The $B \to K\eta'$ Decay Puzzle

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Collaboration with: J. O. Eeg (University of Oslo) and I. Picek (University of Zagreb)

[J.O. Eeg, K.K. and I. Picek, Phys. Lett. **B363** (2003) 87]

Overview

- Introduction to B decays

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- Conclusions

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- ▶ parameters involving 3rd quark family still poorly known

[to exp.]

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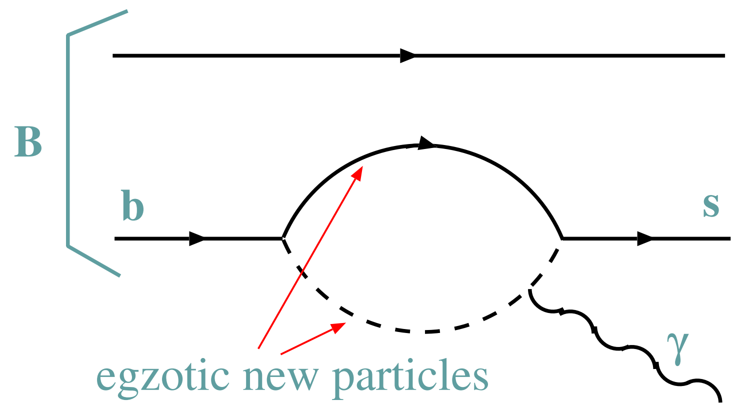

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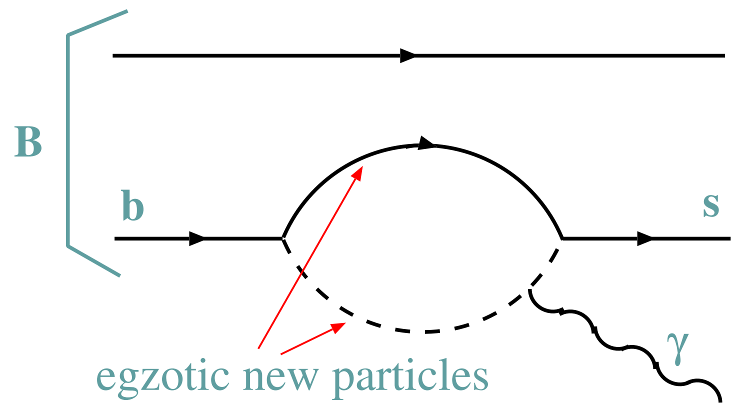
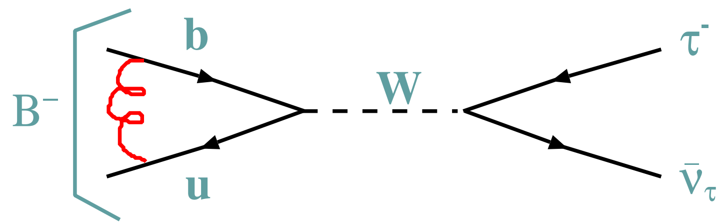


► precision loop calculations are less QCD-polluted because of the large energy scale $\sim m_b$ (asymptotic freedom)

Types of B decays

- leptonic



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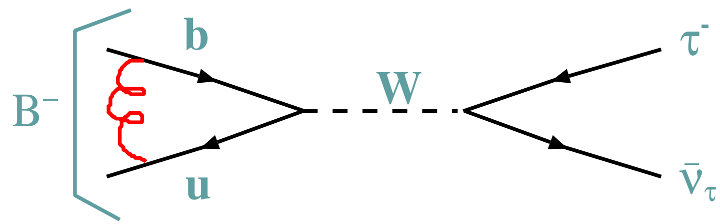

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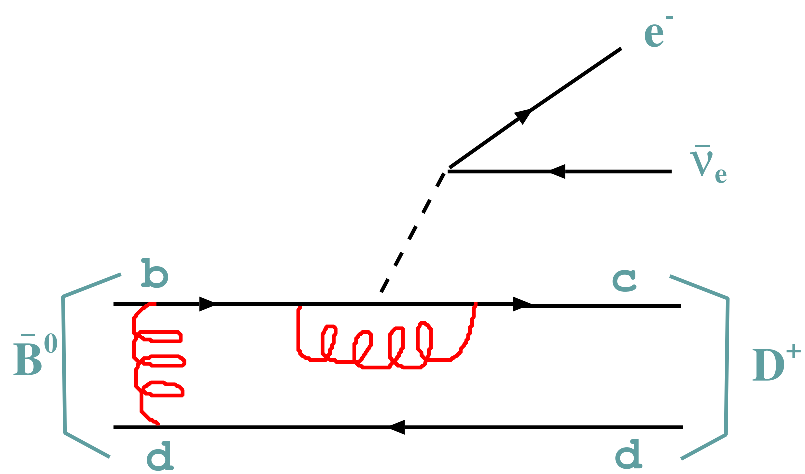
- semi-leptonic



$$\langle D|J^{\text{weak}}_{\text{hadr.}}|B\rangle \propto F_0(q^2), F_1(q^2)$$

Types of B decays (2)

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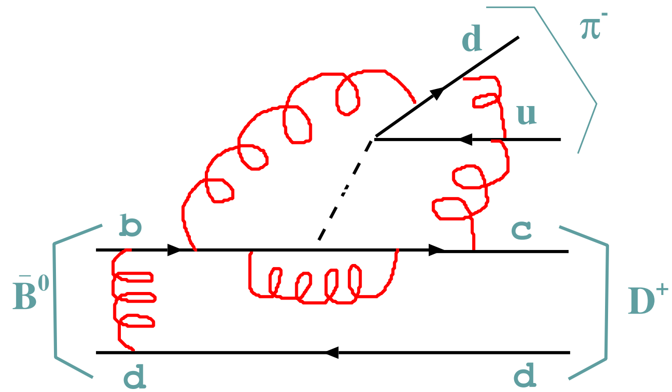

$$\langle D\pi | J^{\text{weak}}_{\text{hadr.}1} J^{\text{weak}}_{\text{hadr.}2} | B \rangle \rightarrow \text{very complicated}$$

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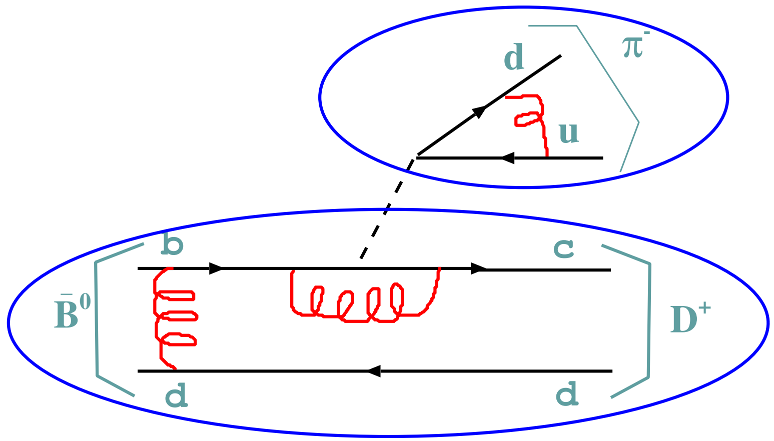

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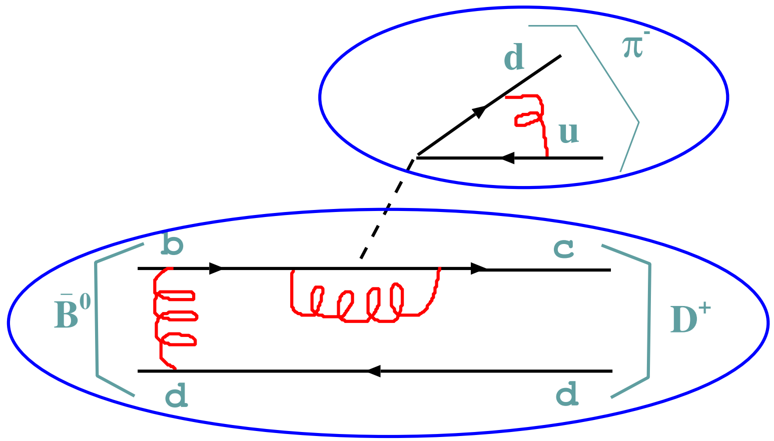

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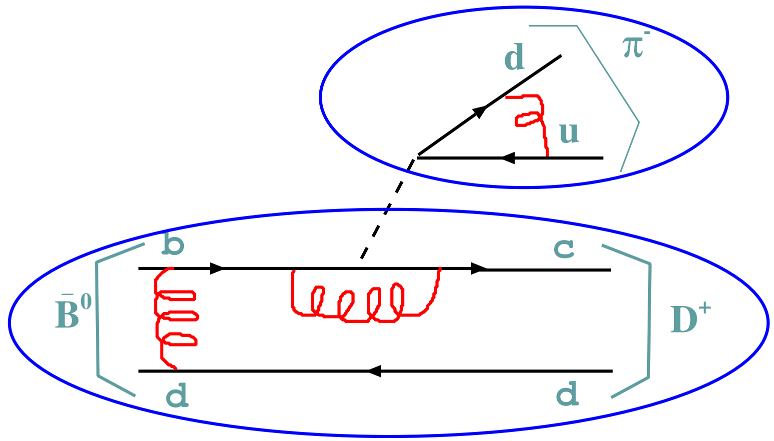

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- improved approaches (QCD factorization, ...)

Experimental data

- CLEO, Belle and BaBar collaborations see a lot of η''s in charmless (rare) hadronic B decays . . .

$$\text{Br}(B^+ \to K^+\eta') = (77 \pm 5) \cdot 10^{-6}$$

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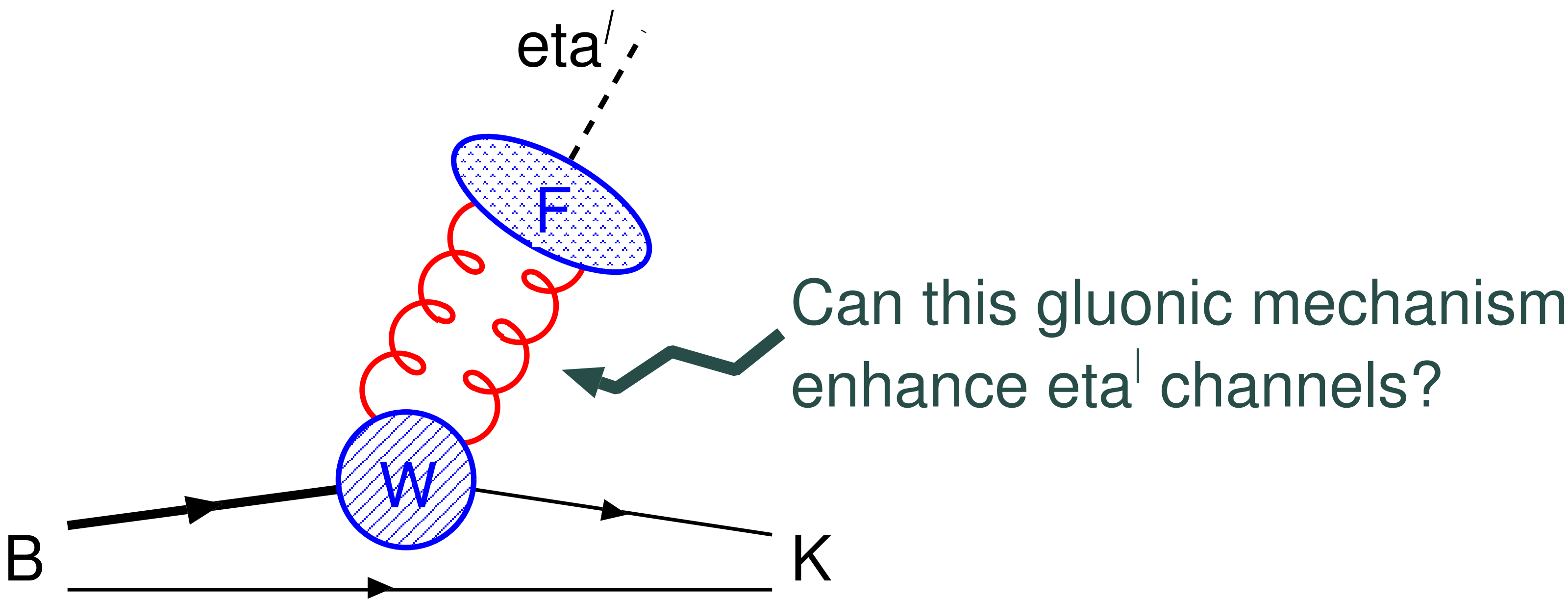

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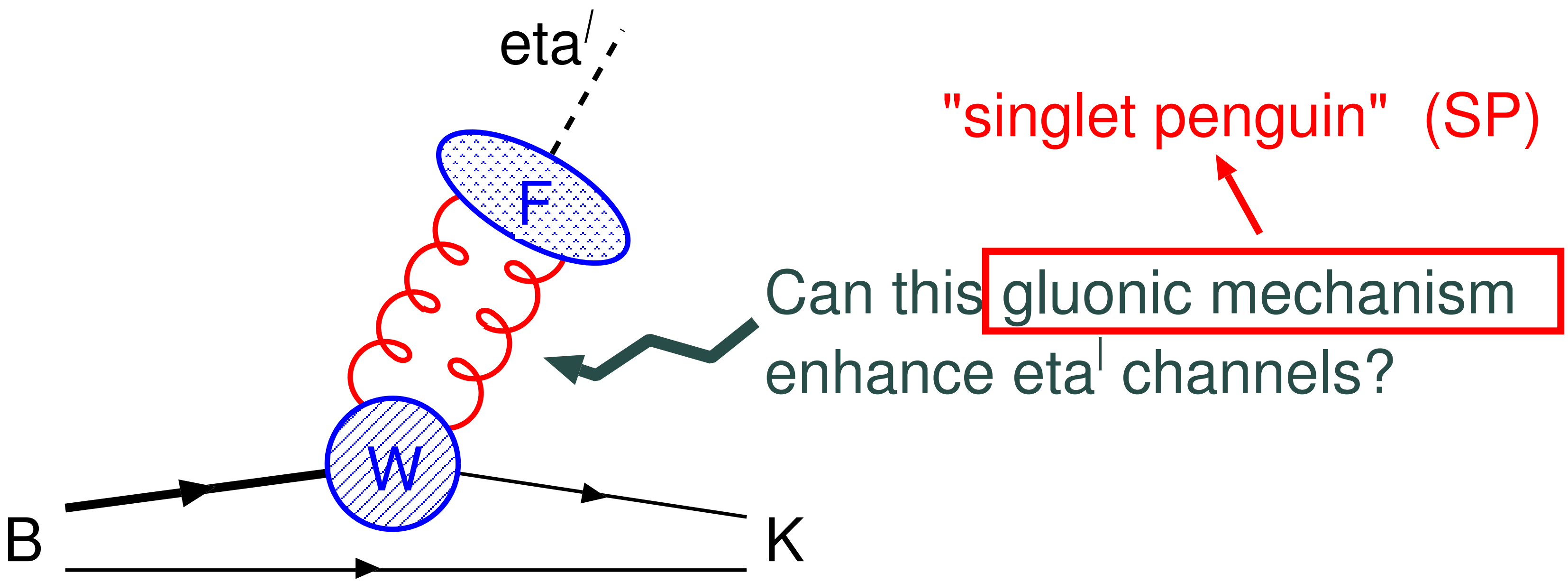

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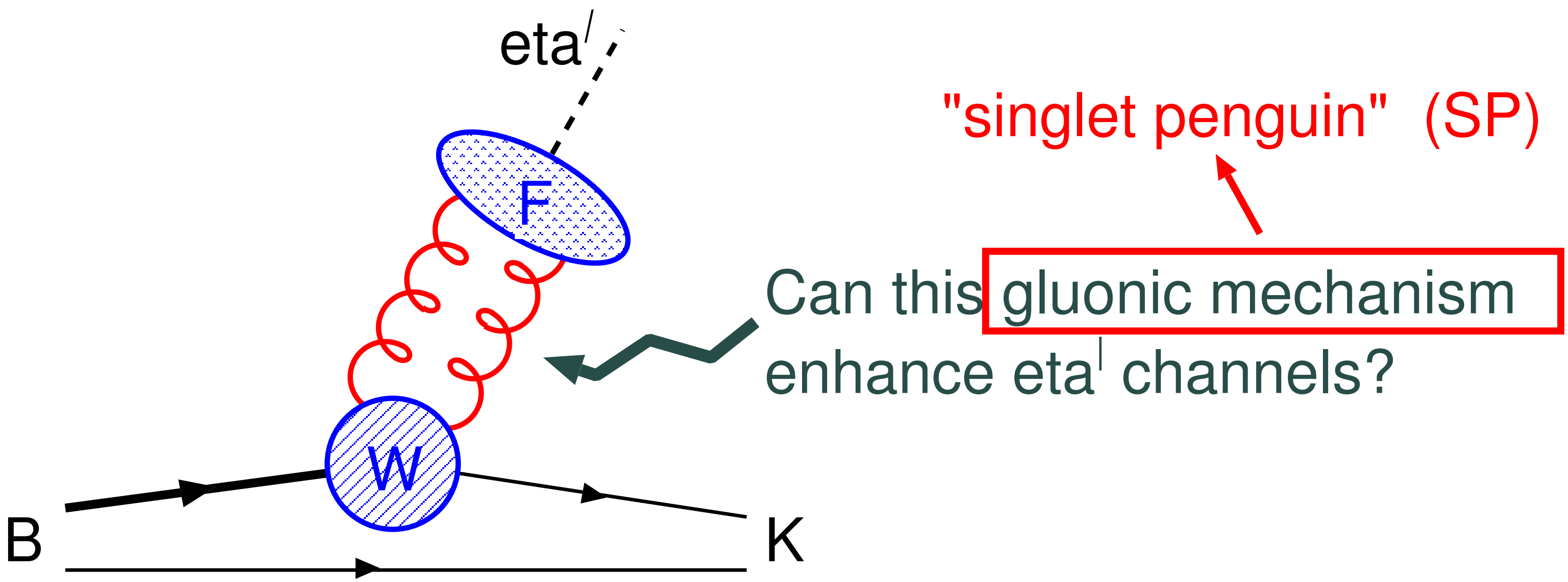

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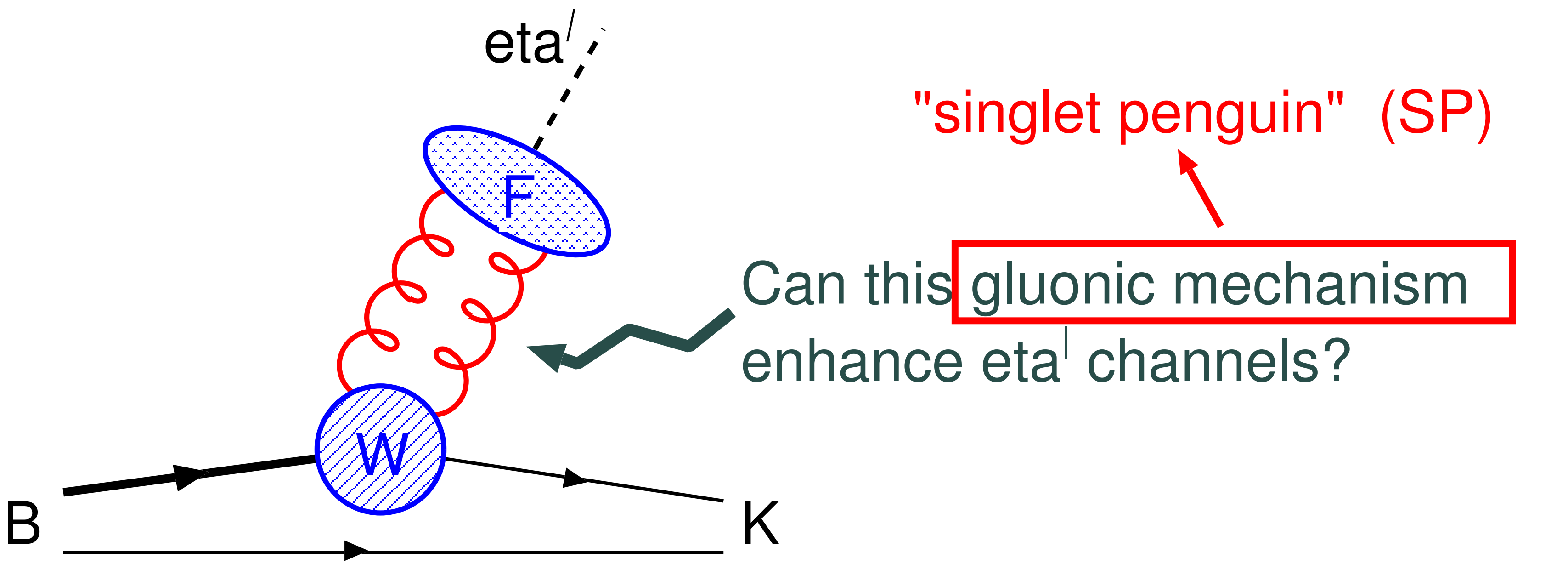

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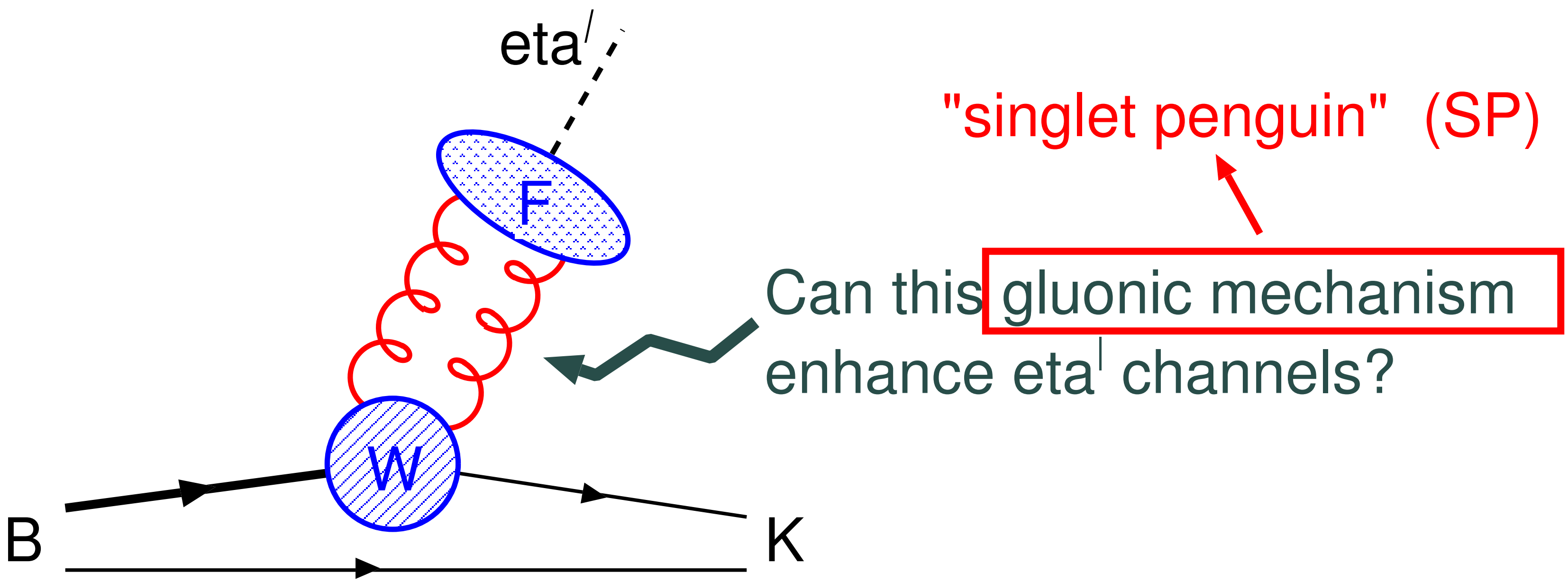

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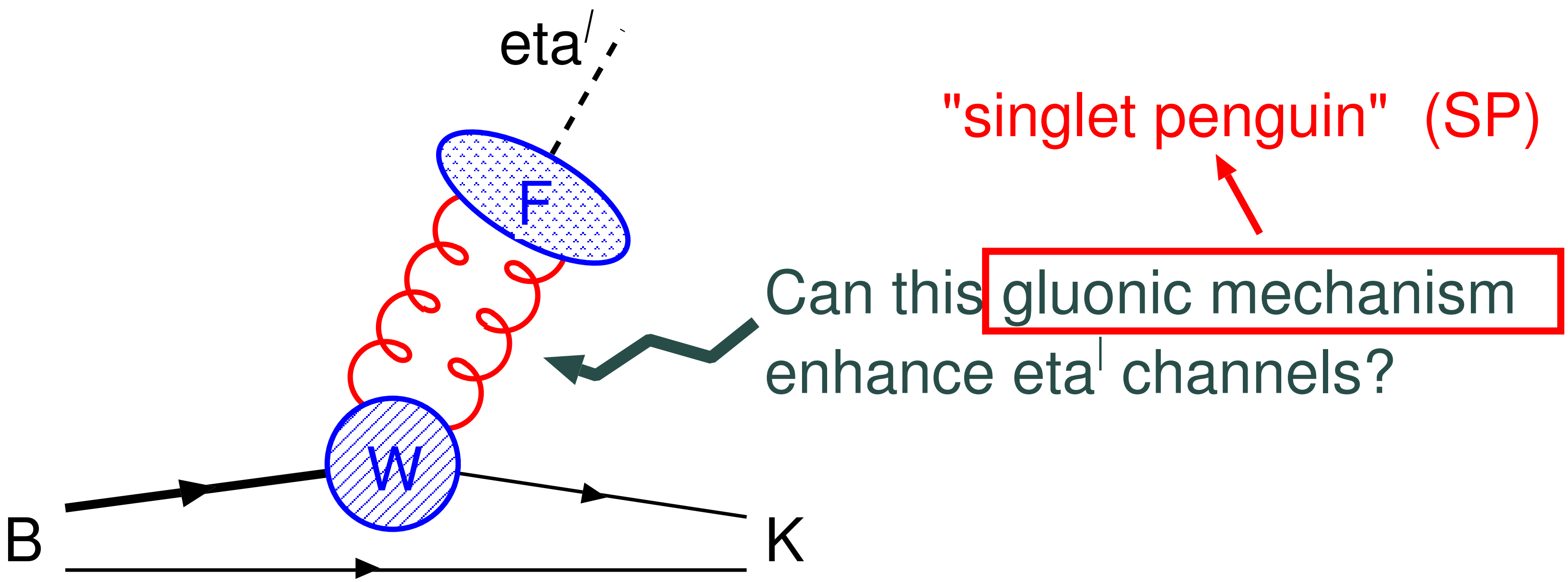

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- 2. perturbative approach $\longrightarrow$ SP part negligible!

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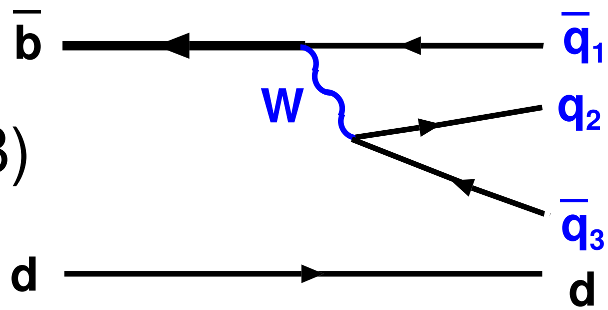
(B)



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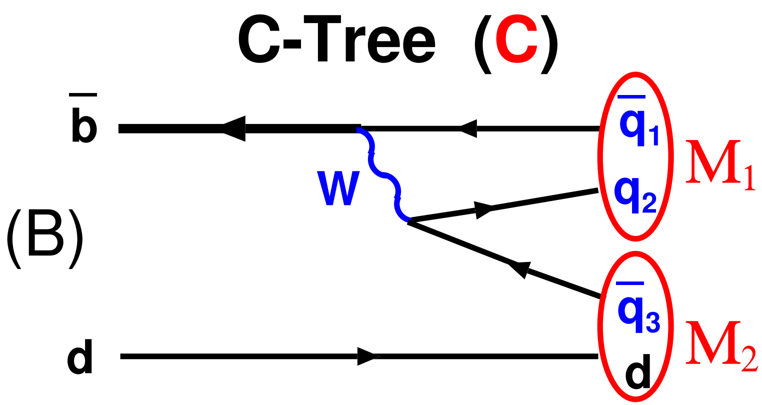

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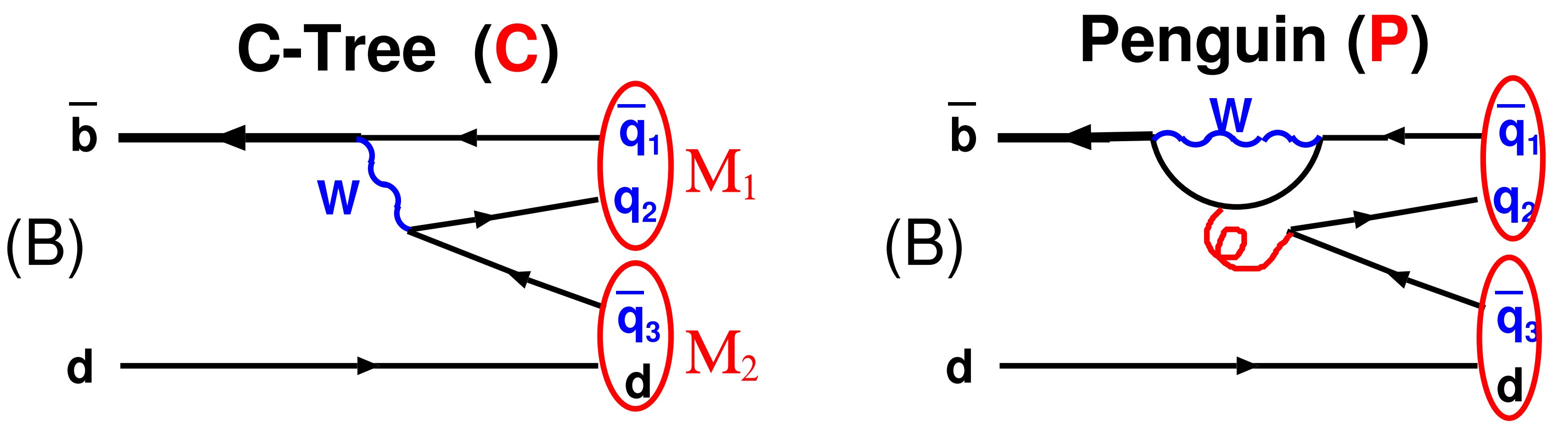

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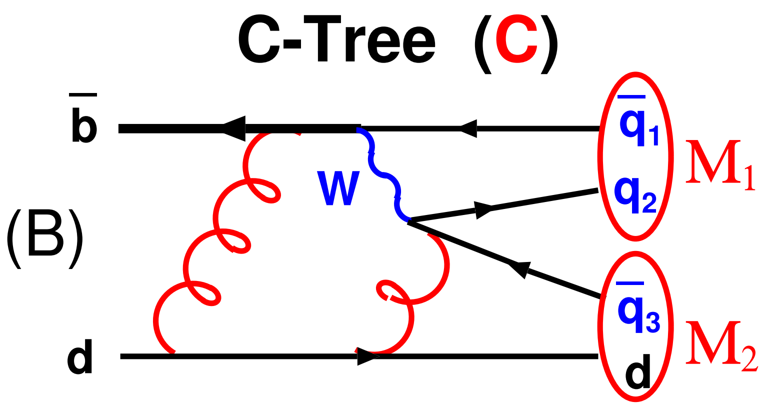

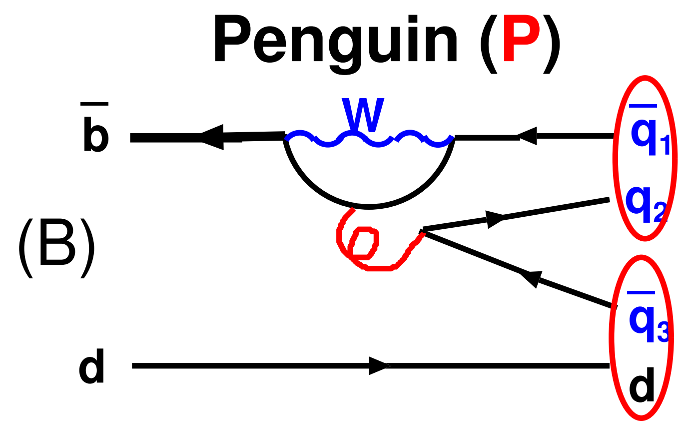

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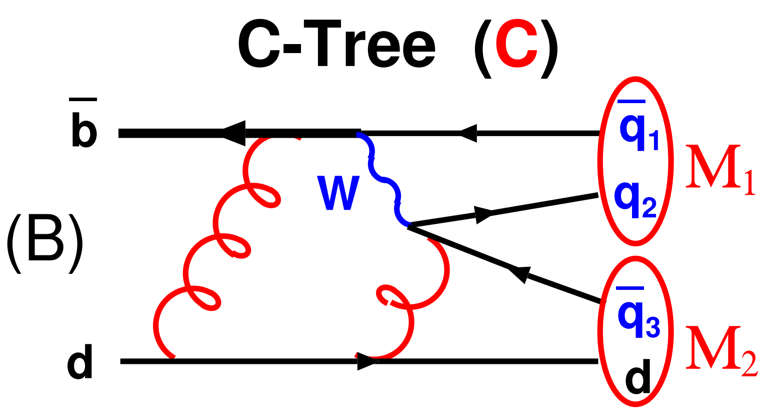

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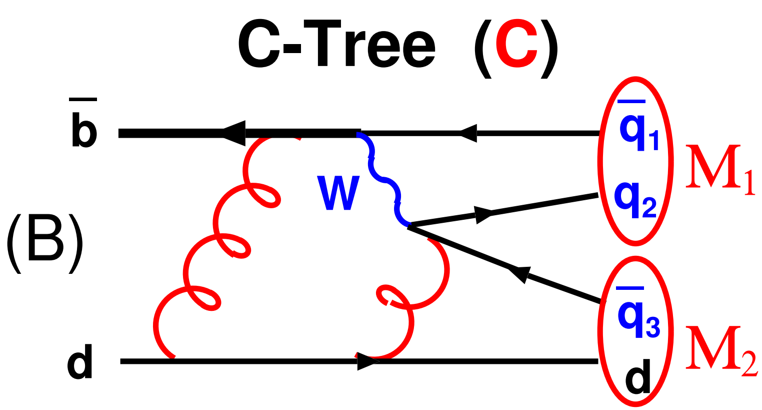

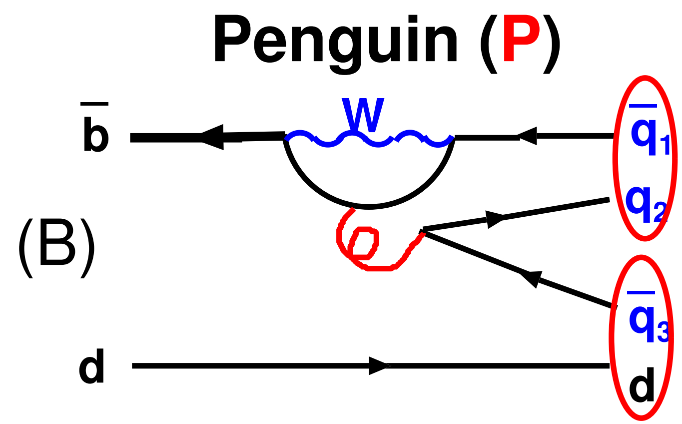

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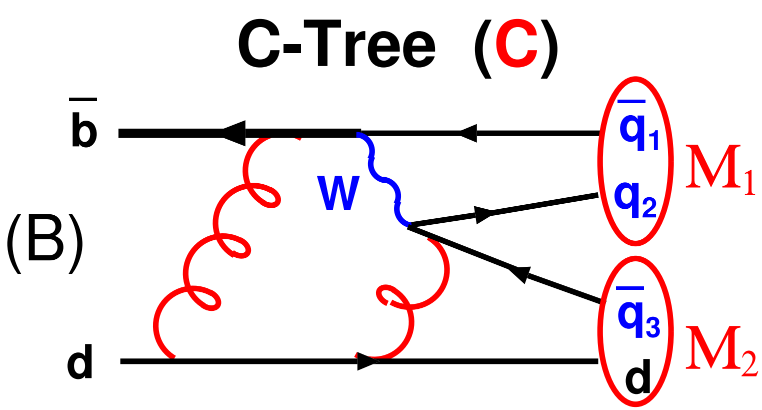

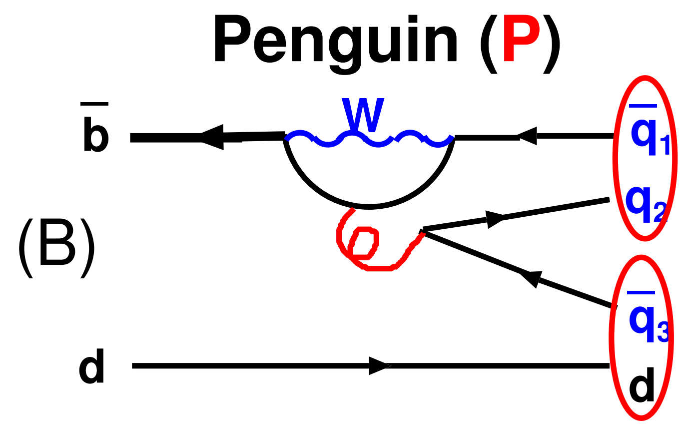

- other topologies: tree (T), exchange (E), annihilation (A), penguin-annihilation (PA), singlet penguin (SP)
- cannot calculate C, T, P, SP, ... but hope that they are invariant under flavour rotations $q_i = u \leftrightarrow d \leftrightarrow s$

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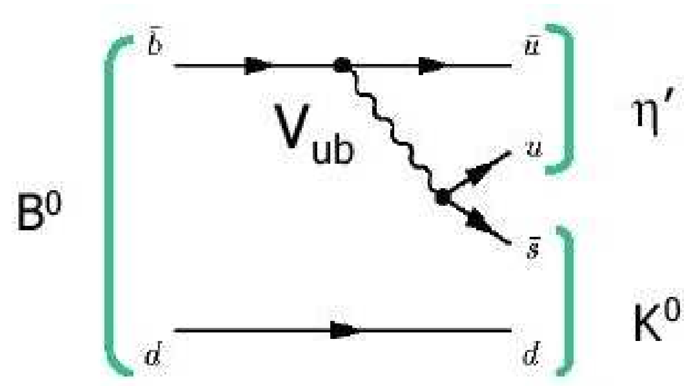

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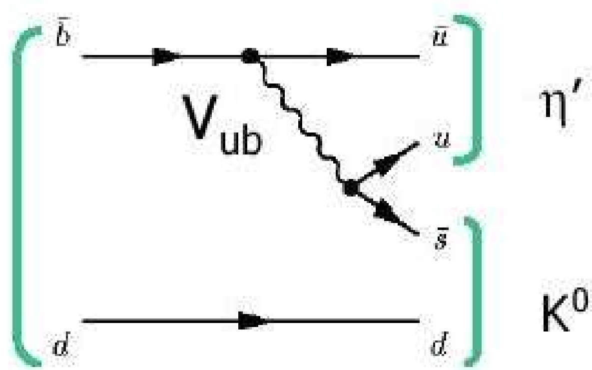

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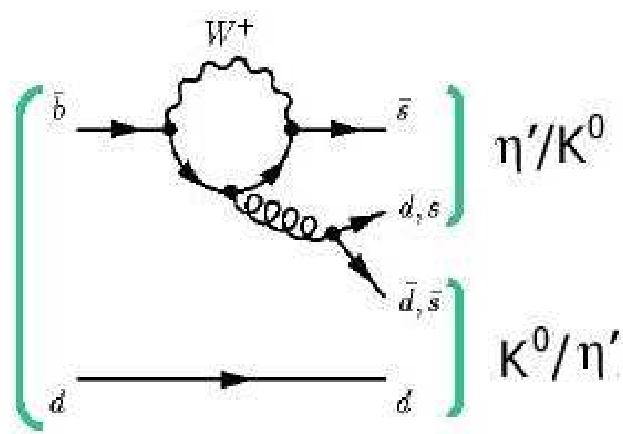

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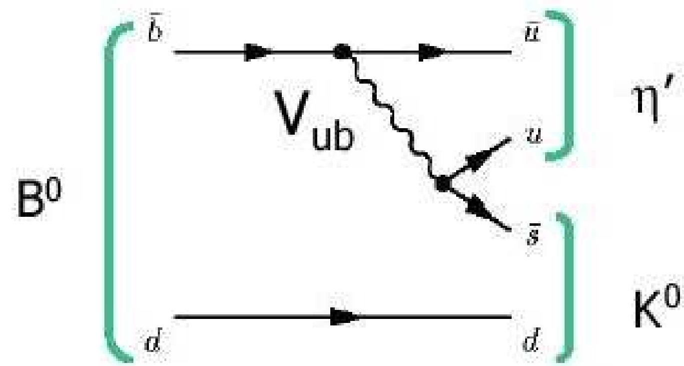

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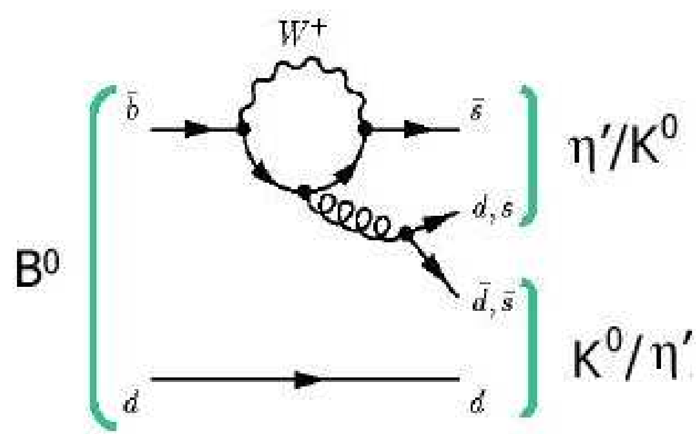
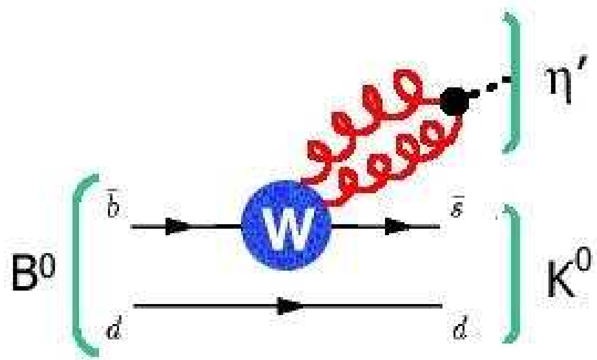


$\longrightarrow$ important contribution (interfer-ence!)

Singlet penguin part

- $SP =$ 

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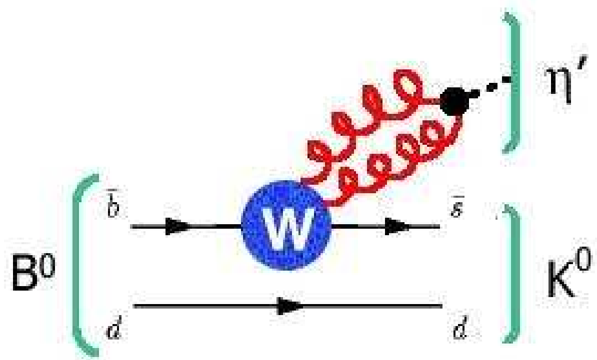

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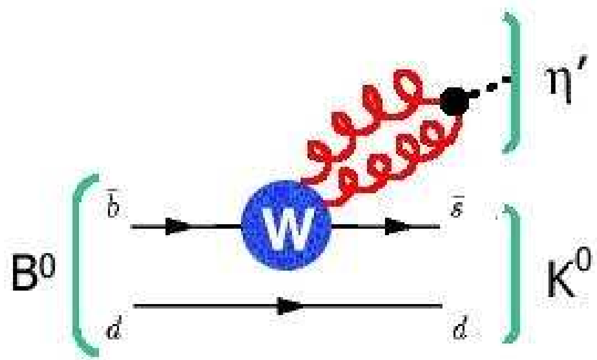
- $SP =$ 

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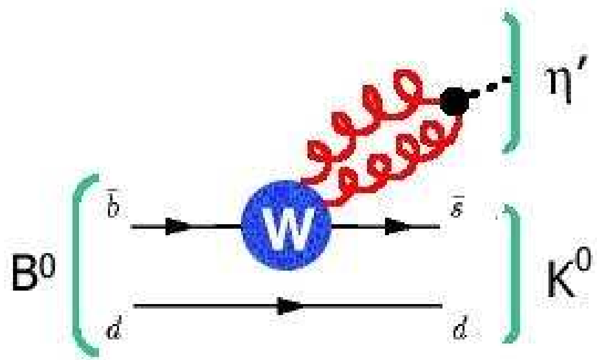

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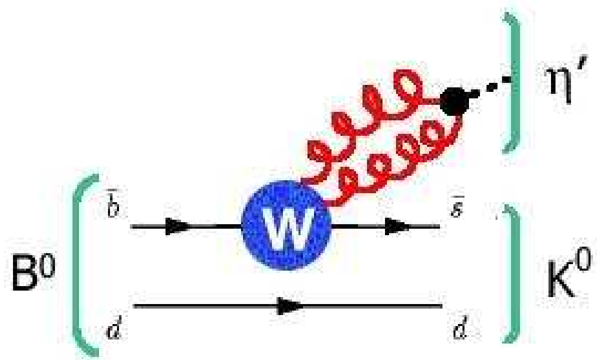
- $SP =$ 

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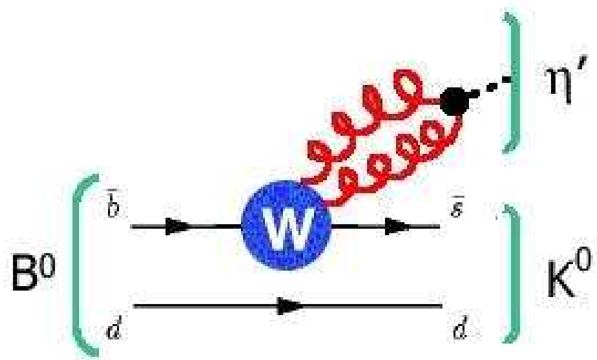

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- Possible objections:
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  - $\eta - \eta'$ mixing implementation
  - Hybrid method (symmetry + quark dynamics), overcomplete basis: more flavour topologies than true $SU(3)_F$ invariants

Alternative flavour symmetry approaches

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$$
\begin{aligned}
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& + d\, B_i H(15)^{ij}_k P^m_j P^k_m + e\, B_i H(15)^{jk}_m P^m_k P^i_j + \tilde{f}\, B_i H(3)^k P^i_k \eta_1 \\
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- we get $\boldsymbol{SP/P = 0.31 - 0.36}$

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- "Pure" $SU(3)_F$ symmetry approach (generalization of [Savage and Wise (1989)])

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$$+ d\, B_i H(15)^{ij}_k P^m_j P^k_m + e\, B_i H(15)^{jk}_m P^m_k P^i_j + \tilde{f}\, B_i H(3)^k P^i_k \eta_1$$
$$+ \tilde{g}\, B_i H(\bar{6})^{ij}_k P^k_j \eta_1 + \tilde{h}\, B_i H(15)^{ij}_k P^k_j \eta_1 + \tilde{s}\, B_k H(3)^k \eta_1 \eta_1$$

- we get $\boldsymbol{SP/P = 0.31 - 0.36}$
- [Fu, He, Hsiao (2003)] $\boldsymbol{SP/P \approx 0.9}$

Perturbative (dynamical) analysis

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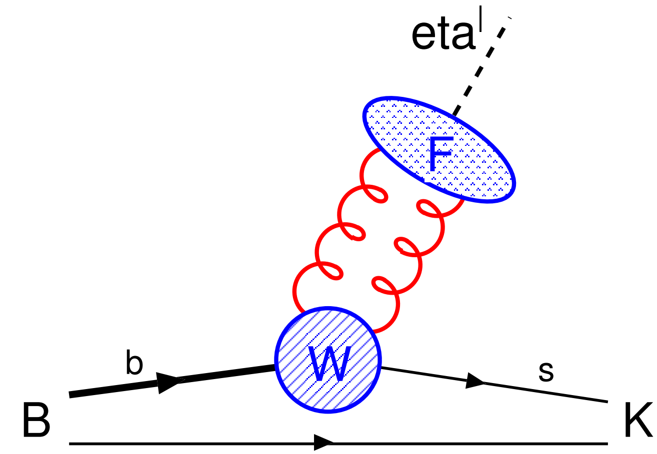
- [Atwood and Soni (1997)]
- [Halperin and Zhitnitsky (1997)]
- [Kagan and Petrov (1997)]
- [Hou and Tseng (1998)]
- [Datta, He and Pakvasa (1998)]
- [Du, Kim and Yang (1998)]
- [Ahmady, Kou and Sugamoto (1998)]
- [Ali, Chay, Greub and Ko (1998)]
- [Kou and Sanda (2002)]
- [Xiao, Chao and Li (2002)]
- [Beneke and Neubert (2002)]
- [Fritzsch and Zhou (2003)]



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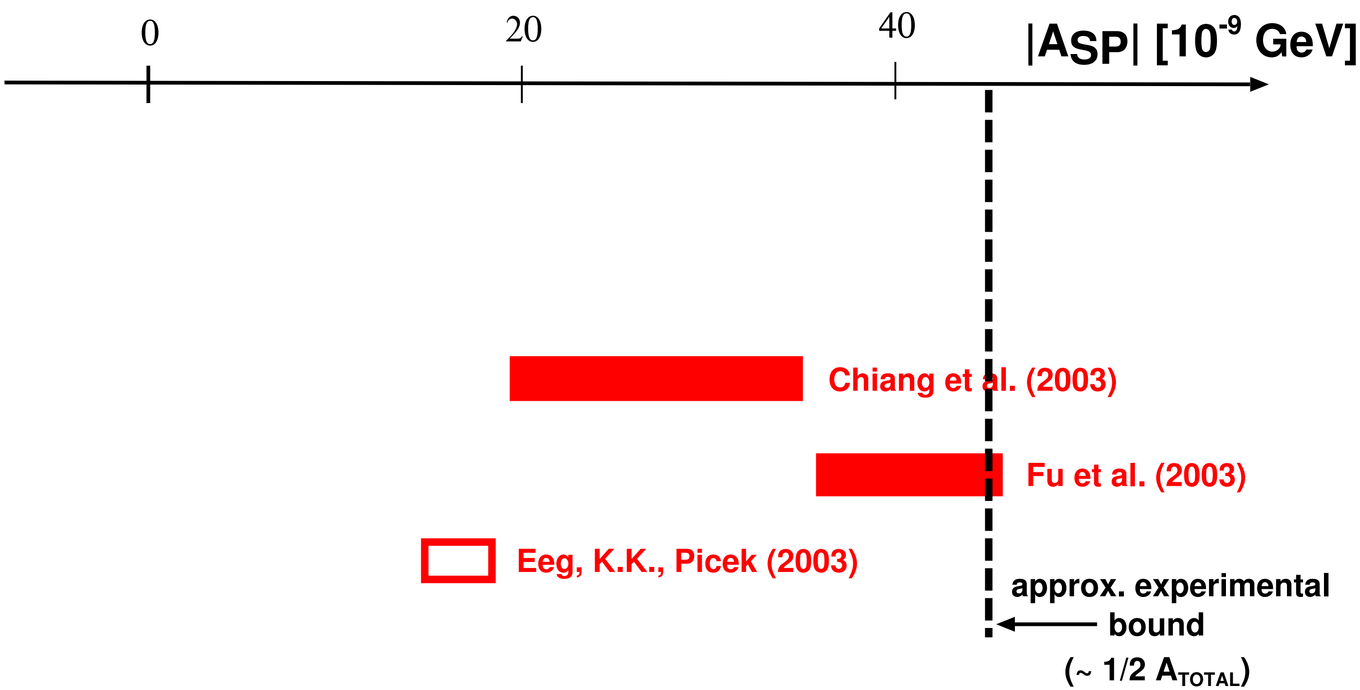
Comparison of two approaches I



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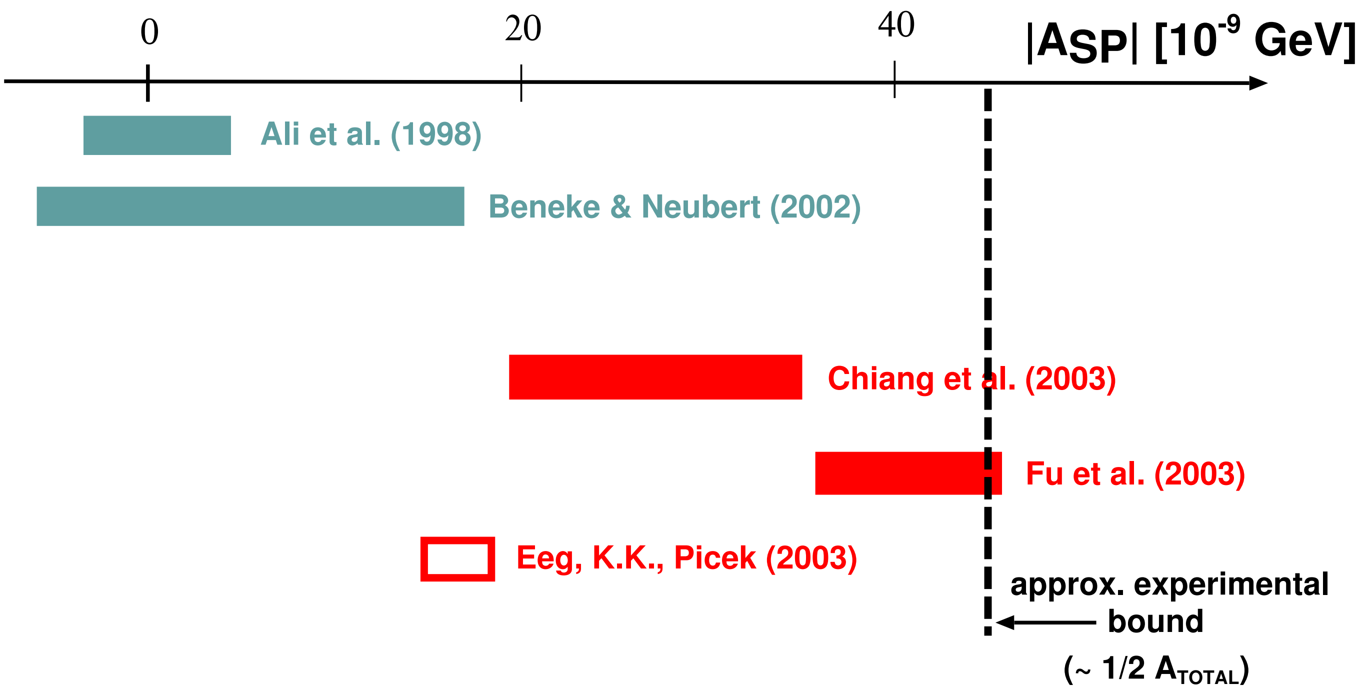

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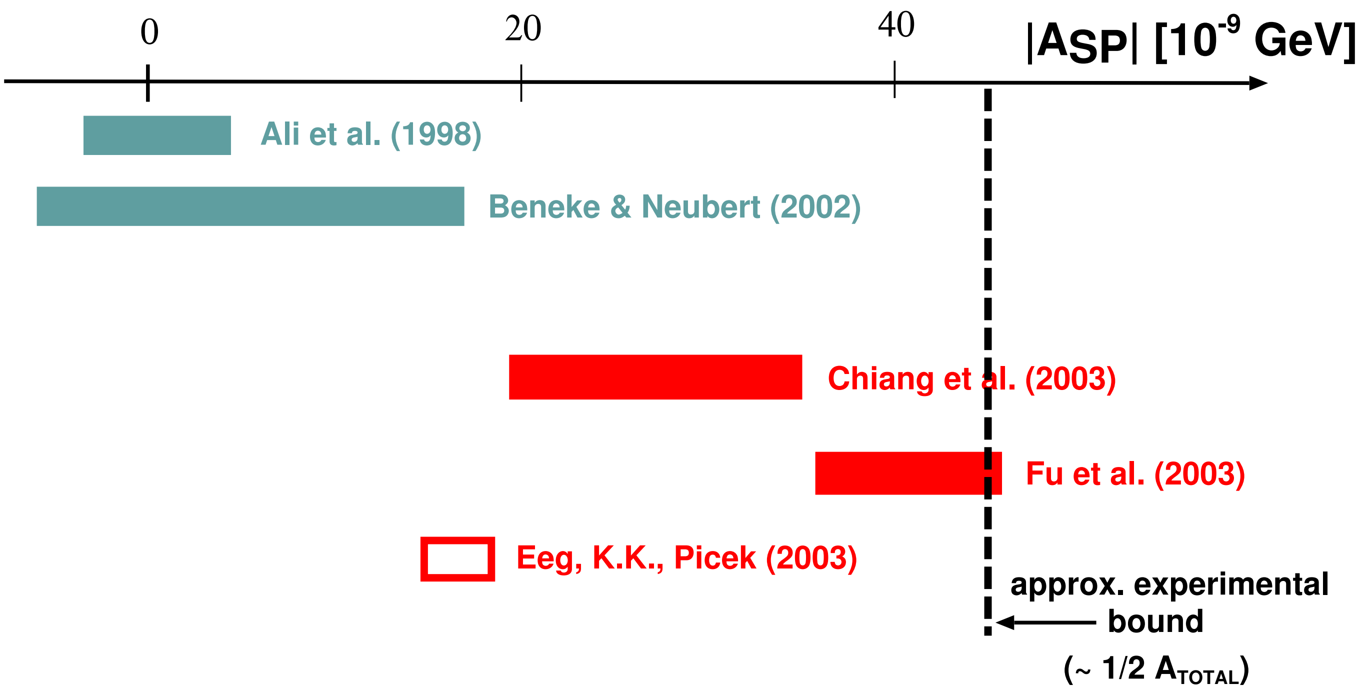

What about hard off-shell gluon contribution? Can it explain the discrepancy?

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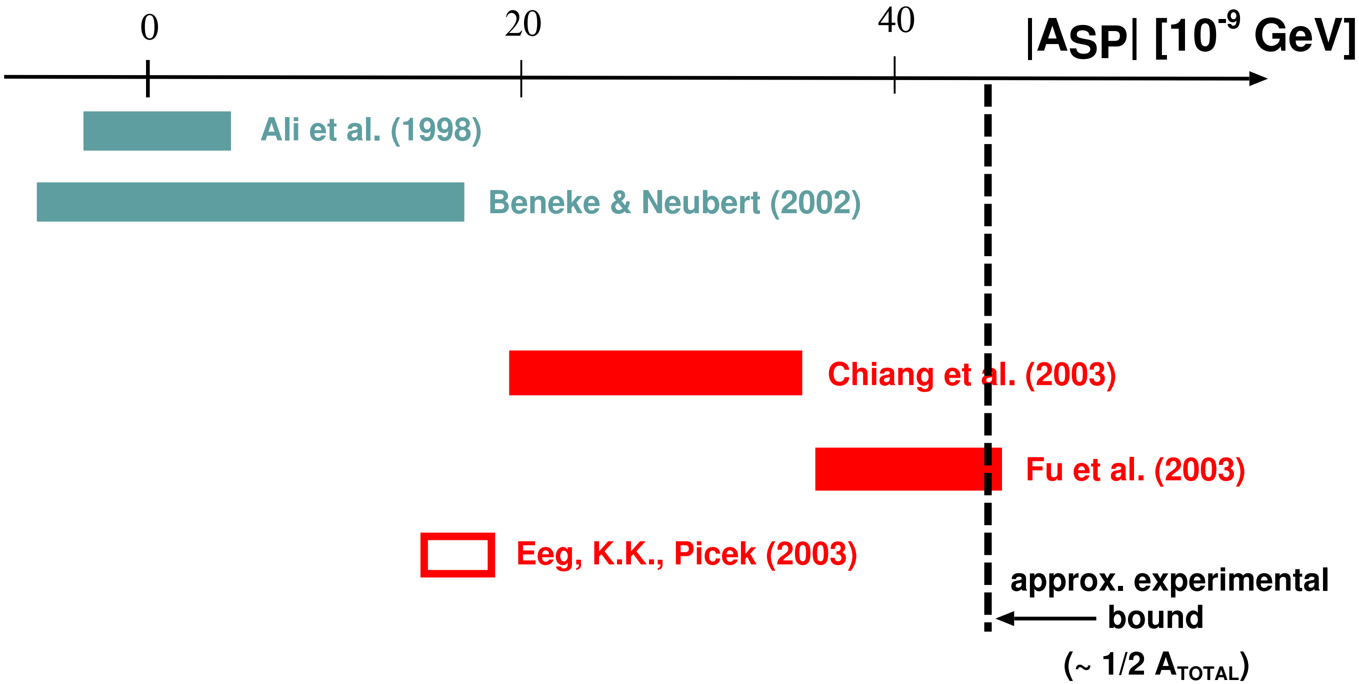

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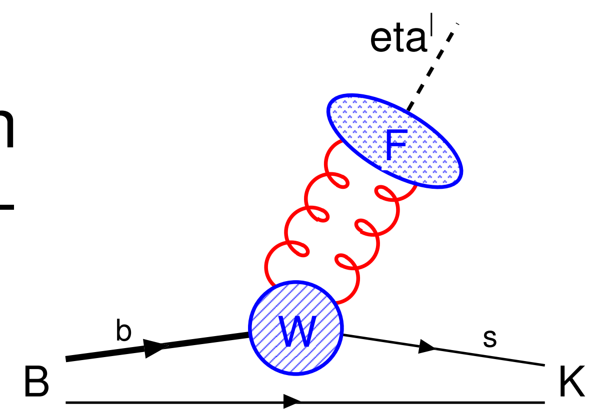

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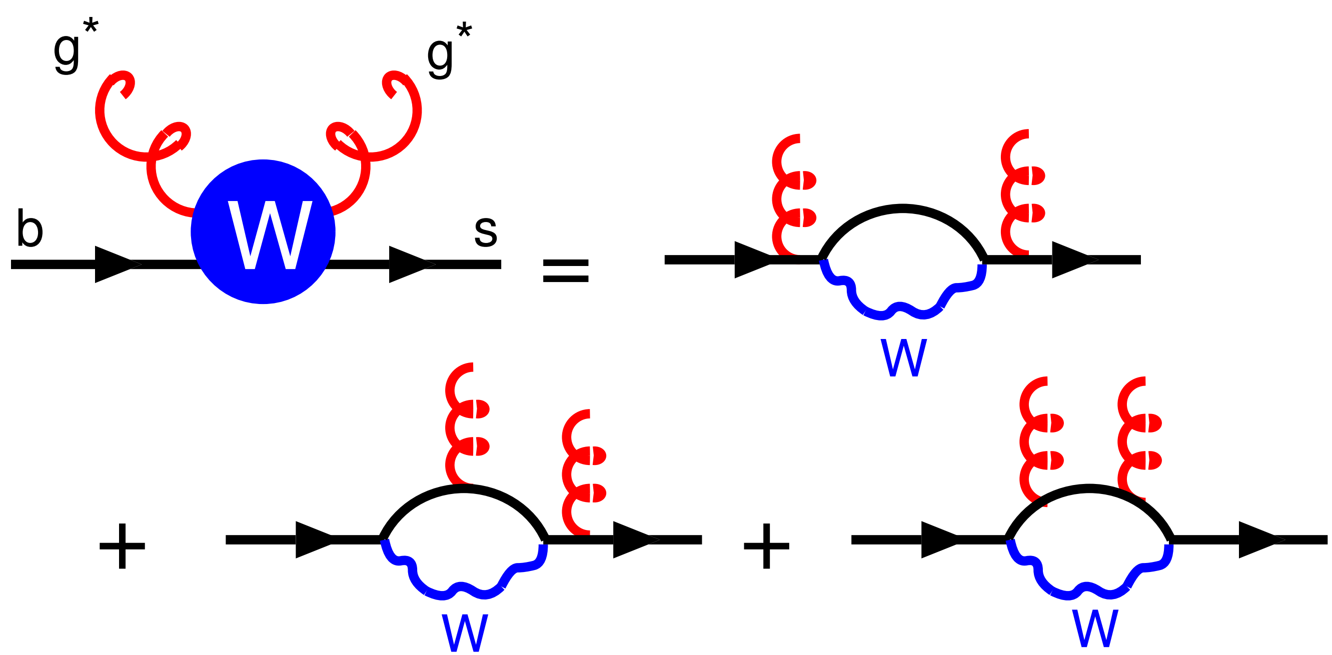
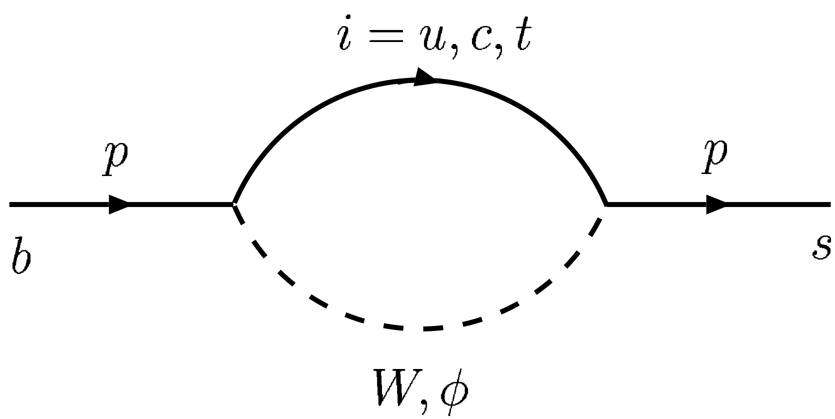
- Building blocks:



$b \to s g^* g^*$ (self-energy)



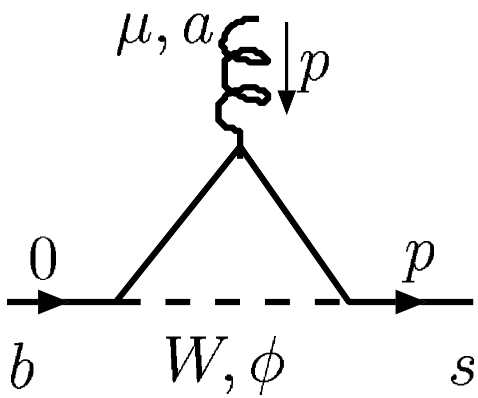
$$= \frac{i}{4\pi^2} \frac{G_F}{\sqrt{2}} \Sigma(p)$$

$$\Sigma(p) = -M_W^2 \not{p} L - 2M_W^2 \left(1 + \frac{m_i^2}{2M_W^2}\right) \not{p} L \int_0^1 \mathrm{d}x (1-x) \ln \frac{D}{\mu_*^2}$$

$$- \int_0^1 \mathrm{d}x \big[(1-x) m_b m_s \not{p} R - m_i^2 (m_b R + m_s L) \big] \ln \frac{D}{\mu_*^2}$$

$$\ln \mu_*^2 = \tfrac{1}{\epsilon} - \gamma_E + \ln 4\pi\mu^2$$

$b \to sg^*g^*$ (Triangle)



$$= \frac{i}{4\pi^2}\frac{G_F}{\sqrt{2}} g_s t^a\, \Gamma^\mu(0, p, -p)$$

$$\Gamma^\mu(0,p,-p) = \frac{4M_W^2}{m_i^2 - M_W^2}\left(1 + \frac{m_i^2}{2M_W^2}\right)(p^2 g^{\mu\nu} - p^\mu p^\nu)\gamma_\nu L \int_0^1 \mathrm{d}x\, x(1-x) \ln \frac{D}{C}$$

$$+ M_W^2 \gamma^\mu L + 2M_W^2 \left(1 + \frac{m_i^2}{2M_W^2}\right)\gamma^\mu L \int_0^1 \mathrm{d}x (1-x) \ln \frac{D}{\mu_*^2}$$

$$D = x m_i^2 + (1-x) M_W^2 - x(1-x) p^2$$

$$C = m_i^2 - x(1-x) p^2$$

$b \to s g^* g^*$ (Box)

$$I^{\mu\nu}(0,0,-p,p) = \frac{2M_W^2}{m_i^2 - M_W^2}\left(1 - \frac{m_i^2}{2M_W^2}\right)(-i\epsilon^{\mu\nu\rho\sigma} p_\sigma \gamma_\rho L) \times$$

$$\times \int_0^1 \mathrm{d}x (1-x) \left\{ (3x-1)\mathbb{Y}_1 + \left[x^2(1-x)p^2 + (x+1)m_i^2\right]\mathbb{Y}_2 \right\}$$

$$+ \frac{2M_W^2}{m_i^2 - M_W^2}\left(1 + \frac{m_i^2}{2M_W^2}\right) \int_0^1 \mathrm{d}x (1-x) \left\{ [-(x+1)\not{p} g^{\mu\nu} - (x-1)(p^\mu\gamma^\nu + p^\nu\gamma^\mu)]\mathbb{Y}_1 \right.$$

$$+ \left(x^2(1-x)\left[-(p^\mu\gamma^\nu + p^\nu\gamma^\mu)p^2 + \not{p}(4p^\mu p^\nu - g^{\mu\nu}p^2)\right] \right.$$

$$\left. \left. + [-(x+1)\not{p} g^{\mu\nu} - (x-1)(p^\mu\gamma^\nu + p^\nu\gamma^\mu)]m_i^2 \right) \mathbb{Y}_2 \right\} L$$

$\mathbb{Y}_{1,2}$ = complicated functions of x, m_i^2, M_W^2, p^2

$b \to sg^*g^*$ (Complete)

$$\mathcal{A} = \mathrm{i}\frac{\alpha_s}{\pi}\frac{G_F}{\sqrt{2}}\bar{s}(0)t^b t^a \sum_i \lambda_i T_{i\mu\nu} b(0)\epsilon_a^{\mu}(-p)\epsilon_b^{\nu}(p) \; + \; \text{(crossed)} \, ,$$

$$T_i^{\mu\nu} = T_{i\text{Box}}^{\mu\nu} + T_{i\text{Triangle}}^{\mu\nu} + T_{i\text{Self-energy}}^{\mu\nu} \, .$$

$$T_i^{\mu\nu} = (-i\epsilon^{\mu\nu\rho\sigma} p_\sigma \gamma_\rho L) \, A_i \; + \; (\mu\text{-}\nu \text{ symmetric part})$$

$$A_i = -\frac{8M_W^2}{m_i^2 - M_W^2}\left(1 + \frac{m_i^2}{2M_W^2}\right)\int_0^1 \mathrm{d}x x(1-x)\ln\frac{D}{C}$$

$$+ \frac{2M_W^2}{m_i^2 - M_W^2}\left(1 - \frac{m_i^2}{2M_W^2}\right)\int_0^1 \mathrm{d}x(1-x)\Big\{(3x-1)\mathbb{Y}_1 + \left[x^2(1-x)p^2 + (x+1)m_i^2\right]\mathbb{Y}_2\Big\}$$

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- $F_{\eta' g^* g^*}$ defined via $\eta' \to g^*(k_1) g^*(k_2)$ amplitude:

$$N_{\mu\nu}^{ab}(\bar{Q}^2, \omega) = -i F_{\eta' g^* g^*}(\bar{Q}^2, \omega)\, \epsilon_{\mu\nu k_1 k_2}\, \delta^{ab} \ ,$$

$$\bar{Q}^2 = -\frac{k_1^2 + k_2^2}{2} \qquad \omega = \frac{k_1^2 - k_2^2}{k_1^2 + k_2^2}$$

$\eta' g^* g^*$ form-factor II

- For $\bar{Q}^2 \gtrsim m_b^2$

$$F_{\eta' g^* g^*}(\bar{Q}^2, 0) = 4\pi\alpha_s(\bar{Q}^2)\frac{f^1_{\eta'}}{\sqrt{3}\bar{Q}^2}\left(1 - \underbrace{\frac{1}{12}B^g_2(\bar{Q}^2)}_{|\eta'\rangle = |gg\rangle}\right)$$

$$f^1_{\eta'} \approx 1.15\sqrt{2} f_\pi$$

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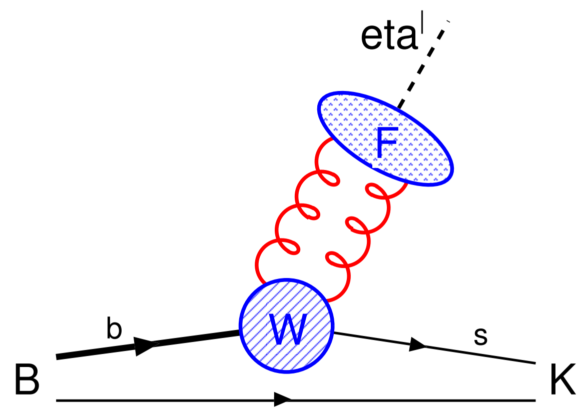
- Double suppression of $F_{\eta' g^* g^*}$:

$$\left.\begin{array}{c} 1/\bar{Q}^2 \\ \alpha_s(\bar{Q}^2) \text{ running} \end{array}\right\} \quad \text{for } \bar{Q}^2 \gg$$

Gluing two pieces together

- Combining amplitudes for $b \longrightarrow sg^*g^*$ and $g^*g^* \longrightarrow \eta'$



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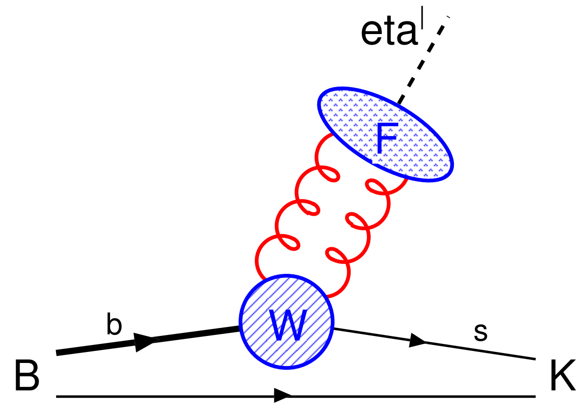

$$\mathcal{A}(b \to s\eta') = \frac{G_F}{8\sqrt{2}\pi^3}\left(\phi_{\eta'}\bar{s}\not{P}_{\eta'}Lb\right)\sum_{i=u,c,t}\lambda_i$$

$$\times \int_{\mu^2\sim m_b^2}^{M_W^2} dQ^2\,\alpha_s(Q^2)\,F_{\eta' g^* g^*}(Q^2)\,A_i(-Q^2)$$

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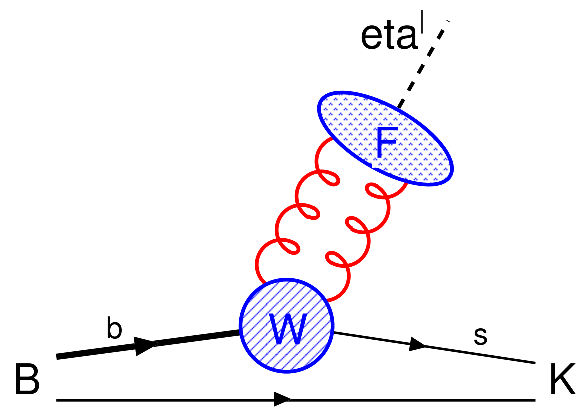

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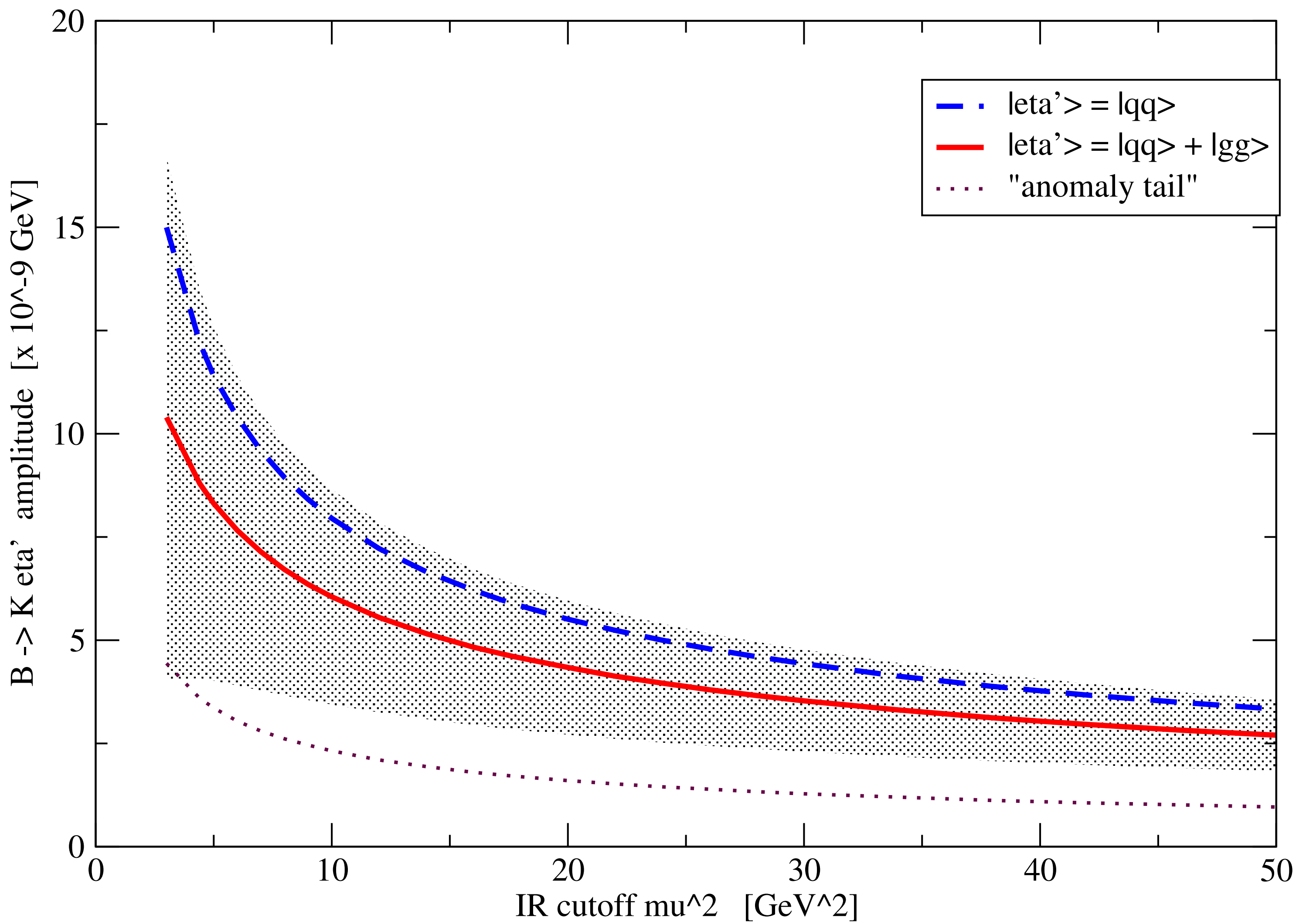
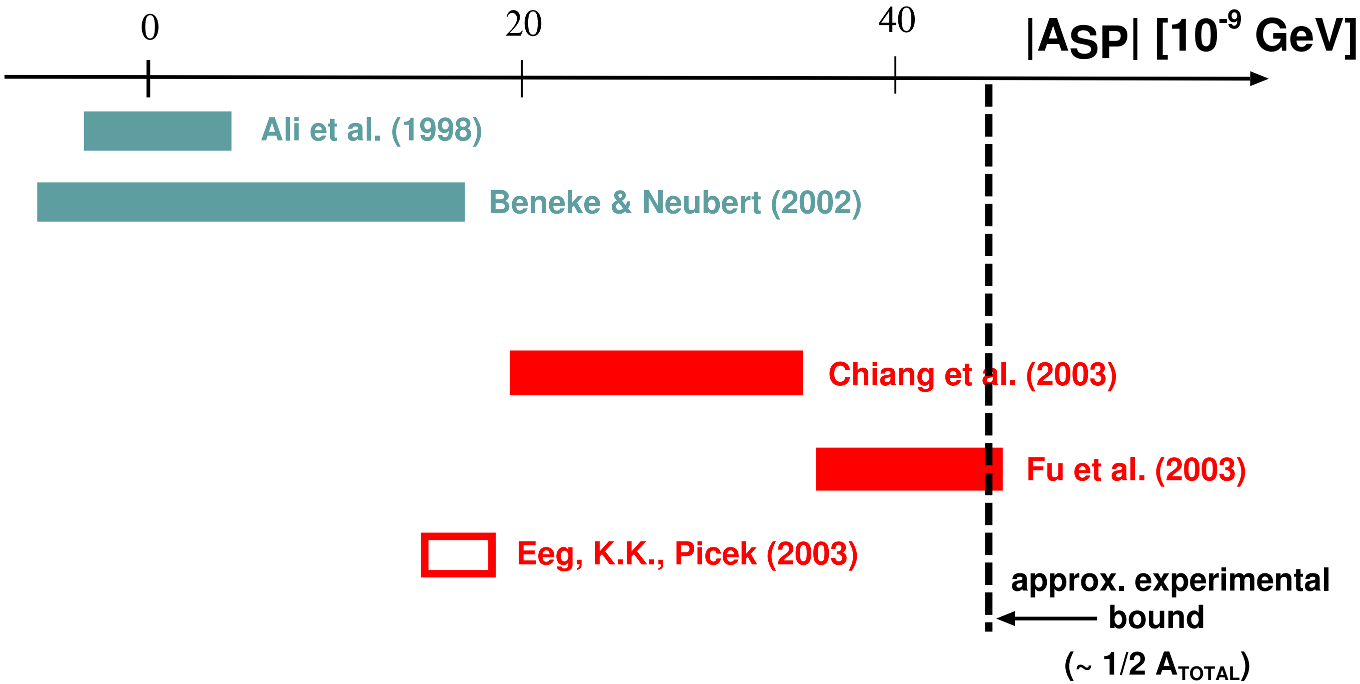
- $\mathcal{A}(b \to s\eta') \;\to\; \mathcal{A}(B \to K\eta')$ via factorization

IR cut-off dependence



Comparison of two approaches II



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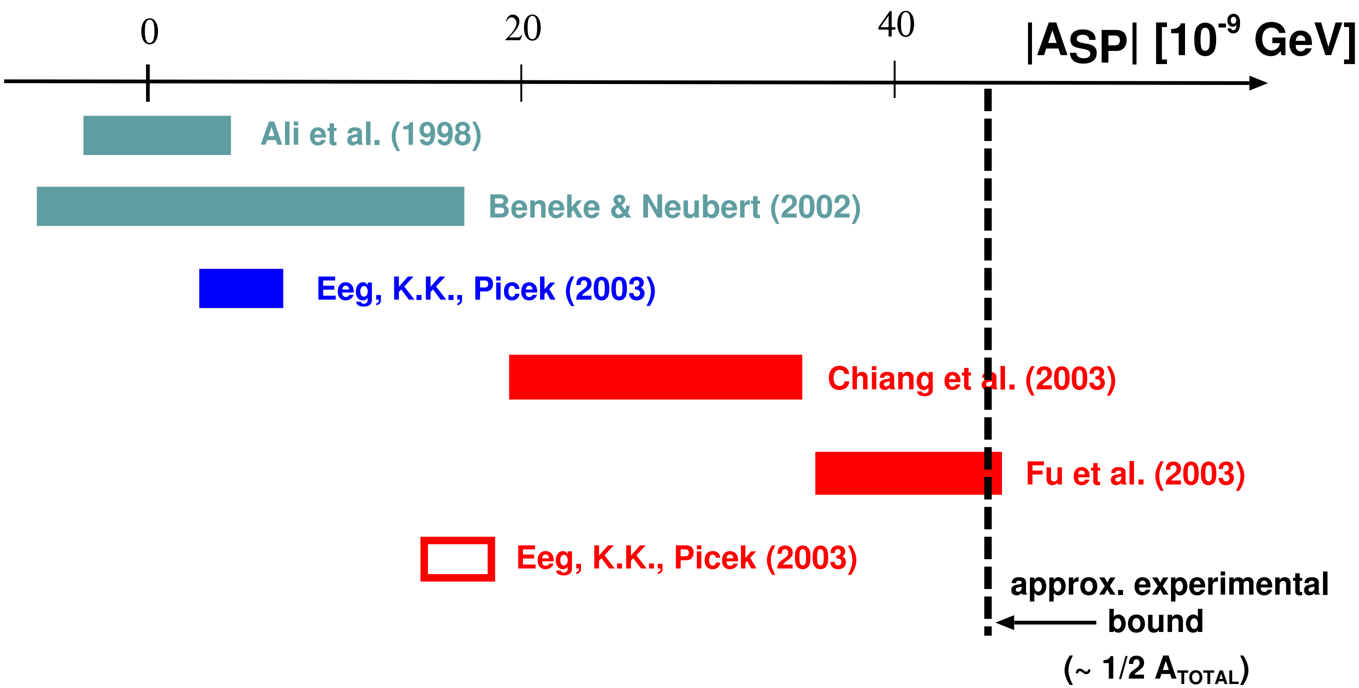

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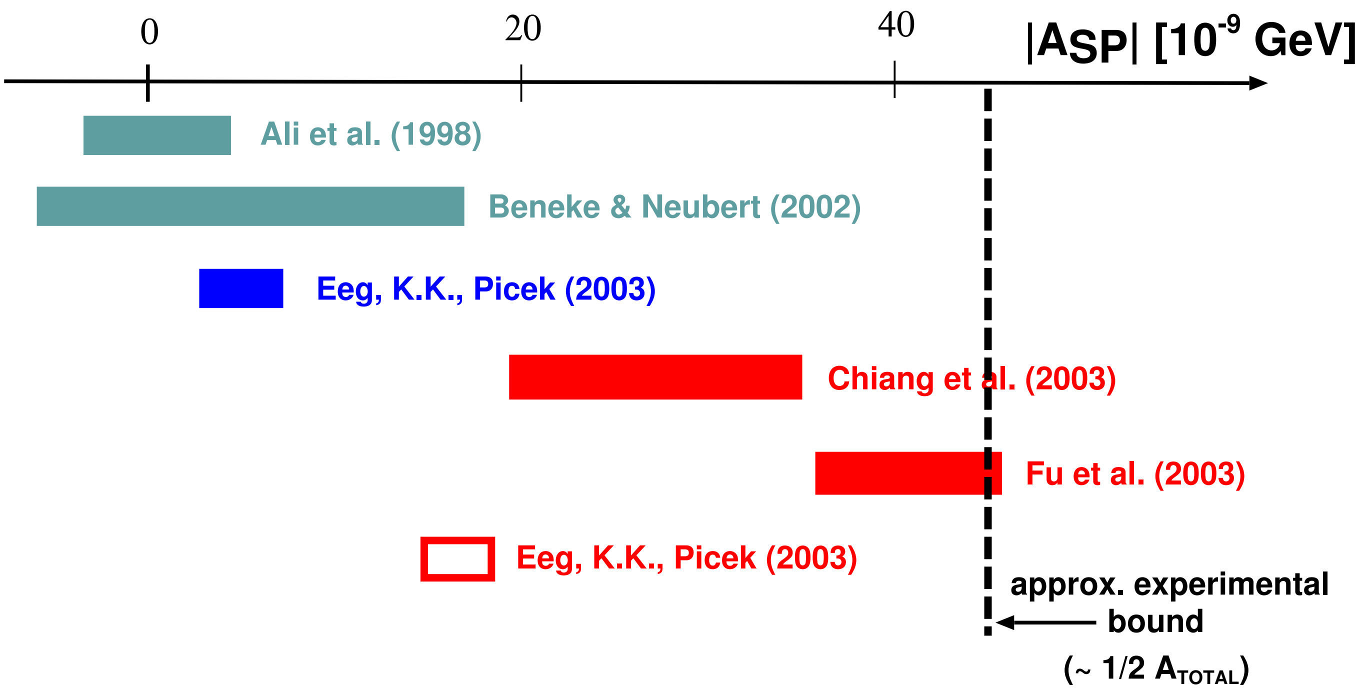

- (One must add SD (blue) on top of LD (gray-blue) and than compare with SU(3) (red).)
- Discrepancy smaller but still exists!

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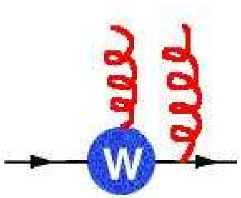
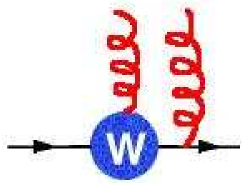
The End

F1-F2 interplay

- 

$\sim F_1(x)(p^2\gamma^\mu - \not{p}p^\mu)L - F_2(x)i\sigma_{\mu\nu}q^\nu m_b R$

F1-F2 interplay

- 

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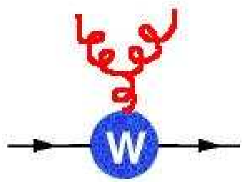
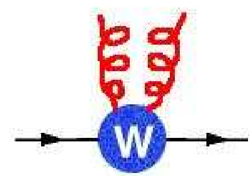
- 

$\sim$ (F_1-terms) + (F_2-terms)



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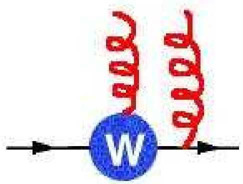
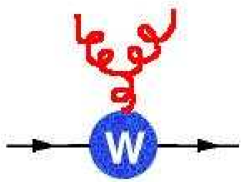
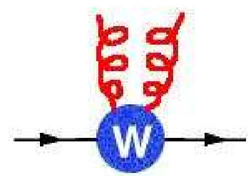
-  $\sim F_1(x)(p^2\gamma^\mu - \not{p}p^\mu)L - F_2(x)i\sigma_{\mu\nu}q^\nu m_b R$

-  $\sim$ (F_1-terms) + (F_2-terms)  $\sim$ (F_1-terms) + (F_2-terms)

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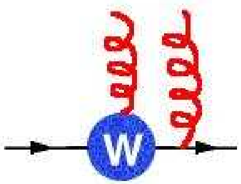
- 

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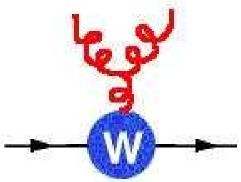
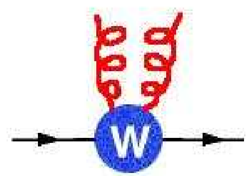
- 

$\sim$ (F_1-terms) + (F_2-terms)   

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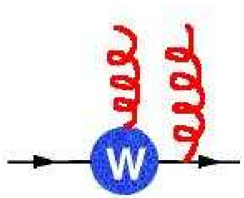
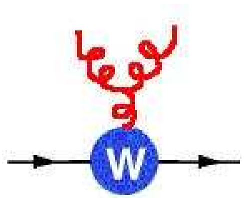
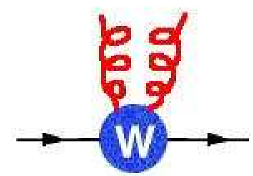
- $x \ll 1 \;\Rightarrow\; (F_1 \sim \ln x) \;\gg\; (F_2 \sim x^2 \ln x)$

- F_1 terms cancel for on-shell or soft gluons (Ward identities, low-energy theorem [Low(1958)]) $\Rightarrow$ suppression

F1-F2 interplay

- 

$\sim F_1(x)(p^2\gamma^\mu - \not{p}p^\mu)L - F_2(x)i\sigma_{\mu\nu}q^\nu m_b R$
- 

$\sim$ (F_1-terms) + (F_2-terms)


$\sim$ (F_1-terms) + (F_2-terms)
- $x \ll 1 \;\Rightarrow\; (F_1 \sim \ln x) \;\gg\; (F_2 \sim x^2 \ln x)$
- F_1 terms cancel for on-shell or soft gluons (Ward identities, low-energy theorem [Low(1958)]) $\Rightarrow$ suppression
- but not for hard off-shell gluons ([Witten (1977)])!

Hiperindex