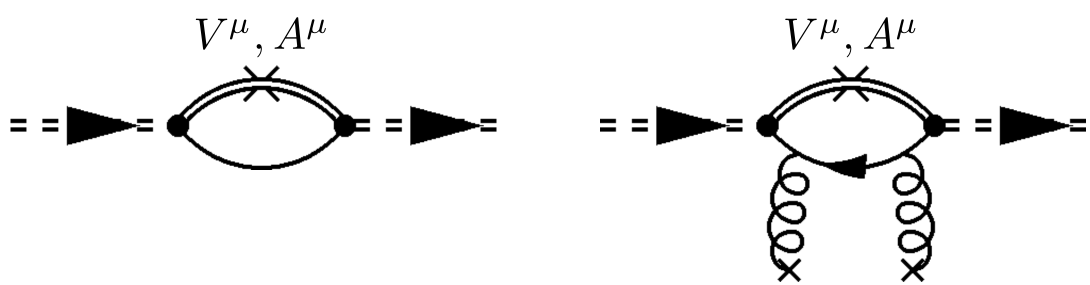
- Determined by diagrams



$$V^\mu = \gamma^\mu\ , \quad A^\mu = \gamma^\mu\gamma_5$$

Isgur-Wise function — results

- Result:

$$\xi(\omega) = \frac{2}{1+\omega}(1-\rho) + \rho r(\omega) + \frac{\rho\langle 0|\frac{\alpha_s}{\pi}G^2|0\rangle}{24m^2 f_\pi^2}\frac{1-r(\omega)\omega}{1+\omega}$$

where

$$\rho = \frac{(1+3g_A) + \frac{\pi}{32}\frac{G_H^2}{m^3}\langle 0|\frac{\alpha_s}{\pi}G^2|0\rangle}{4(1+\frac{N_c m^2}{8\pi f^2})}$$

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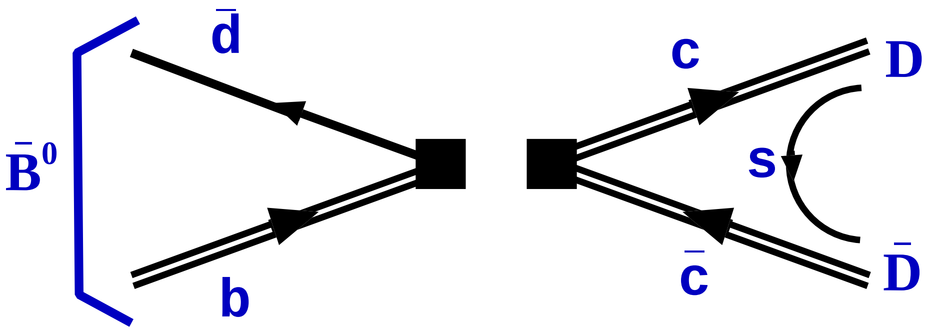
- Numerically, expanding around no-recoil point $\omega = 1$:

$$\xi(\omega) = 1 - 0.64(\omega - 1) + \cdots$$

Application: $B_d \longrightarrow D_s\bar{D}_s$

- [Eeg, Fajfer, Prapotnik (2005); Eeg, K.K. (2005)]
- factorized amplitude



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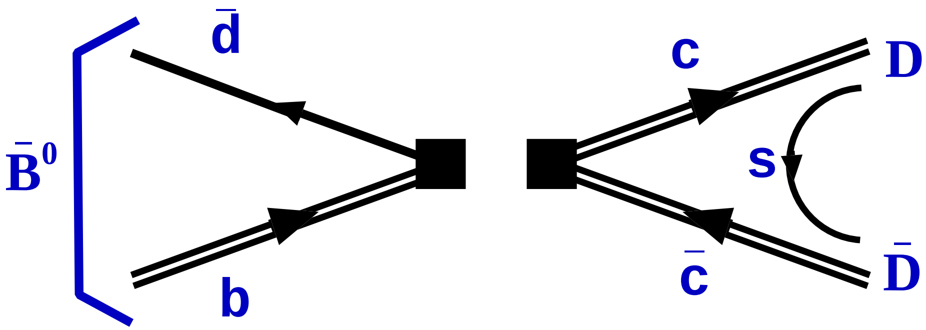

$$\underbrace{\langle 0 | J^{\mu}_{(V-A)} | \bar{B} \rangle}_{\sim f_B p_B^{\mu}} \quad \times \quad \langle D_s \bar{D}_s | J_{(V-A)_\mu} | 0 \rangle$$

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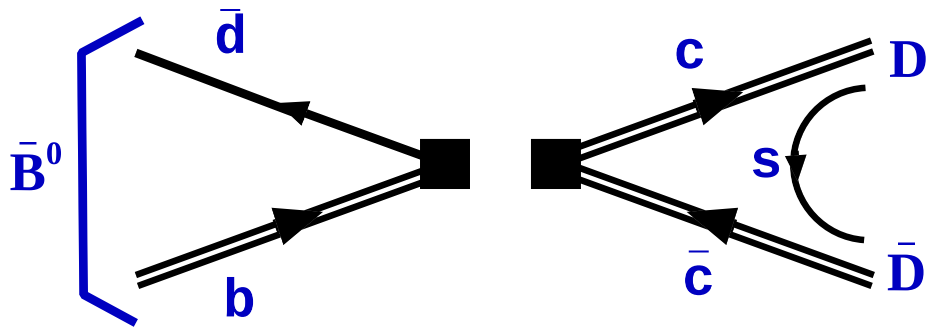

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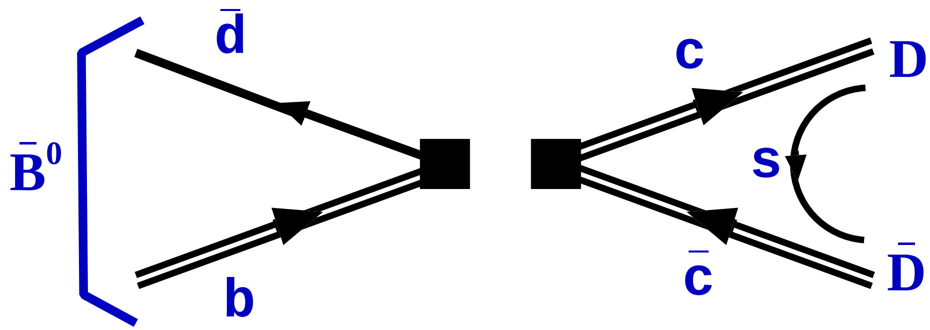

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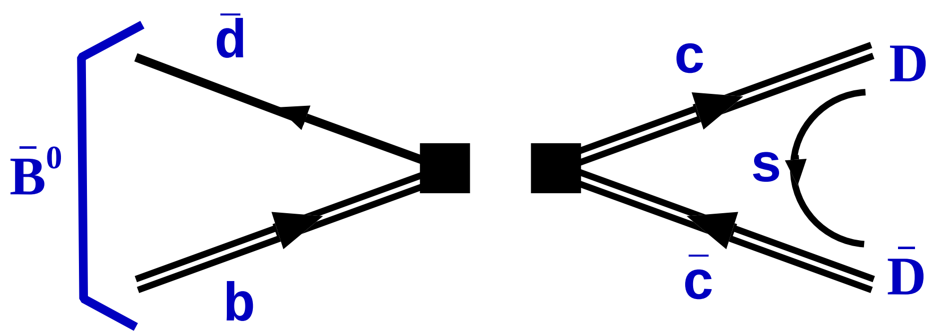

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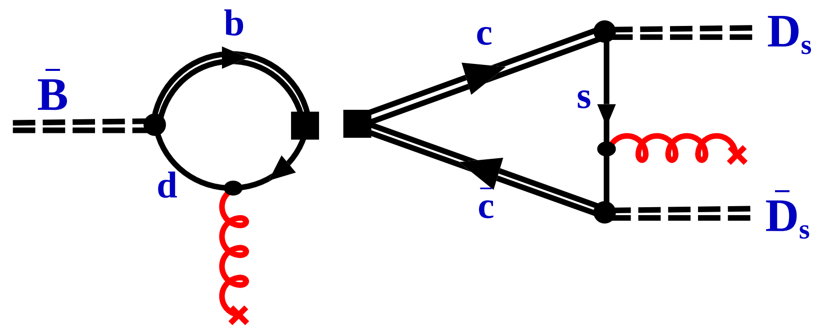
- non-factorizable contributions dominant

Gluon Condensate Contributions



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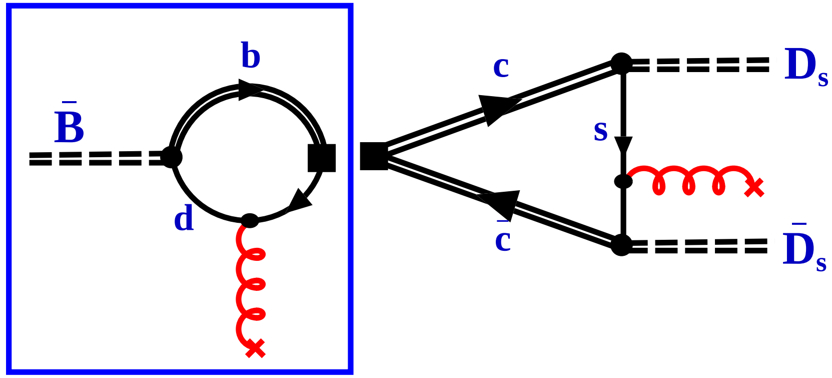

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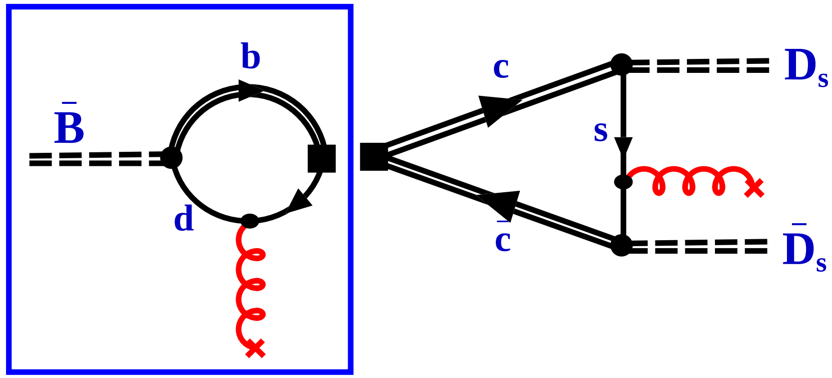

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$$\left(\overline{q_L}\, t^a\, \gamma^\alpha\, Q_{v_b}^{(+)}\right)_{1G} \rightarrow$$

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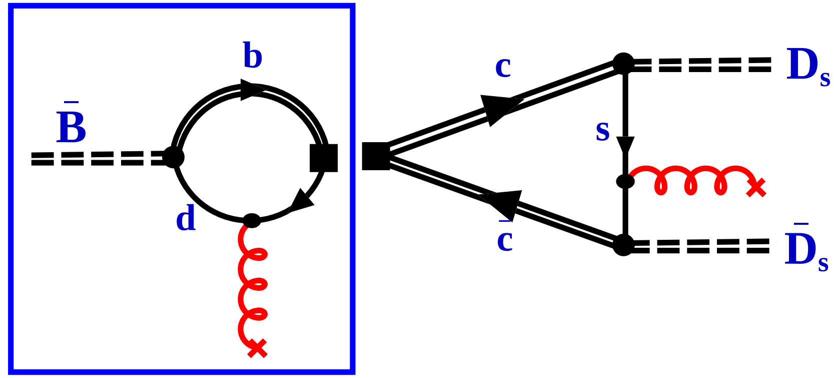

- Bosonisation of left-hand-side

$$\left(\overline{q_L}\, t^a\, \gamma^\alpha\, Q_{v_b}^{(+)}\right)_{1G} \rightarrow$$

$$\rightarrow \frac{G_H\, g_s}{64\pi}\, G^a_{\mu\nu}\, \mathrm{Tr}\left[\xi^\dagger \gamma^\alpha L\, H_b^{(+)} \left(\sigma^{\mu\nu} \;-\; F\,\{\sigma^{\mu\nu}, \not{v}_b\}\,\right)\right]$$

$$F \;\equiv\; \frac{2\pi f_\pi^2}{m^2\, N_c} \approx \frac{1}{3}$$

Gluon Condensate Contributions (2)

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$$\left(\overline{Q_{v_c}^{(+)}}\, t^a\, \gamma^\alpha\, L Q_{\bar{v}}^{(-)}\right)_{1G} \rightarrow$$

$$\rightarrow \frac{G_H^2\, g_s}{128\pi m(\lambda - 1)}\, G^a_{\mu\nu}$$

$$\times\, \mathrm{Tr}\left[\overline{H_c^{(+)}}\, \gamma^\alpha\, L\, H_{\bar{c}}^{(-)} \left(X \sigma^{\mu\nu}\, \{\sigma^{\mu\nu}, \not{v}_c - \not{\bar{v}}\}\right)\right]$$

$$X \equiv \frac{4}{\pi}(\lambda - 1)\, r(-\lambda)\ ; \qquad \lambda \equiv \bar{v} \cdot v_c$$

$$r(x) = \frac{1}{\sqrt{x^2 - 1}} \ln\left(x + \sqrt{x^2 - 1}\right)$$

Gluon Condensate Contributions (3)

- Multiplying LHS $\times$ RHS, and doing traces one obtains the amplitude

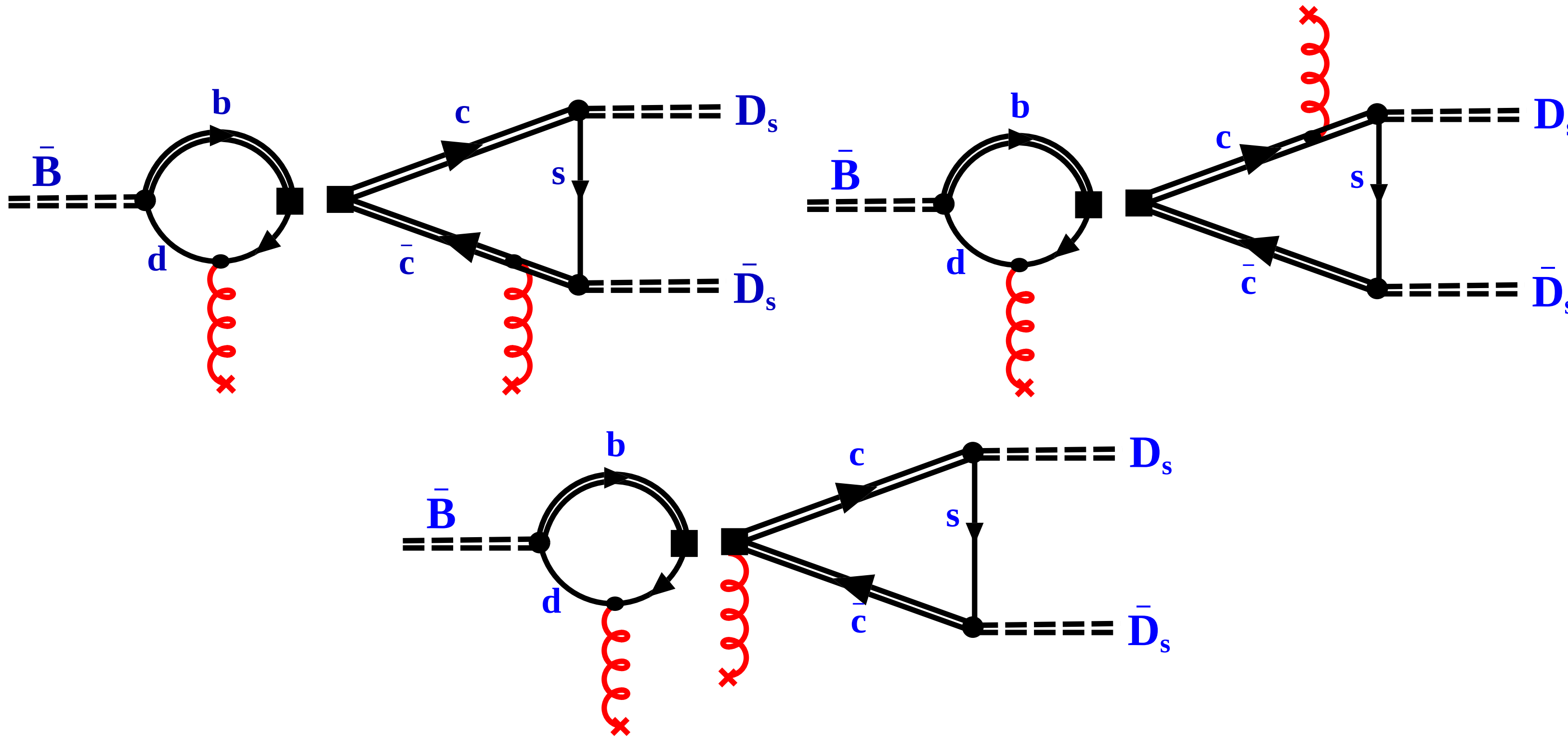
$$\mathcal{A}(B \to D\bar{D})_{\text{gluon condensate}} =$$

$$\frac{G_F}{\sqrt{2}} V_{cb} V_{cd}^* \, a_2 \, \frac{G_H^3 \sqrt{M_B^3}}{3m(\lambda - 1)2^8} \, \langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle$$

$$\times \quad \left[-\frac{3}{4}(2F + 1)X + \lambda - 1 \right]$$

$$a_2 = 1.29 + 0.08i$$ Wilson coefficient

$1/m_c$ corrections



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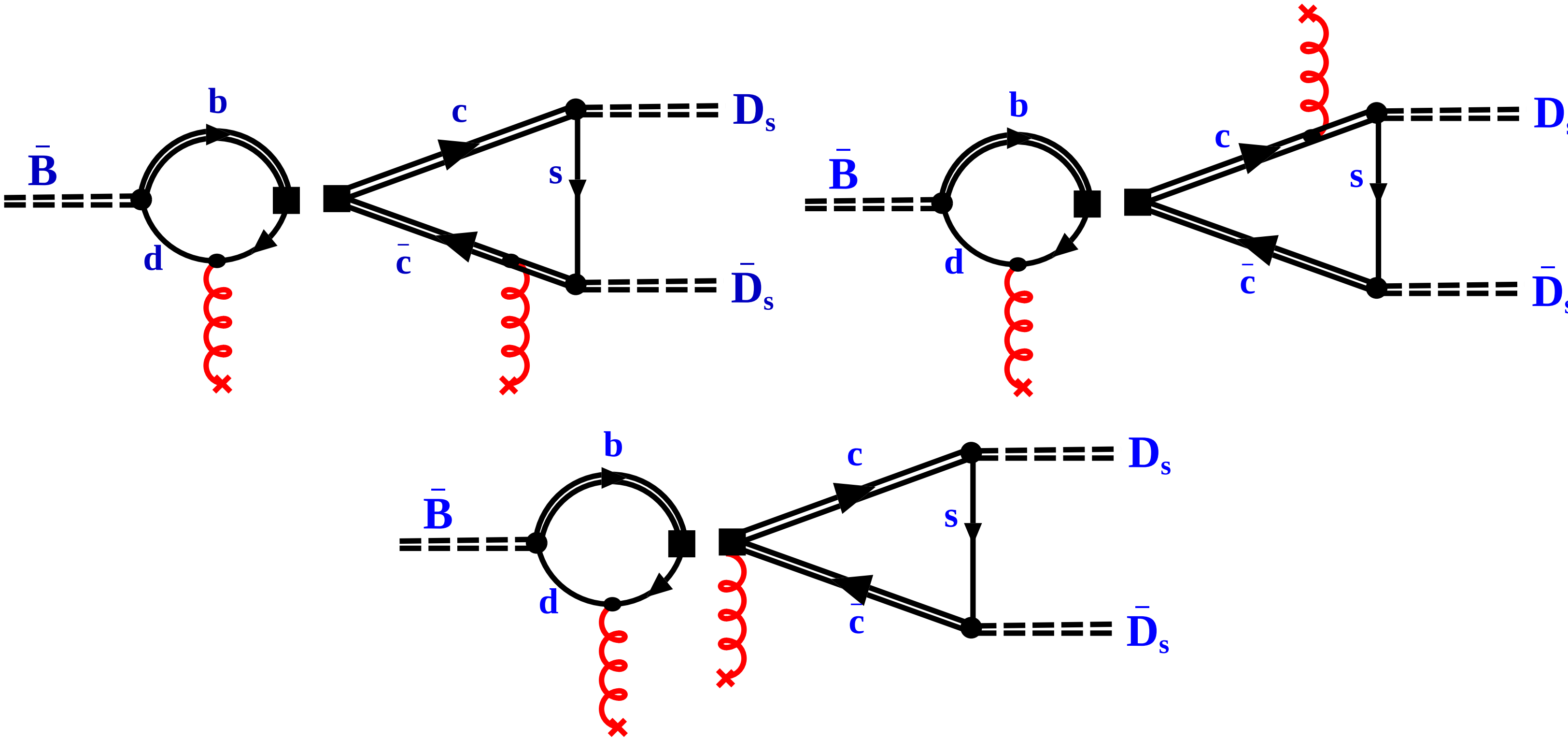

- $c \to e^{-im_c v\cdot x}\left(Q_v^{(+)} + \frac{1}{2m_c}\frac{1-\not{v}}{2} i\not{D} Q_v^{(+)}\right)$ in HQET

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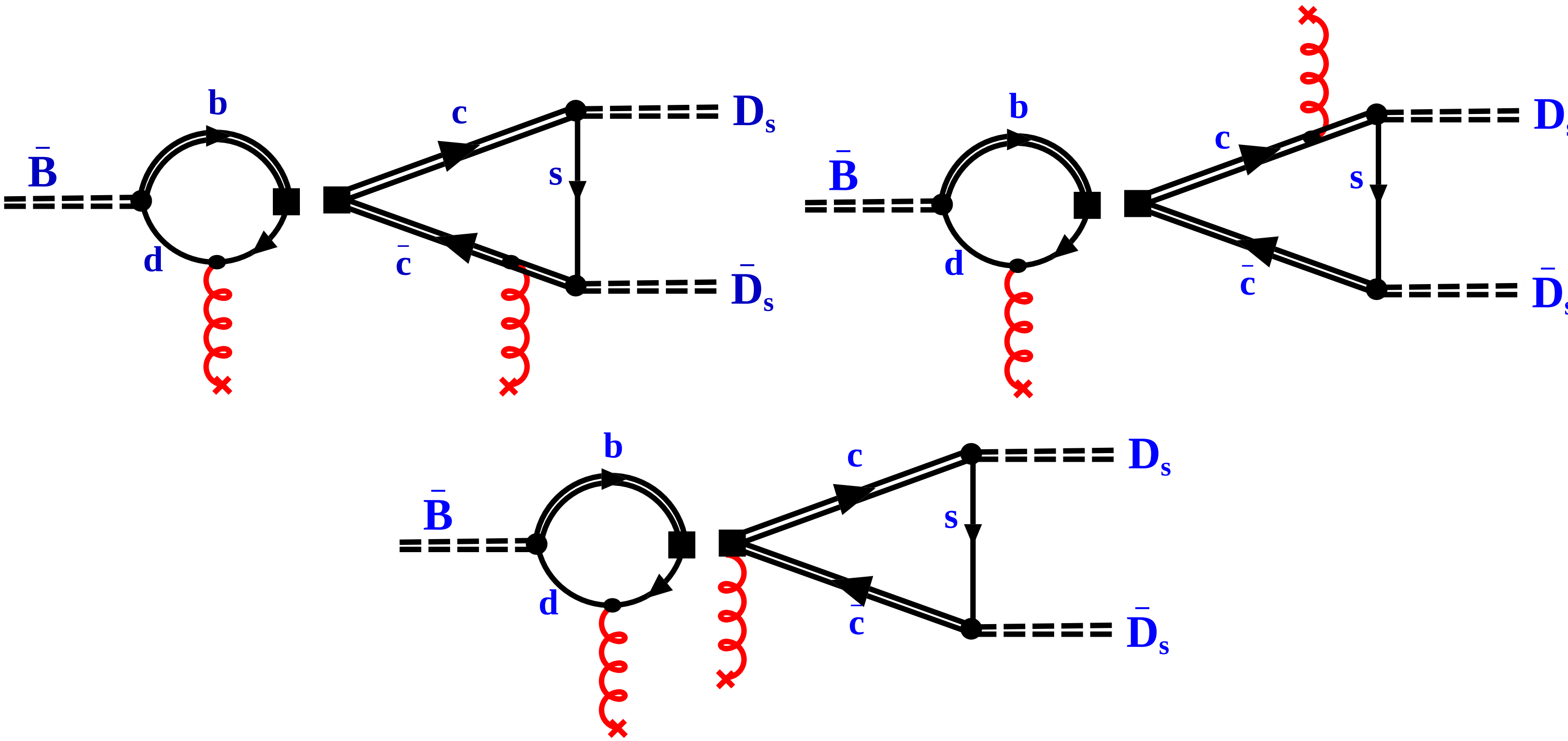

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- amounts to $\approx 30\%$ correction

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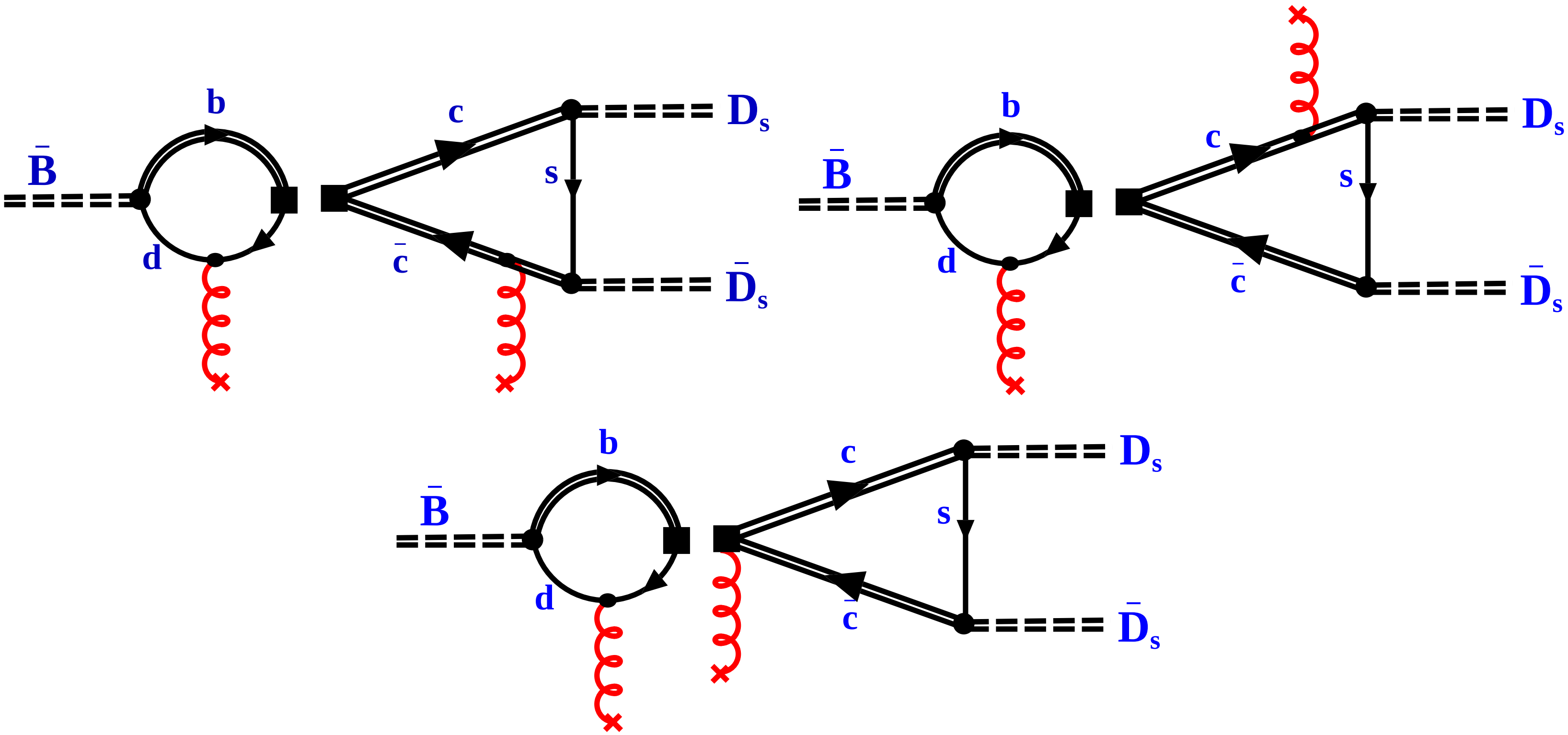

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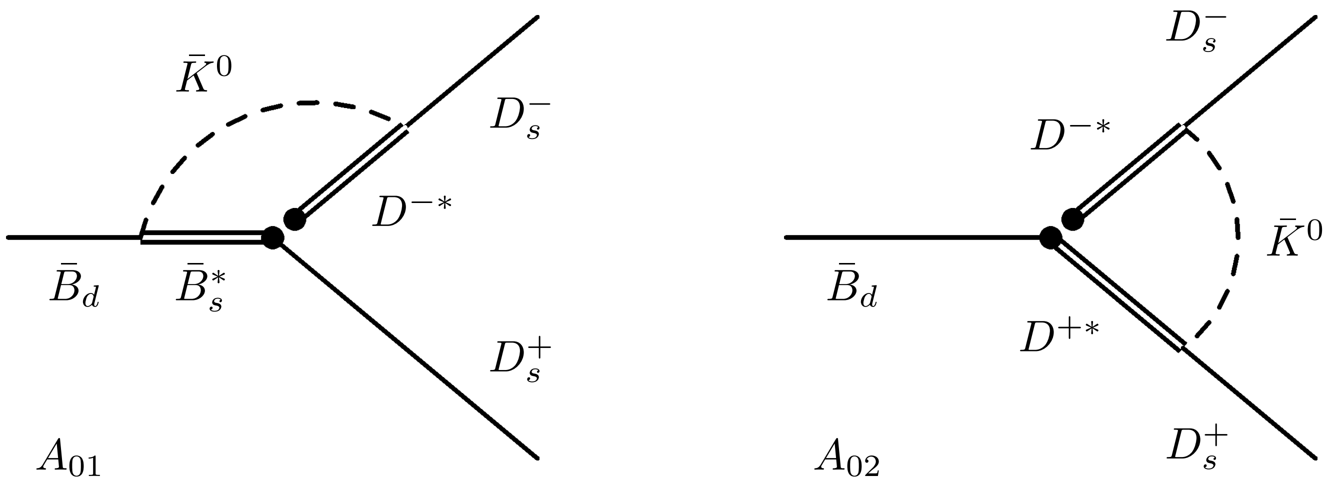
- $1/m_b$ corrections much smaller (but calculated as well)

Chiral loop contributions



- Generated by heavy-light chiral perturbation theory — also non-factorizable

Predictions for $B_d \longrightarrow D_s^{(*)} \overline{D_s^{(*)}}$

- (Numbers without $1/m_c$ corrections; $\approx 30\%$)

$$Br(\bar{B}^0 \to D_s^+ D_s^-) = 2.5 \times 10^{-4}$$

$$Br(\bar{B}_s^0 \to D^+ D^-) = 4.5 \times 10^{-3}$$

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- Decays involving vector D^* first to be measured?

Application: $B \to K\eta'$

- CLEO, Belle and BaBar collaborations see a lot of η''s in charmless (rare) hadronic B decays . . .

$$\text{Br}(B^+ \to K^+\eta') = (77 \pm 5) \cdot 10^{-6}$$

$$\text{Br}(B^0 \to K^0\eta') = (61 \pm 6) \cdot 10^{-6}$$

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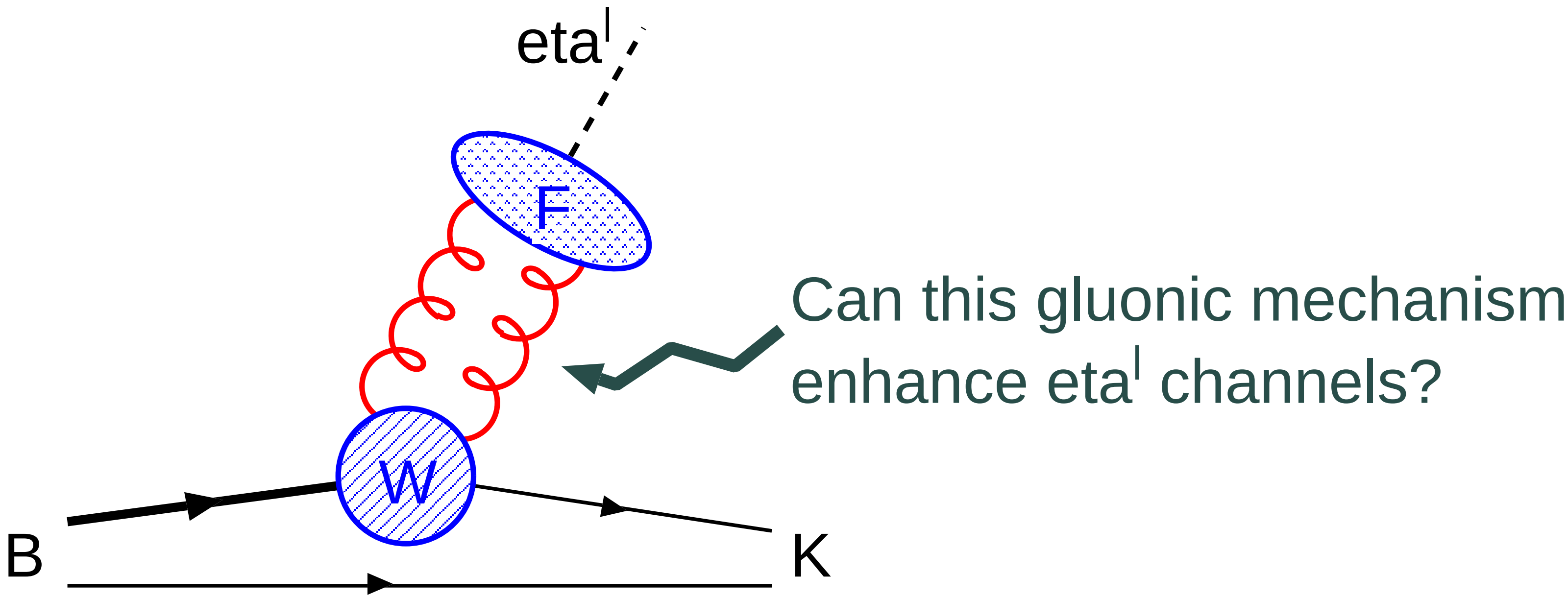

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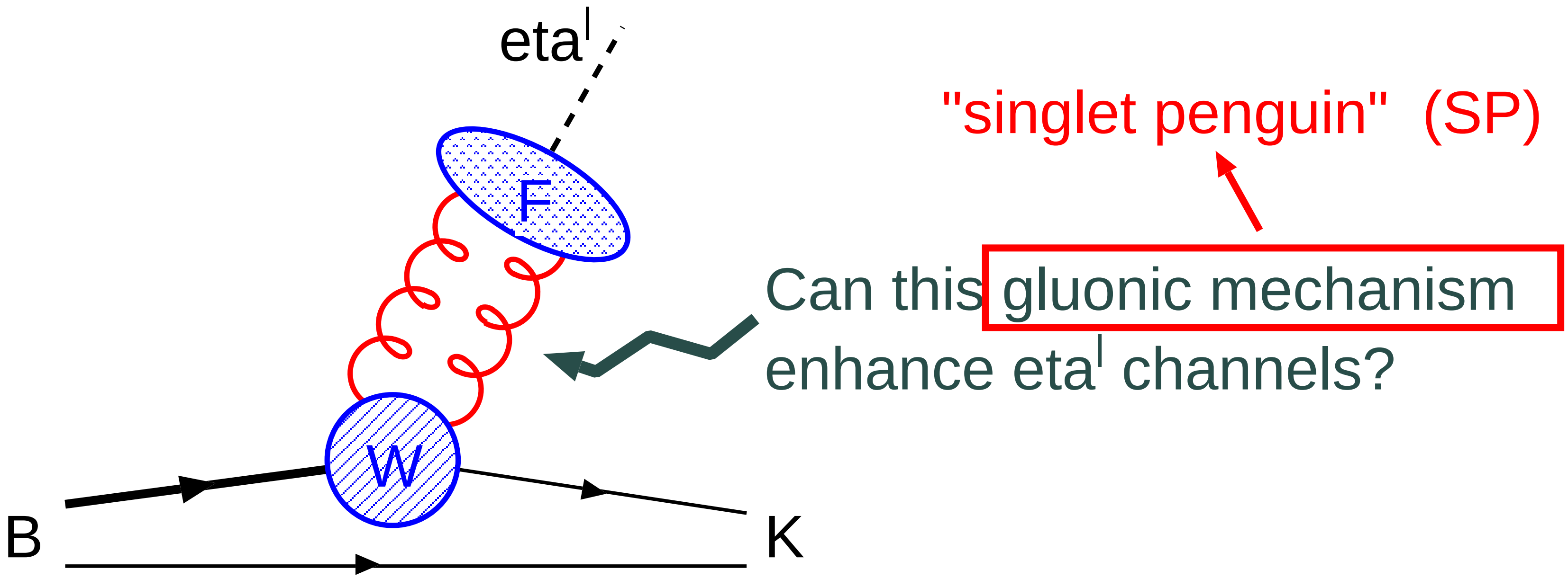

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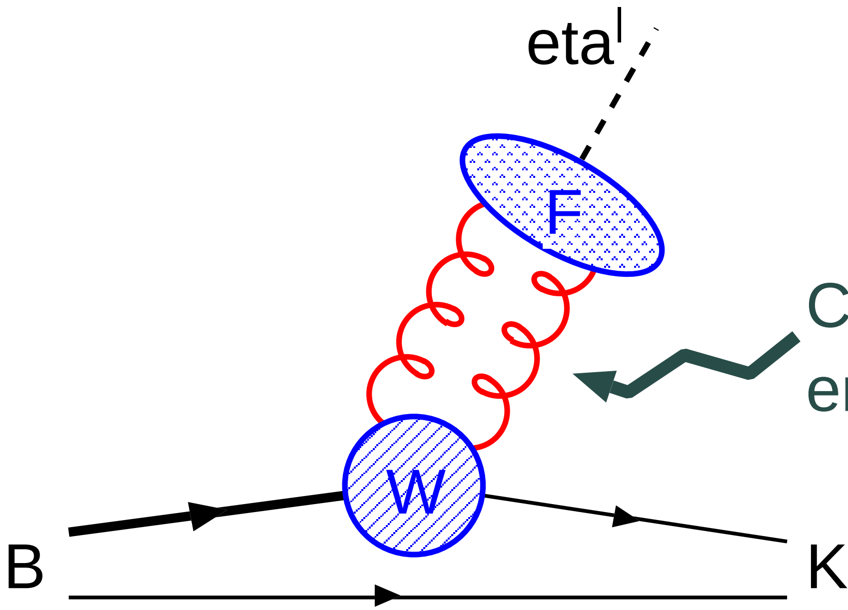

"singlet penguin" (SP)

Can this gluonic mechanism enhance eta| channels?

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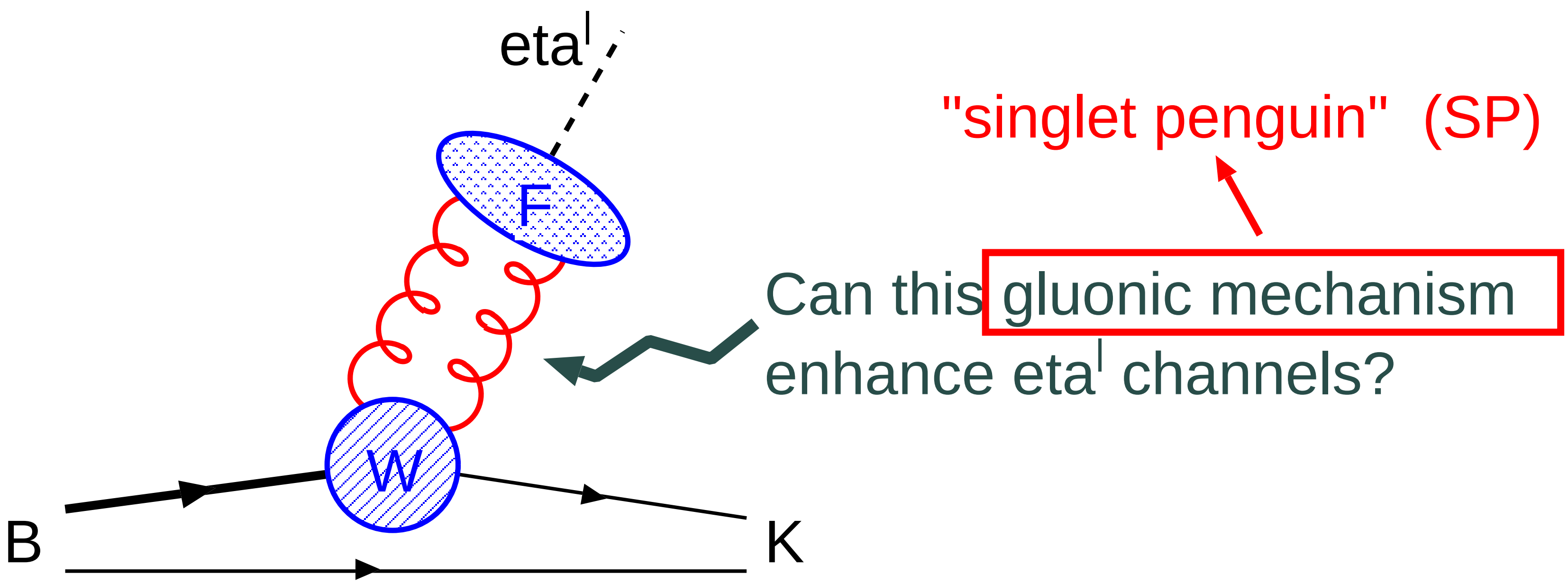

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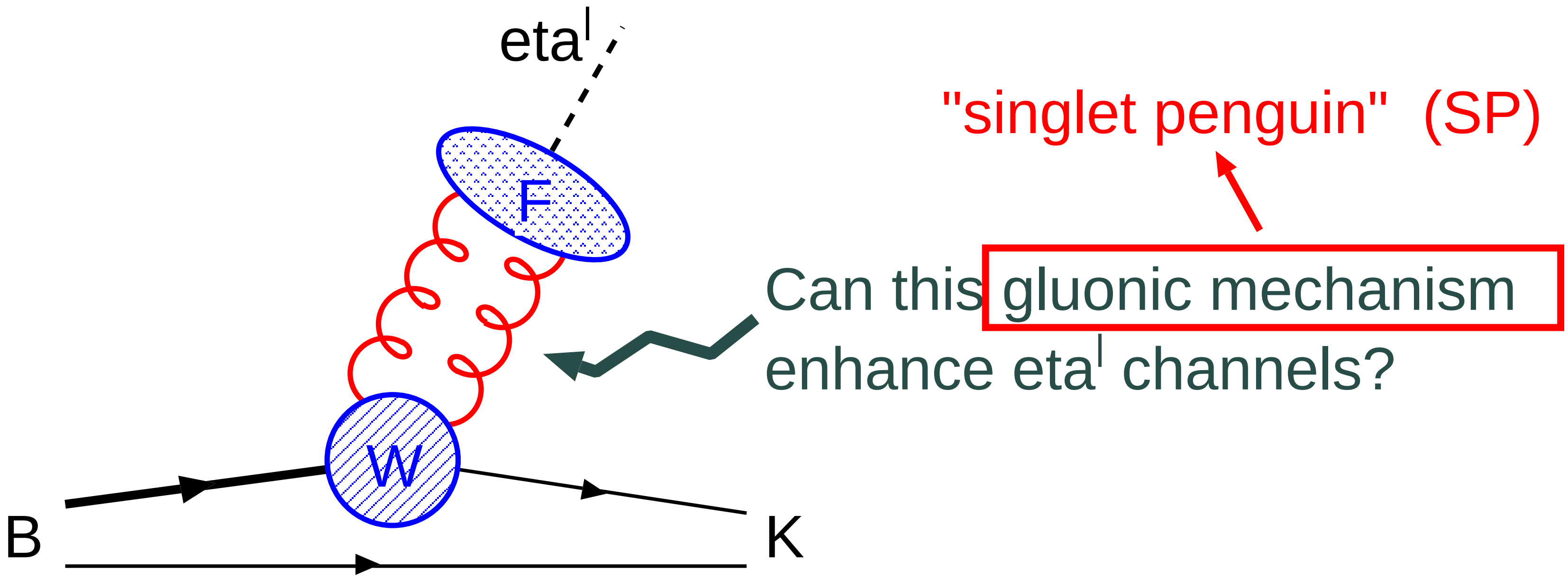

- $SU(3)_F$ symm. approach $\rightarrow SP > 25 \times 10^{-9}$ GeV .
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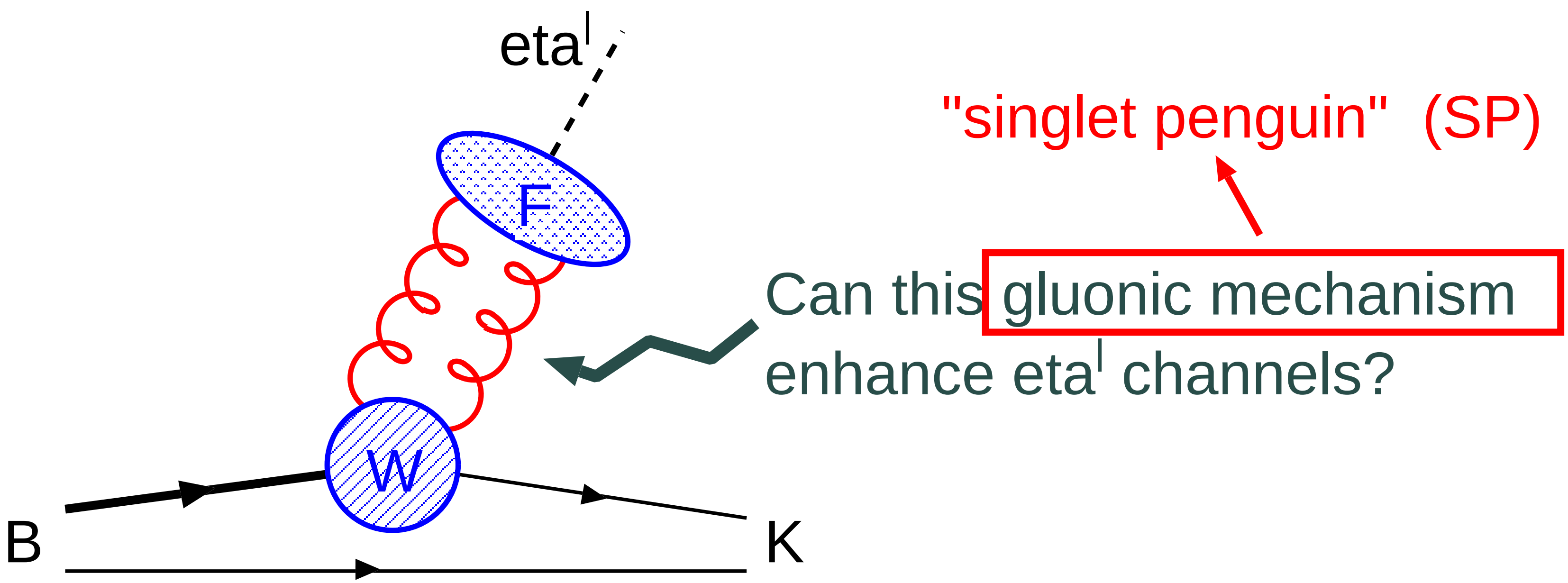
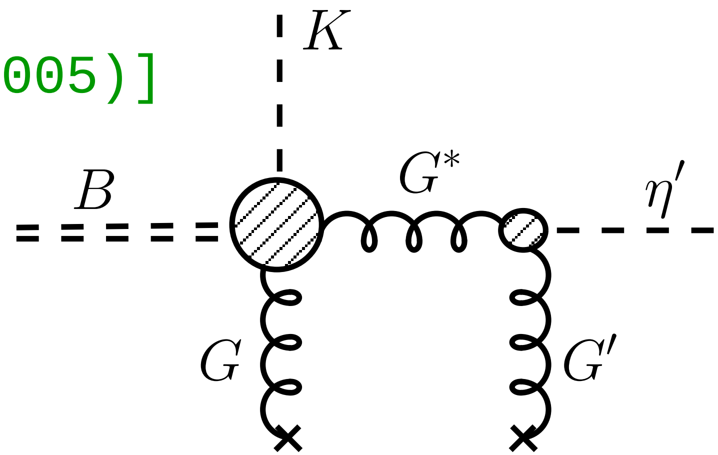


- SU(3)$_F$ symm. approach $\longrightarrow$ $SP > 25 \times 10^{-9}$ GeV .
- perturbative approach $\longrightarrow$ $SP(5 \pm 8) \times 10^{-9}$ GeV

$HL\chi QM$ approach

- [Eeg, K.K., Picek (2005)]
- Schematically:



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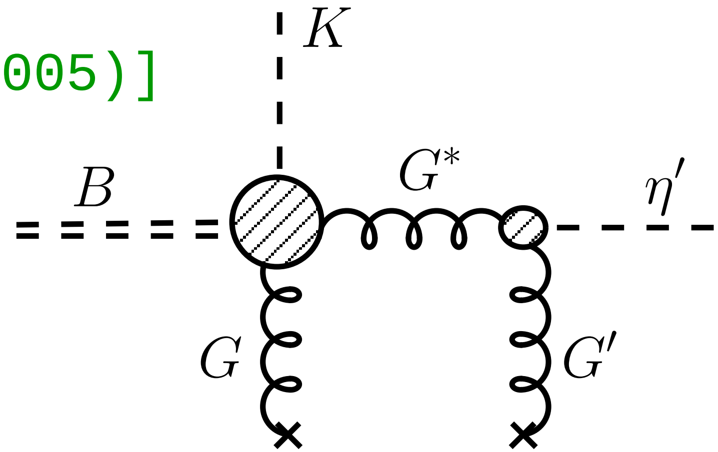

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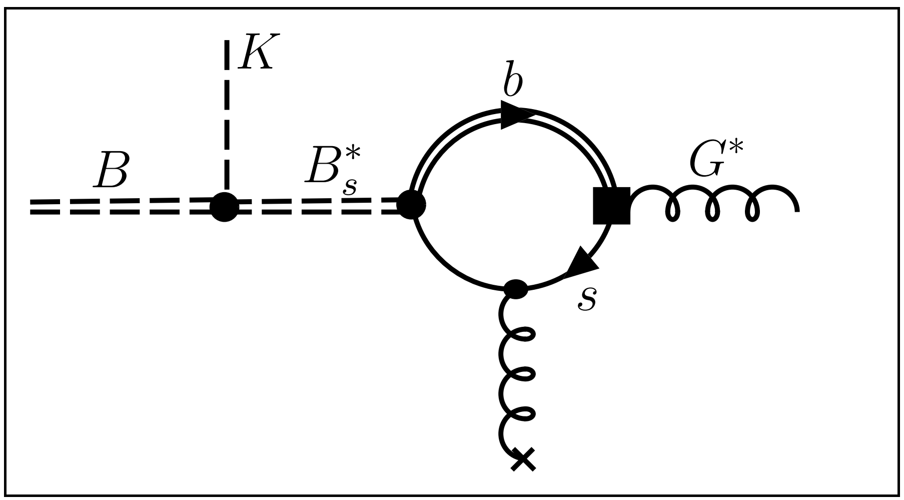

$\times$ $F_{\eta' G G^*}(q^2)$ form factor

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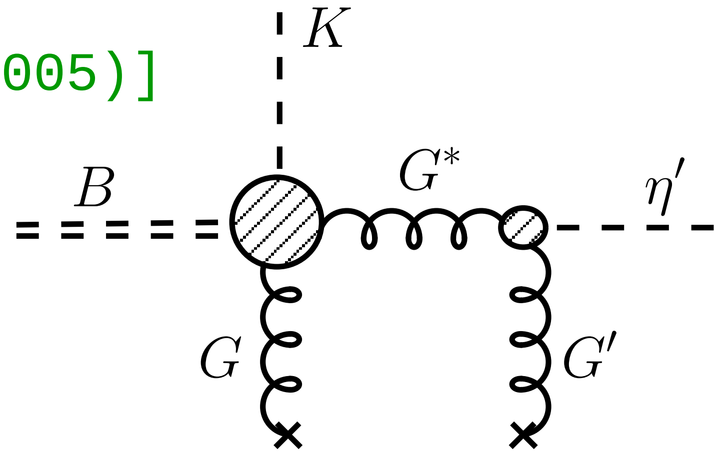

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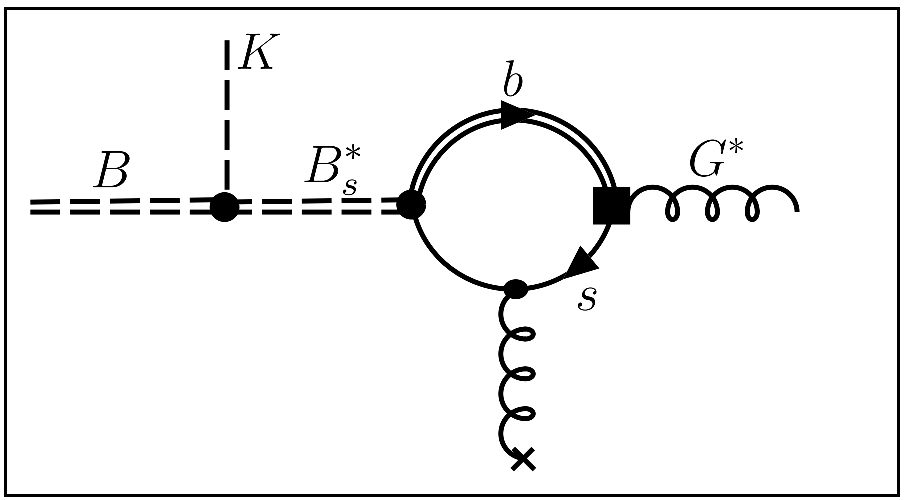

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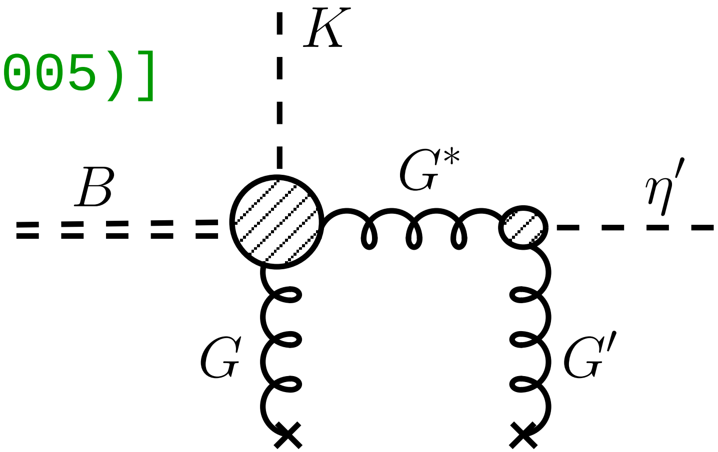

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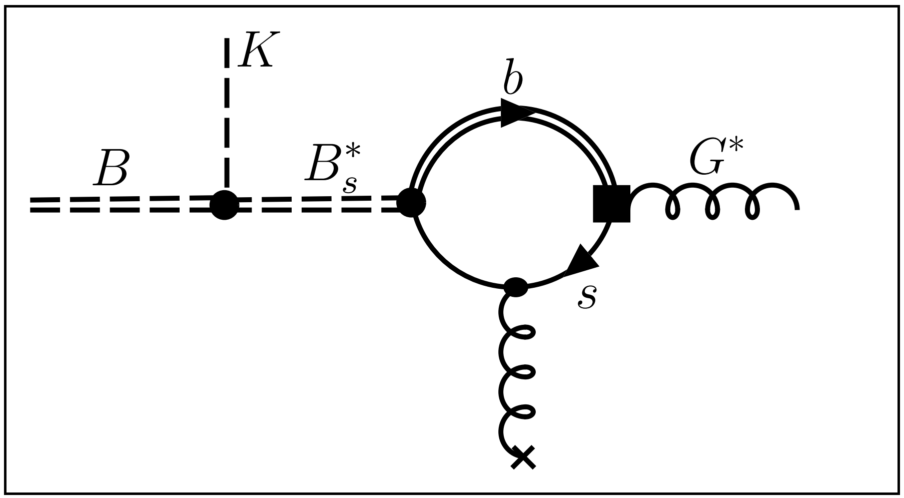

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- RHS: $F_{\eta' G G^*}$ by [Ali and Parkhomenko (2002,2003); Kroll and Passek-Kumerički (2003)]

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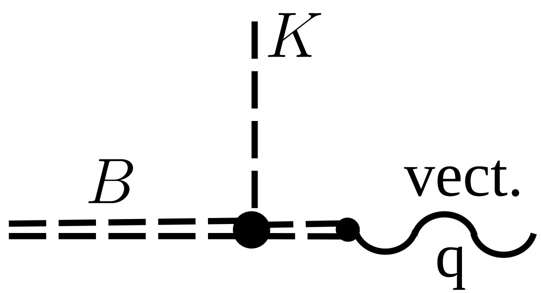

$$: \quad F_K(q^2) = \frac{F_K(0)}{\left(1 - \frac{q^2}{M^2_{B_s^*}}\right)\left(1 - \frac{q^2}{\lambda M^2_{B_s^*}}\right)}$$

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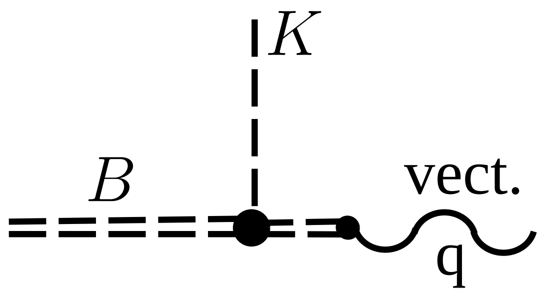

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- Result:

$$M(B \to K\eta')_{\langle G^2 \rangle} = \frac{G_F}{4\sqrt{2}} V_{ts}^* V_{tb}\, \Delta F_1 \langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle$$

$$\times \frac{G_H\, F_K(m^2_{\eta'})\, F_{\eta' G G^*}(m^2_{\eta'})}{3 f_B \sqrt{M_B}} \frac{M_B^2}{32} \left\{ \frac{f_\pi^2}{m^2 N_c} + \frac{3}{2\pi} \right\}$$

Singlet penguin $B \to K\eta'$ (results)

- Singlet penguin amplitude within HLχQM:

$$SP(B \to K\eta')_{\langle G^2 \rangle} = (8 \pm 3) \times 10^{-9} \text{ GeV}$$

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- . . . helps (but not completely) in explaining the difference against SU(3) symmetry approach, where

$$SP(B \to K\eta')_{\mathrm{SU(3)}} > 25 \times 10^{-9} \text{ GeV} .$$

Singlet penguin $B \to K\eta'$ (results)

- Singlet penguin amplitude within HLχQM:

$$SP(B \to K\eta')_{\langle G^2 \rangle} = (8 \pm 3) \times 10^{-9}\ \text{GeV}$$

  with uncertainty coming mostly from uncertainty of $\langle 0|\frac{\alpha_s}{\pi}G^2|0\rangle$ and $F_{\eta' G G^*}$

- Combining this with complementary QCD factorization result [Beneke and Neubert (2002)] ...

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- Singlet penguin apparently not dominant mechanism behind $B \to K\eta'$ rate enhancement.

Conclusions

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The End

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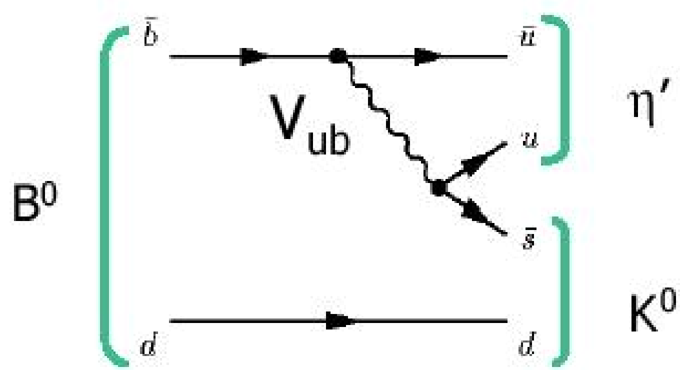
C-Tree $C =$ 

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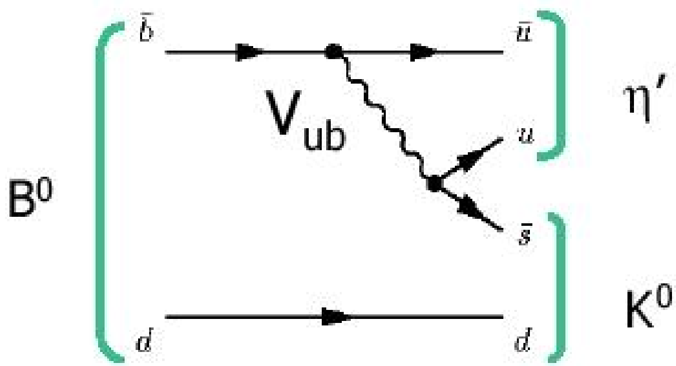

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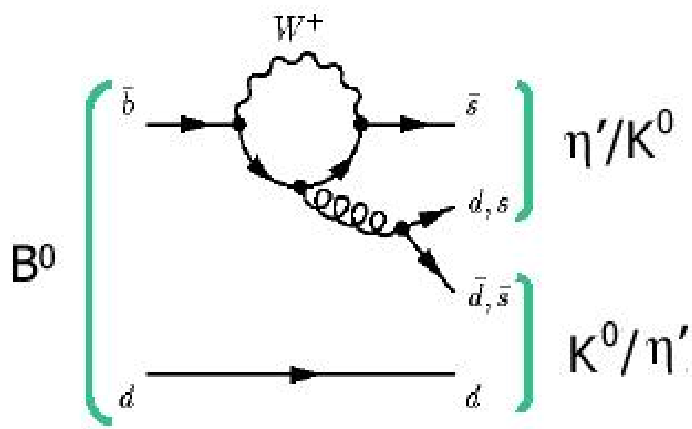
Penguin $P =$



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$SU(3)_F$ flavour symmetry approach

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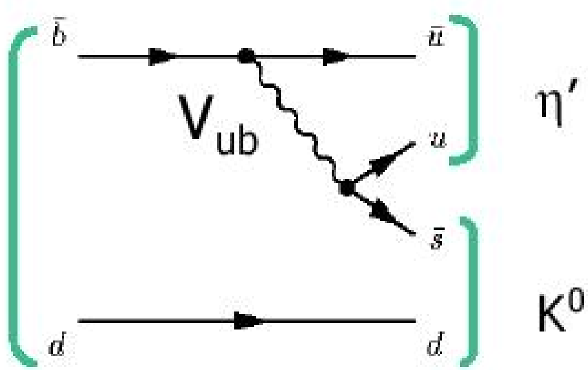
C-Tree $C =$ B^0



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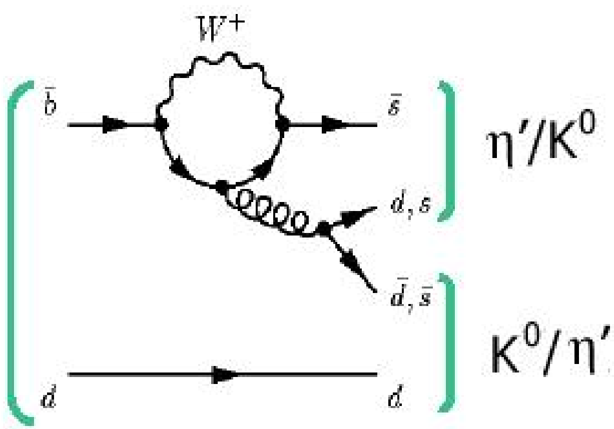
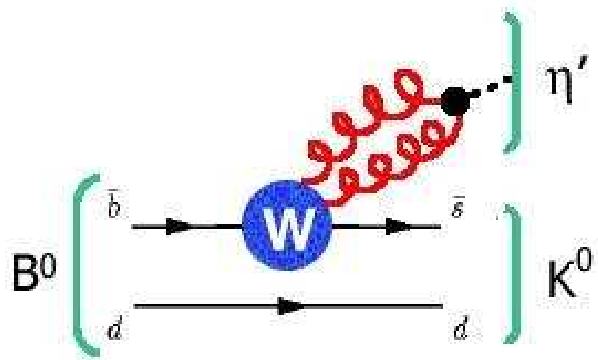


$\longrightarrow$ important contribution (interference!)

Singlet penguin part

- $SP =$ 

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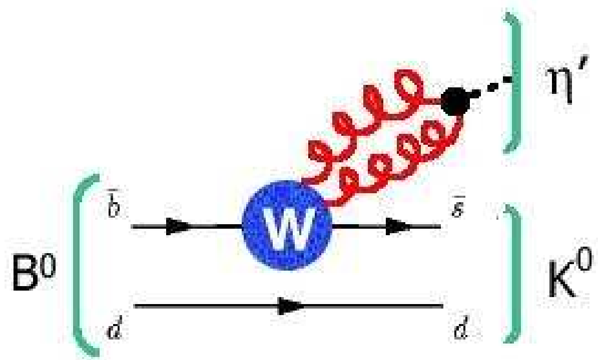
- $SP =$ 

- [Chiang, Gronau, Rosner (2003)]: $SP/P \approx 0.4 - 0.8$
- Possible objections:
  - $\eta - \eta'$ mixing implementation
  - Hybrid method (symmetry + quark dynamics), overcomplete basis: more flavour topologies than true $SU(3)_F$ invariants

Alternative flavour symmetry approaches

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  $\longrightarrow$ results practically unchanged

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$$H_{\text{eff}} = a\, B_k H(3)^k P^j_i P^i_j + b\, B_i H(3)^k P^j_k P^i_j + c\, B_i H(\bar{6})^{ij}_k P^m_j P^k_m$$
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- we get $\boldsymbol{SP/P = 0.31 - 0.36}$
- $\Rightarrow$ result (large $\boldsymbol{SP}$) is not sensitive to details of $SU(3)_F$ symmetry implementation

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