Holographic imaging of nucleon via deeply virtual Compton scattering and conformal symmetry

Krešimir Kumerički

Department of Physics
University of Zagreb

Collaboration with:

Dieter Müller (Regensburg),
Kornelija Passek-Kumerički (Regensburg, Zagreb),
Andreas Schäfer (Regensburg)

Institut “Jožef Stefan”
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Outline

Introduction to Generalized Parton Distributions (GPDs)
- Proton Structure
- Definition and properties of GPDs
- Deeply virtual Compton scattering (DVCS)

Conformal Approach to DVCS Beyond NLO
- Conformal Approach
- DVCS at NNLO perturbative QCD

Results
- Choice of GPD Ansatz
- Size of Radiative Corrections
- Fitting GPDs to Data
- 3D image of proton

Summary
Parton distribution functions

- Deeply inelastic scattering, \(-q_1^2 \to \infty\), 
  \[ x_{BJ} \equiv \frac{-q_1^2}{2p \cdot q_1} \to \text{const} \]
Parton distribution functions

- Deeply inelastic scattering, \(-q_1^2 \rightarrow \infty\), \(x_{BJ} \equiv \frac{-q_1^2}{2p \cdot q_1} \rightarrow \text{const}\)
Parton distribution functions

- Deeply inelastic scattering, $-q_1^2 \to \infty$, $x_{BJ} \equiv \frac{-q_1^2}{2p \cdot q_1} \to \text{const}$

\[
\sum_X \frac{\gamma^*}{xp} \rightarrow \frac{2}{pdf q(x)} \rightarrow \text{const}
\]

$q(x)$ – probability that parton $q$ has momentum $xp$
Parton distribution functions

• Deeply inelastic scattering, \(-q_1^2 \rightarrow \infty\), \(x_{BJ} \equiv \frac{-q_1^2}{2p \cdot q_1} \rightarrow \text{const}\)

\[
\sum_X x_p \gamma^* p q_1 \rightarrow \gamma^* x p X \rightarrow \gamma^* x p p
\]

\[
PDF q(x) - \text{probability that parton } q \text{ has momentum } x p
\]

• no information on spatial distribution of partons
Electromagnetic form factors

- Dirac and Pauli form factors:
  \[ F_{1,2}(t = q_1^2) \]
Electromagnetic form factors

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\[ q(b_\perp) \sim \int db_\perp e^{i q_1 \cdot b} F_1(t = q_1^2) \]
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Electromagnetic form factors

- Dirac and Pauli form factors:
  \[ q(b_\perp) \sim \int db_\perp e^{iq_1 \cdot b_\perp} F_1(t = q_1^2) \]

- GPD: \[ H^q(x, 0, t = \Delta^2) = \int db_\perp e^{i\Delta \cdot b_\perp} q(x, b_\perp) \]
Probing the proton with two photons

- Deeply virtual Compton scattering [Müller '92, et al. '94]
Probing the proton with two photons

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\[ P = P_1 + P_2 \quad q = \frac{(q_1 + q_2)}{2} \]

Generalized Bjorken limit:

\[-q^2 \simeq \frac{Q^2}{2} \to \infty \]

\[ \xi = \frac{-q^2}{2P \cdot q} \to \text{const} \]

- QCD: factorization of short- and long-distance physics
Probing the proton with two photons

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- QCD: factorization of short- and long-distance physics

\[ \langle P_2 | \bar{q}(-z-) \gamma^+ q(z-) | P_1 \rangle \]
**Definition of GPDs**

- In QCD GPDs are defined as [Müller '92, et al. '94, Ji, Radyushkin '96]

\[
F^q(x, \eta, \Delta^2) = \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle P_2 | \bar{q}(-z) \gamma^+ q(z) | P_1 \right\rangle \bigg|_{z^+=0, z_\perp=0}
\]

\[
F^g(x, \eta, \Delta^2) = \frac{4}{P^+} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle P_2 | G^+_{a\mu}(-z) G^{-\mu}_a(z) | P_1 \right\rangle \bigg|_{z^+=0, z_\perp=0}
\]

Forward limit
\[
x + \eta \frac{P^+}{2} \quad x - \eta \frac{P^+}{2} \quad \frac{1 + \eta}{2} P^+ \quad \frac{1 - \eta}{2} P^+
\]

\[
P = P_1 + P_2 ; \quad \Delta = P_2 - P_1 ; \quad \eta = -\frac{\Delta^+}{P^+} \text{ (skewedness)}
\]
Properties of GPDs

- Decomposing into helicity conserving and non-conserving part:

\[ F^a = \frac{\bar{u}(P_2)\gamma^+ u(P_1)}{P^+} H^a + \frac{\bar{u}(P_2)i\sigma^{+\nu} u(P_1)\Delta_\nu}{2MP^+} E^a \quad a = q, g \]
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• Forward limit (\(\Delta \to 0\)): \(\Rightarrow\) GPD \(\to\) PDF

\[
F^q(x, 0, 0) = H^q(x, 0, 0) = \theta(x)q(x) - \theta(-x)\bar{q}(-x)
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\]

- Sum rules:

\[
\int_{-1}^{1} dx \left\{ \begin{array}{c}
H^q(x, \eta, \Delta^2) \\
E^q(x, \eta, \Delta^2) \\
\end{array} \right\} = \left\{ \begin{array}{c}
F^q_1(\Delta^2) \\
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- Possibility of flavour decomposition of proton spin

\[
\frac{1}{2} \int_{-1}^{1} dx x \left[ H^q(x, \eta, \Delta^2) + E^q(x, \eta, \Delta^2) \right] = J^q(\Delta^2) \quad [\text{Ji '96}]
\]
Relevance of GPDs for collider physics

- heavy particle production $\Rightarrow$ collision is more central
  $\Rightarrow$ larger probability for multiple parton collisions

- [Frankfurt, Strikman and Weiss ’04]
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- heavy particle production $\Rightarrow$ collision is more central $\Rightarrow$ larger probability for multiple parton collisions

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- No influence on total inclusive cross sections, but event structure is sensitive to transversal parton distributions.

- Relevant for LHC?
Deeply virtual Compton scattering (I)

- Measured in leptoproduction of a real photon:
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\[ \sigma \propto |T_{DVCS}|^2 + |T_{BH}|^2 + T_{DVCS}^\ast T_{BH} + T_{DVCS}T_{BH}^\ast \]

Using \( T_{BH} \) as a referent "source" enables measurement of the phase of \( T_{DVCS} \) → proton "holography" [Belitsky and Müller '02]
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\gamma^* P_1 P_2 \rightarrow F_{1,2}(\Delta) \]

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\[ P = P_1 + P_2 \quad q = (q_1 + q_2)/2 \]
\[ \Delta = P_2 - P_1 \]

\[ q_1^2 = Q^2 \quad q_2^2 = 0 \]

\[ -q^2 \approx Q^2/2 \to \infty \]
\[ \xi = \frac{-q^2}{2P \cdot q} \to \text{const} \]

\[ A(\xi, \Delta^2, Q^2) = \sum_i \int dx \ C_i(x, \xi, Q^2/\mu^2) \ GPD_i(x, \eta = \xi, \Delta^2, \mu^2) \]
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- Measurements at DESY, JLab, CERN (COMPASS)
- At large energies, flavour singlet GPDs dominate
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- \( \Rightarrow \) need NNLO to stabilize perturbation series and investigate convergence \( \Rightarrow \) conformal approach
Operator Product Expansion

\[ J_{em}(x)J_{em}(0) \rightarrow \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \left( \frac{1}{x^2} \right)^2 x_{n+k+1} C_{n,k} O_{n,k} \]

\[ O_{n,k} \equiv (i \partial_+)^k \bar{\psi} \gamma^+ (i \leftrightarrow D_+)^n \psi \]

\[ D_+ \equiv \vec{D}_+ - \vec{D}_+ \]
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- \( C_{n,0} \) and \( \gamma_n \) of \( O_{n,0} \) are well known from DIS up to NNLO.
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- \( \gamma_{n,k} \neq 0 \Rightarrow \) operators \( O_{n,k} \) mix under evolution.
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• Choosing operator basis in which \( \gamma_{n,k} \) is diagonal would help. But how to diagonalize unknown matrix?!
Introduction to GPDs

Conformal Approach to DVCS Beyond NLO

Results

Summary

Operator Product Expansion

\[ J_{em}(x) J_{em}(0) \longrightarrow \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \left( \frac{1}{x^2} \right)^2 x_{-n+k+1}^n C_{n,k} O_{n,k} \]

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- Choosing operator basis in which \( \gamma_{n,k} \) is diagonal would help. But how to diagonalize unknown matrix?!
- (At least) to LO answer is: use conformal operators.
Conformal operators

\[ \mathbb{O}_{n,n+k} = (i\partial^+)^{n+k} \bar{\psi} \gamma^+ C_n^{3/2} \left( \frac{D^+}{\partial^+} \right) \psi \]

- they have well-defined conformal spin \( j = n + 2 \)
- massless QCD is conformally symmetric at the tree level
  \( \Rightarrow \) conformal spin is conserved
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- conformal symmetry broken at the loop level (renormalization introduces mass scale, dimensional transmutation) \( \Rightarrow \)
  - running of the coupling constant \( \partial g / \partial \ln \mu \equiv \beta \neq 0 \)
  - anomalous dimensions of operators \( \gamma_{jk} = \delta_{jk} \gamma_j + \gamma_{jk}^{\text{ND}} \)
Conformal operators

\[ \bigodot_{n,n+k} = (i\partial^+)^{n+k} \bar{\psi} \gamma^+ C_n^{3/2} \left( \frac{D^+}{\partial^+} \right) \psi \]

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\( \Rightarrow \bigodot_{n,n+k} \) start to mix at NLO
Conformal Towers

\[ \text{spin} = n + k + 1 \]

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These mix at NLO

\[ \text{Diagonalize in artificial } \beta = 0 \text{ theory by changing scheme} \]

\[ O_{CS} = B - 1 O_{MS} \]

\[ \gamma_{CS,jk} = \delta_{jk} \gamma_k \]
Conformal Towers

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so that

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Conformal Towers

• Diagonalize in artificial $\beta = 0$ theory by changing scheme

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Conformal Towers

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\[ \Omega_{\text{CS}} = B^{-1} \Omega_{\text{MS}} \]

so that

\[ \gamma_{jk}^{\text{CS}} = \delta_{jk} \gamma_k \]

- \( C_{n,k} = (-1)^k \frac{(n+2)_k}{k!(2n+4)_k} C_{n,0} \) ⇒ summing complete tower
In full QCD $\beta \neq 0$ and NLO diagonalization is spoiled:

$$\gamma_{jk}^{CS} = \delta_{jk}\gamma_k + \frac{\beta}{g}\Delta_{jk}$$
\( \beta \neq 0 \) (I)

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\gamma^{\text{CS}}_{jk} = \delta_{jk} \gamma_k + \frac{\beta}{g} \Delta_{jk}
\]

- However, there is also ambiguity in \( \overline{\text{MS}} \to \text{CS} \) rotation matrix:

\[
B = B^{(\beta=0)} + \frac{\beta}{g} \delta B
\]
\( \beta \neq 0 \quad (1) \)

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- By judicious choice of \( \delta B \) one can “push” mixing to NNLO (\( \overline{\text{CS}} \) scheme, [Melić et al.]).
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- By judicious choice of $\delta B$ one can “push” mixing to NNLO ($\overline{\text{CS}}$ scheme, [Melić et al.]).

- But how to calculate rotation matrix $B$? This is problem equivalent to calculation of $\gamma_{j,k}$. 
\[ \beta \neq 0 \text{ (II)} \]

- The \( B^{(\beta=0)} \) is constrained by conformal Ward identities . . .

\[
B_{jk}^{(\beta=0)\text{NLO}} = \delta_{jk} - \frac{\alpha_s}{2\pi} \theta(j > k) \frac{\gamma_{jk}^{\text{SCT, LO}}}{a_{jk}} \quad (a_{jk} \text{ — known matrix})
\]

SCT \equiv \text{special conformal transformation}
\[ \beta \neq 0 \ (II) \]

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(SCT ≡ special conformal transformation)

- . . . and, as a consequence

\[
\overline{\text{MS}}_{\gamma_{jk}}^{\text{ND},(1)} = \left[ \gamma_{j}^{\text{SCT, (0)}} - \beta_0 \frac{b}{g}, \gamma_{k}^{(0)} \right]_{jk} \]

\( a_{jk} \) — known matrix

[ Müller '93]
\[ \beta \neq 0 \ (II) \]

- The \( B^{(\beta=0)} \) is constrained by conformal Ward identities ... 

\[
B^{(\beta=0)\text{NLO}}_{jk} = \delta_{jk} - \frac{\alpha_s}{2\pi} \theta(j > k) \frac{\gamma^{\text{SCT}, \text{LO}}_{jk}}{a_{jk}}
\]

\( \gamma^{\text{SCT}} \equiv \text{special conformal transformation} \)

- ... and, as a consequence

\[
\overline{\text{MS}}^{\text{ND}, (1)}_{\gamma_{jk}} = \left[ \gamma^{\text{SCT}, (0)} - \beta_0 \frac{b}{g}, \gamma^{(0)} \right]_{jk} \frac{a_{jk}}{a_{jk}} 
\]

- Final result:

\( n \)-loop DIS (diagonal) result + \((n - 1)\)-loop SCT anomaly = \( n \)-loop non-diagonal prediction
NNLO DVCS (I)

- DVCS amplitude in terms of conformal moments:

\[ S^H(\xi, \Delta^2, Q^2) = 2 \sum_{j=0}^{\infty} \xi^{-j-1} C_j(Q^2/\mu^2, \alpha_s(\mu)) \mathcal{H}_j(\xi = \eta, \Delta^2, \mu^2) \]

\[ H^q_j(\eta, \ldots) = \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} dx \eta^{j-1} C_j^{3/2}(x/\eta) H^q(x, \eta, \ldots) \]
NNLO DVCS (I)

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- ... analogous to Mellin moments in DIS: \( x^n \rightarrow C^{3/2}_n(x) \)
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• ...analogous to Mellin moments in DIS: \( x^n \rightarrow C_n^{3/2}(x) \)
• Here, Wilson coefficients \( C_j \) read ...
NNLO DVCS (II)

\[ C_j(Q^2/\mu^2, Q^2/\mu^*^2, \alpha_s(\mu)) = \sum_{k=j}^{\infty} C_k(1, \alpha_s(Q)) \mathcal{P} \exp \left\{ \int_{Q}^{\mu} \frac{d\mu'}{\mu'} \left[ \gamma_j(\alpha_s(\mu')) \delta_{kj} + \frac{\beta}{g} \Delta_{kj}(\alpha_s(\mu'), \mu'/\mu^*) \right] \right\} \]

with

\[ C_j(1, \alpha_s(Q)) = \frac{2^{1+j+\gamma_j(\alpha_s)/2} \Gamma \left( \frac{5}{2} + j + \gamma_j(\alpha_s)/2 \right)}{\Gamma(3/2) \Gamma(3 + j + \gamma_j(\alpha_s)/2)} c_j^{\overline{\text{MS}},\text{DIS}}(\alpha_s) \]

- \[ \frac{2^{\cdots} \Gamma(\cdots)}{\Gamma(3/2) \Gamma(\cdots)} \] is result of resumming the conformal tower \( j \)
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- \( c_j^{\overrightarrow{\text{MS,DIS}}}(\alpha_s) \) from [Zijlstra, v. Neerven '92,'94, v. Neerven and Vogt '00]

- Finally, evolution of conformal moments is given by ...
NNLO DVCS (III)

\[ \mu \frac{d}{d\mu} H_j(\cdots, \mu^2) = -\gamma_j(\alpha_s(\mu)) H_j(\cdots, \mu^2) \]

\[ - \frac{\beta(\alpha_s(\mu))}{g(\mu)} \sum_{k=0}^{j-2} \eta^{j-k} \Delta_{jk}(\alpha_s(\mu), \frac{\mu}{\mu^*}) H_k(\cdots, \mu^2) \]

- \( \Delta_{jk} \) — unknown correction, starts at NNLO, and can be suppressed by choice of initial condition — neglected
- \( \gamma_j \) from [Vogt, Moch and Vermaseren '04]
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- We have used these expressions to
  1. investigate size of NNLO corrections to non-singlet [Müller '05] and singlet [K.K., Müller, Passek-Kumerički and Schäfer '06] Compton form factors
  2. perform fits to DVCS (and DIS) data and extract information about GPDs [K.K., Müller and Passek-Kumerički '07]
Results on NNLO DVCS

- We use simple Regge-inspired ansatz for GPDs . . .

\[ H_j(\xi, \Delta^2, Q_0^2) = \begin{pmatrix} N'_\Sigma F_\Sigma(\Delta^2) B\left(1 + j - \alpha_\Sigma(\Delta^2), 8\right) \\ N'_G F_G(\Delta^2) B\left(1 + j - \alpha_G(\Delta^2), 6\right) \end{pmatrix} \]

\[ \alpha_a(\Delta^2) = \alpha_a(0) + 0.25\Delta^2 \quad F_a(\Delta^2) = \left(1 - \frac{\Delta^2}{m_a^2}\right)^{-3} \]
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• . . . corresponding in forward case (\(\Delta = 0\)) to PDFs of form

\[ \Sigma(x) = N'_\Sigma x^{-\alpha_\Sigma(0)} (1 - x)^7 ; \quad G(x) = N'_G x^{-\alpha_G(0)} (1 - x)^5 \]
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- We calculate \(K\)-factors

\[ K^P_{\text{abs}} = \left| \frac{S\mathcal{H}^{NP\text{LO}}}{S\mathcal{H}^{NP-1\text{LO}}} \right| \quad K^P_{\text{arg}} = \frac{\text{arg}(S\mathcal{H}^{NP\text{LO}})}{\text{arg}(S\mathcal{H}^{NP-1\text{LO}})} \]
Size of Radiative Corrections - Modulus

- NLO: up to 40–70% (\(\overline{\text{MS}}\)); up to 30–55% (\(\overline{\text{CS}}\)) ["hard"]
- NNLO: 8–14% ("hard"); 1-4% ("soft") [\(\overline{\text{CS}}\)]
Scale Dependence

Same $K$-factors, but with $\mathcal{H} \rightarrow d\mathcal{H}/d\ln Q^2$

- **NLO**: even 100%
- **NNLO**: somewhat smaller, but acceptable only for larger $\xi$
Fast fitting routine

- $N_\Sigma = 0.143$, $\alpha_\Sigma(0) = 1.10$, $m_\Sigma = 1.26$, $N_G = 0.549$, $\alpha_G(0) = 0.915$, $m_G = 1.66$, $Q_0^2 = 2.5$ GeV$^2$

- $\chi^2/(\text{number of degrees of freedom}) = 54/64$
Example of final fit result

- $d\sigma(\gamma^* p \rightarrow \gamma p)/dt$ [nb/GeV$^2$]
- $\sigma(\gamma^* p \rightarrow \gamma p)$ [nb]
- $F_2$

**Graphs:**
- $Q^2$ vs. $-t$ [GeV$^2$]
- $W$ vs. $Q^2$ [GeV$^2$]
- $W$ vs. $t$ [GeV$^2$]
- $F_2$ vs. $Q^2$ [GeV$^2$]
**Parton probability density**

- Fourier transform of GPD for $\eta = 0$ can be interpreted as probability density depending on $x$ and transversal distance $b$
  
  \[ H(x, \vec{b}) = \int \frac{d^2 \vec{\Delta}}{(2\pi)^2} e^{-i\vec{b} \cdot \vec{\Delta}} H(x, \eta = 0, \Delta^2 = -\vec{\Delta}^2) , \]

- Average transversal distance:
  
  \[ \langle \vec{b}^2 \rangle(x, Q^2) = \frac{\int d\vec{b} \, b^2 H(x, \vec{b}, Q^2)}{\int d\vec{b} \, H(x, \vec{b}, Q^2)} = 4B(x, Q^2) , \]

\[ (at \quad Q^2 = 4 \text{ GeV}^2) \]

\[ \langle \vec{b}^2 \rangle_{\text{gluon}}(\xi = 10^{-3}) = 0.30^{+0.07}_{-0.04} \text{ fm}^2 \]
Three-dimensional image of a proton

Quarks:

Gluons:

$H(x, b)$

$H(x, b) \times 10^{-5}$

$H(x, b) \times 10^{-3}$

$H(x, b) \times 10^{-1}$

$H(x, b)$ normalized
Summary

- Generalized parton distributions offer unified description of the proton structure. They are experimentally accessible via DVCS.
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The End
Relation to distribution amplitudes

- In QCD GPDs are defined as [Müller '92, et al. '94, Ji, Radyushkin '96]

\[
F^q(x, \eta, \Delta^2) = \int \frac{dz^-}{2\pi} e^{ix P^+ z^-} \langle P_2 | \bar{q}(-z) \gamma^+ q(z) | P_1 \rangle \bigg|_{z^+=0, z_\perp=0}
\]

\[
F^g(x, \eta, \Delta^2) = \frac{4}{P^+} \int \frac{dz^-}{2\pi} e^{ix P^+ z^-} \langle P_2 | G_a^+(-z) G_{a\mu}^+(z) | P_1 \rangle \bigg|_{...}
\]

\[
P = P_1 + P_2 ; \quad \Delta = P_2 - P_1 ; \quad \eta = -\frac{\Delta^+}{P^+} \text{ (skewedness)}
\]
Conformal algebra

- Conformal group restricted to light-cone \( \sim O(2,1) \)

\[
L_+ = -iP_+ \\
[L_0, L_\mp] = \mp L_\mp \\
L_\pm = \frac{i}{2} K_\pm \\
[L_-, L_+] = -2L_0 \\
L_0 = \frac{i}{2} (D + M_{-+}) \\
L^2 = L_0^2 - L_0 + L_- L_+ \\
(L^2, \mathcal{O}_{n,n+k}) = j(j-1) \mathcal{O}_{n,k}
\]

\((D — \text{dilatations, } K_\pm — \text{special conformal transformation (SCT)})\)
Appendix

Size of Radiative Corrections - phase

\[ \Delta^2 = 0 \]

- Thick lines: "hard" gluon
  \[ N_G = 0.4 \]
  \[ \alpha_G(0) = \alpha_S(0) + 0.1 \]
- Thin lines: "soft" gluon
  \[ N_G = 0.3 \]
  \[ \alpha_G(0) = \alpha_S(0) \]

- NLO: up to 24% (\(\overline{\text{MS}}\)); up to 13% (\(\overline{\text{CS}}\))
- NNLO and "soft" NLO — less than 5%

["hard"]
Appendix

Scale Dependence - Modulus

- **NLO**: even 100%
- **NNLO**: smaller (largest for “hard” gluons)

Thick lines:
“hard” gluon
\[ N_G = 0.4 \]
\[ \alpha_G(0) = \alpha_S(0) + 0.1 \]

Thin lines:
“soft” gluon
\[ N_G = 0.3 \]
\[ \alpha_G(0) = \alpha_S(0) \]