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Quark off-shell contributions to $K_L \rightarrow \gamma\gamma$ in a bound-state approach

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Abstract

We present a new piece of evidence in favour of the importance of the quark off-shellness in the kaon. The matrix elements of the flavour-changing operators for $K_L \rightarrow \gamma\gamma$ comply with the general behaviour of the matrix elements expected in the pairing bound-state model used here. The present calculation in essence agrees with previous chiral-quark results. The off-shell contribution turns out to be dominant (the model on-shell amplitude being at the 10% level). Compared with the chiral perturbation-theory approach, our off-shell contribution is an entirely new $\mathcal{O}(p^4)$ *direct-decay* piece, whereas the non-diagonal magnetic-moment term belongs to the order $\mathcal{O}(p^6)$.

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1 Introduction

There is a long-standing need to calculate hadronic matrix elements properly – a problem that involves departure from the parton (free-quark mass-shell) regime. A recent attempt to partially fulfill this programme has focused on the study of quark off-shell effects in radiative K^0 -decays [1, 2]. These off-shell contributions were revealed by Eeg and Picek first in the significant CP-violating $K_L \rightarrow \gamma\gamma$ amplitude [1] and more recently in the “anomalous part” of the CP-conserving $K_L \rightarrow \gamma\gamma$ amplitude [2]. The theoretical framework in which off-shell quarks were handled was an effective low-energy QCD model [3], the “chiral quark model” providing the meson-quark coupling. Such a framework departs from the perturbative (partonic) QCD regime, where one faces the vanishing of the matrix elements of those operators [4] which are zero if QCD equations of motion (EOM) are used. However, presently it is possible to go beyond the perturbative analysis of Politzer and Simma [4] only in the model dependent way. Therefore the off-shell effect calculated in the chiral quark model should be studied in other approaches accounting for the nonperturbative QCD.

The purpose of this paper is to investigate off-shell effects in a completely different type of model where mesons are not treated as elementary fields, but are represented by quark-antiquark bound states. Such a model is a natural environment for studying off-shell effects, as quarks are by definition off-shell in the bound states, and especially so in such strongly bound, highly relativistic systems as light pseudoscalar mesons. Another advantage is that bound-state solutions are not pointlike but extended, and this will by itself ensure that the quark loops in our calculations do not diverge. As has often been pointed out (e. g. recently by Ball and Ripka [5]), *ad hoc* regularization procedures often lead to inconsistencies and spurious results. In the present approach, no *ad hoc* regularizations or cut-offs are necessary, because the momentum dependence of the non-local bound-state meson vertices provides a natural regularization.

The calculation of the matrix elements of relevant quark operators will proceed along the same track as the previous calculation of pion and kaon decay constants and the $\pi^0 \rightarrow \gamma\gamma$ amplitude in the quark bilocal bound-state model [6]. Using $\pi^0 \rightarrow \gamma\gamma$ as a “monitoring process” has the advantage of decoding the anomaly part in $K_L \rightarrow \gamma\gamma$ in the same way in which the comparative consideration of these processes in variants of the effective low-energy QCD enabled us to isolate the anomalous contribution in [2].

As in [2], let us again parametrize the $K_L \rightarrow \gamma\gamma$ and $\pi^0 \rightarrow \gamma\gamma$ processes by an effective interaction of order $\mathcal{O}(\alpha = e^2/4\pi)$:

$$\mathcal{L}_P = \alpha C_P \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} \phi_P , \quad (1)$$

where, for $P = K_L \simeq K_2 = \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0)$ or $P = \pi^0$, the measured widths require

$$|C_{K_2}| = 5.9 \times 10^{-11} \text{MeV}^{-1} \quad ; \quad |C_{\pi^0}| = 4.3 \times 10^{-4} \text{MeV}^{-1} \quad .$$

The low-energy QCD calculation of ref. [2] accounted for the full C_{π^0} (axial anomaly) amplitude. In this reference the authors were able to isolate the anomalous part of the $K_L \rightarrow \gamma\gamma$ amplitude and showed that it accounted for roughly a quarter¹ of the empirical $|C_{K_2}|$. This amplitude, resembling very much the famous anomalous pionic one, is essentially a *direct* contribution, as opposed to possible *reducible* pole contributions, which is out of scope of both the previous [1, 2] and the present work. Actually, the anomalous part of the $K_L \rightarrow \gamma\gamma$ amplitude was represented by the off-shell contribution from the point of view of the effective quark-operator evaluation.

Let us now recall the appearance of the off-shellness [1, 2] in $K_L \rightarrow \gamma\gamma$. The essential point is to overbridge the nonperturbative QCD ($\lesssim 1$ GeV) and the electroweak ($\sim M_W$) scale. At the latter, the flavour change (FC) $s \rightarrow d$ in the presence of external photons results (after integrating out heavy loop particles) in an effective lagrangian [1]

$$\mathcal{L}(s \rightarrow d)_\gamma = B \epsilon^{\mu\nu\lambda\rho} F_{\mu\nu} (\bar{d} i \overleftrightarrow{D}_\lambda \gamma_\rho L s) , \quad (2)$$

where quarks are interacting fields with respect to QCD. This fundamental fields are chiral fermions ($L = \frac{1-\gamma_5}{2}$, $R = \frac{1+\gamma_5}{2}$ projections). As explained previously [1], dealing with free quarks at such a high-energy scale results in cancellation of 1PI and 1PR graphs for $s \rightarrow d\gamma\gamma$, induced by $V_{1\gamma}$ and $V_{2\gamma}$ vertices contained in (2) and explicated in eqs. (10) and (11) below. However, when bringing these vertices below the GeV scale, they operate on the off-shell (bound) quark states and the cancellation can be lost. In order to follow the fate of the off-shell contribution, it is convenient to rewrite (2) in the form

$$\mathcal{L}(s \rightarrow d)_\gamma = \mathcal{L}_F + \mathcal{L}_\sigma \quad , \quad (3)$$

where

$$\mathcal{L}_F = B \bar{d} [(i\gamma \cdot D - m_d) \sigma_{\mu\nu} F^{\mu\nu} L + \sigma_{\mu\nu} F^{\mu\nu} R (i\gamma \cdot D - m_s)] s \quad (4)$$

represents the piece which would vanish on-shell by applying the QCD equations of motion, and

$$\mathcal{L}_\sigma = B \bar{d} (m_s \sigma_{\mu\nu} F^{\mu\nu} R + m_d \sigma_{\mu\nu} F^{\mu\nu} L) s \quad (5)$$

¹ Instead of the half quoted in ref. [2]. There was a superfluous factor of 2 in eq. (8) of this reference.

is the off-diagonal magnetic-moment term which vanishes in the chiral limit. The quantity $B \sim eG_F$,

$$B = \frac{G_F}{\sqrt{2}} \frac{e}{4\pi^2} \lambda_u \hat{B}_u \quad , \quad (6)$$

contains the Kobayashi–Maskawa factor of the value $\lambda_u = V_{ud}V_{us}^* \simeq 0.215$, whereas \hat{B}_u incorporates the perturbative, short-distance QCD corrections. The precise value of this \hat{B}_u factor depends on the renormalization scale μ [7], increasing from 0.16 for $\mu = 0.7$ GeV to 0.66 for $\mu = 0.3$ GeV for the local operator in (4), and ranging between 0.14 for $\mu = 0.7$ GeV and 0.32 for $\mu = 0.3$ GeV for the local operator of the nondiagonal magnetic-moment transition in (5). Somewhat different behaviour of these two operators at lower-energy scales is due to the different anomalous dimensions they have.

In the present paper the calculation in the chiral limit suffices to extract the chiral-anomaly contribution (which is known to be an overwhelming contribution to the $\pi^0 \rightarrow \gamma\gamma$ decay). However, the kaon decay requires investigation beyond the chiral limit. Therefore, in the following we first present the calculation in the $SU(3)_f$ limit ($m_q = m_{u,d} = m_s$), suitable for extracting the anomalous contribution in the chiral limit. After that we proceed with the study of $K_L \rightarrow \gamma\gamma$ beyond the chiral limit and for non-degenerate quark masses. This enables us to find a share of the non-anomalous part in the direct $K_L \rightarrow \gamma\gamma$ decay amplitude.

2 Bound-state evaluation of the $K_L \rightarrow \gamma\gamma$ amplitude

To evaluate the hadronic matrix elements of the above effective operators (2)–(6), we use the variant [8, 9] of an effective meson bilocal theory [9–12] in which the related decay $\pi^0 \rightarrow \gamma\gamma$ was computed by two of us [6], along with the pion and kaon spectrum and the decay constants² F_π and F_K .

Varying the effective bilocal action yields [9–12, 7] the Schwinger–Dyson equation (SDE) for the dressed quark propagator (whose self-mass $\Sigma(q)$ is thereby generated dynamically), and the Bethe-Salpeter equation (BSE) for the bilocal bound-state meson-vertex function. The meson mass M_P results as the corresponding BSE eigenvalue.

Since we are interested in the qualitative issue of the existence and the importance of off-shell effects and not in the precise quantitative description of hadrons as bound states, we choose a very simplified instantaneous quark-quark interaction kernel $K(x, y)$ leading to a potential model with very

² $F_K = \sqrt{2}f_K$, where $f_K = f_\pi$ ($f_\pi^{expt} = 92.4 MeV$) in the $SU(3)_f$ symmetry limit.

tractable SDE and BSE. Concretely, we use the special form [8, 9]

$$K^\eta(x - y) = K^\eta(z, X) = \eta_\mu \gamma^\mu V(z_\perp) \delta(z_P) \eta_\nu \gamma^\nu \quad , \quad (7)$$

where $z = x - y$, $X = (x + y)/2$ and $\eta^\mu = P^\mu / \sqrt{P^2}$, whereas the decomposition of four-vectors into components parallel and perpendicular to the total meson momentum P^μ is given by $x_\parallel^\mu = \eta^\mu x_P$, $x_P = x \cdot \eta$ and $x_\perp^\mu = x^\mu - x_\parallel^\mu$. $V(r)$ is a scalar function of $r = z_\perp$.

Choosing the model harmonic interaction, $V(r) = (4/3)V_0 r^2$, $V_0 = const$, just as in ref. [6], we are able to use the SDE and BSE solutions for the pion and the kaon, which were obtained in [6], and use them here in the calculation of $\bar{K}^0 \rightarrow \gamma\gamma$. The case of the “funnel” (Coulomb+linear) potential has also been solved [14], but we do not use it here to avoid complexities of the renormalization of the divergences appearing in the bound-state equations [15, 16, 14] for this choice of the potential. On the other hand, when $V(r)$ is chosen to be the Nambu-Jona-Lasinio contact potential, a UV cut-off is needed. An additional motivation for choosing the harmonic potential is therefore the fact that both the SDE and the BSE are then divergence-free. Namely, besides the absence of divergences in quark-loop integrals commented on in the Introduction, we also avoid divergences in the bound-state equations themselves, so that no regularizations or cut-offs are necessary.

Since in the presently calculated matrix elements we need only one bilocal, it is of course most convenient to work in its rest frame. We point out that in this frame the special ansatz (7) for the interaction kernel reduces [$\eta = (1, 0, 0, 0)$] to the ordinary $\gamma^0 V(r) \gamma^0$ type of interaction used in many calculations for mesons in the “pairing” (Nambu–Jona-Lasinio–inspired) approach, *e.g.* refs. [15, 17, 16]. Among these, we may point out Le Yaouanc *et al.* [17] as a paradigmatic example because they studied the harmonic binding, $V \sim r^2$, in detail. However, the problems they pointed out as induced by non-covariance (ambiguities in the definition of F_π , the wrong dispersion relation) are avoided by the usage of the ansatz kernel (7) since it leads to boost-invariant SDE and BSE. Of course, the important features of pseudoscalar meson physics established in ref. [17], such as the dynamical chiral symmetry breaking (D χ SB) by generating the dynamical quark mass and the appearance of the pion as a Goldstone boson in the chiral limit, were also present in ref. [6] and are therefore also present in this work. This is especially important here where we want to elucidate some aspects of long-distance, non-perturbative QCD effects in $\bar{K}^0 \rightarrow \gamma\gamma$.

The transitions of bilocal mesons are due to the interaction part of the effective bilocal action W used in ref. [6], which we call W_{trans} . It is obtained [12, 8, 9] by integrating out the fermions in the generating functional, which results in the fermion determinant, *i. e.* a trace-log form whose expansion

has infinitely many terms:

$$W_{trans}[\mathcal{M}, L] = iN_c \sum_{n=2}^{\infty} \frac{1}{n} \text{Tr} \Phi^n \quad , \quad (8)$$

where “Tr” also includes the integration, and Φ is defined through the meson bilocal \mathcal{M} which must now be “shifted” [6, 18] by the transition-inducing external operator L , coupled locally to the internal quark lines represented by the dressed propagator G_Σ :

$$\Phi(x, y) = \int d^4z G_\Sigma(x, z) [\mathcal{M}(z, y) - L(z) \delta^{(4)}(x - y)] \quad . \quad (9)$$

For the problem at hand, $L(x)$ represents a sum of operators describing electromagnetic and weak radiative decays of mesons (whose hadronic structure is described by the bilocal \mathcal{M} , the solution of the bound-state equations governed by the quark interaction kernel $K(x, y)$).

For leptonic weak decays (e. g. when calculating F_π or F_K in ref. [6]), $L(x)$ was simply the leptonic current coupled to the V–A quark current. Here, in order to induce $\bar{K}^0 \rightarrow \gamma\gamma$, $L(x)$ includes both the ordinary flavour diagonal photonic coupling $eQ\gamma^\mu A_\mu(x)$ and the electroweak FC vertices contained in (2):

$$V_{1\gamma} = B\varepsilon^{\mu\nu\lambda\rho} F_{\mu\nu}(x) i \overleftrightarrow{\partial}_\lambda(x) \gamma_\rho \frac{1}{2}(1 - \gamma_5) \quad ; \quad \overleftrightarrow{\partial} = \overrightarrow{\partial} - \overleftarrow{\partial} \quad , \quad (10)$$

$$V_{2\gamma} = 2e_D B\varepsilon^{\mu\nu\lambda\rho} F_{\mu\nu}(x) A_\lambda(x) \gamma_\rho \frac{1}{2}(1 - \gamma_5) \quad . \quad (11)$$

Here $Q = \frac{1}{3} \text{diag}(2, -1, -1)$ and $e_D = eQ_D = -e/3$.

The transition matrix element is thus

$$\mathcal{A}_{\bar{K}^0\gamma\gamma} = \langle \gamma(k, \sigma), \gamma(k', \sigma') | W_{trans}[\mathcal{M} - eQ\cancel{A} - V_{1\gamma} - V_{2\gamma}] | \bar{K}^0(p) \rangle \quad , \quad (12)$$

where p, k, k' are the kaon and photon momenta, respectively, and σ, σ' are photon polarizations.

We recall that in ref. [6] the $\pi^0 \rightarrow \gamma\gamma$ transition was caused by the cubic ($n = 3$) term from W_{trans} , because it contained subterms with one meson bilocal \mathcal{M} and two photon fields A_μ , yielding the amplitude corresponding to the triangle graph:

$$\mathcal{A}_{\pi^0\gamma\gamma} = \langle \gamma(k, \sigma), \gamma(k', \sigma') | iN_c \text{Tr}(\mathcal{M} G_\Sigma eQ\cancel{A} G_\Sigma eQ\cancel{A} G_\Sigma) | \pi^0(p) \rangle \quad . \quad (13)$$

Similarly, for strangeness-changing transitions, the cubic ($n = 3$) term will contribute to $\bar{K}^0 \rightarrow \gamma\gamma$ through subterms containing one meson bilocal \mathcal{M} ,

one pure electromagnetic and one single-photon effective electroweak vertex $V_{1\gamma}$:

$$\mathcal{A}_{\bar{K}^0\gamma\gamma}^{(1)} = \langle \gamma\gamma | iN_c \text{Tr}(\mathcal{M}G_\Sigma V_{1\gamma} G_\Sigma eQ\mathcal{A}G_\Sigma + \mathcal{M}G_\Sigma eQ\mathcal{A}G_\Sigma V_{1\gamma} G_\Sigma) | \bar{K}^0 \rangle \quad . \quad (14)$$

This corresponds to the triangle graphs in fig. 1 and their crossed counterparts.

On the other hand, the existence of the FC two-photon vertex leads to the contribution to the total amplitude from the quadratic ($n = 2$) term:

$$\mathcal{A}_{\bar{K}^0\gamma\gamma}^{(2)} = \langle \gamma(k, \sigma), \gamma(k', \sigma') | iN_c \text{Tr}(\mathcal{M}G_\Sigma V_{2\gamma} G_\Sigma) | \bar{K}^0(p) \rangle \quad , \quad (15)$$

corresponding to the graph in fig. 2. Obviously, this graph is in essence given by the kaon decay constant F_K as calculated in ref. [6].

In order to keep the notation close to that of refs. [1, 2], let us first relate the $\bar{K}^0 \rightarrow \gamma\gamma$ matrix elements to the $C_{\bar{K}^0}$ coupling and after that consider the physical C_{K_2} in (1).

Observing that the sum of the amplitudes (14) and (15) should match the matrix element of the corresponding operator (1), we can write

$$\begin{aligned} 8\alpha C_{\bar{K}^0} &= \mathcal{A}_{\bar{K}^0\gamma\gamma}^{(2)} - \mathcal{A}_{\bar{K}^0\gamma\gamma}^{(1)} \\ &= -4e_D B(F_K - F_{LD}) \quad . \end{aligned} \quad (16)$$

Here the effect is expressed by the familiar decay constant F_K and by the generalization of the pion-decay triangle loop amplitude F_{LD} . This way of expressing the amplitude is very transparent and enables one to understand the effect both qualitatively and quantitatively.

The amplitudes of definite strangeness in (16) are then the building blocks needed to construct the physical K_L -decay amplitude. Restricting ourselves to the overwhelming CP-conserving amplitude from the $K_2 = \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0)$, CP=-1 eigenstate, we obtain the sought decay strength as

$$C_{K_2} = G_F \lambda_u \frac{\hat{B}_u}{6\pi} (F_K - F_{LD}) \quad . \quad (17)$$

The main discussion in the concluding section is devoted to the amplitudes C_{K_2} and $C_{K_2}^\sigma$. Whereas C_{K_2} results from the full FC effective Lagrangian \mathcal{L}_γ in (2), $C_{K_2}^\sigma$ is the amplitude resulting just from its on-shell remnant \mathcal{L}_σ in (5).

3 Results and discussion

We present our numerical results in two tables, where the potential strength V_0 and the current quark masses m_q are the only input model parameters.

The other quantities are model outputs, except \hat{B}_u , which lies within an interval of values, depending on the scale parameter μ at which the operators in (2)–(5) are brought from the original M_W scale. Therefore we used two choices which can be inferred from the discussion in the Introduction. The choice (I) of using a common value ($\hat{B}_u = 0.2$) as appropriate at higher μ -values (~ 0.7 GeV) in fact maximizes the on-shell contribution. The choice (II) of low values of μ ($= 0.3$ GeV) implies larger but also significantly different \hat{B}_u 's for different operators because of their different anomalous dimensions, namely $\hat{B}_u = 0.66$ and $\hat{B}_u = 0.32$ for \mathcal{L}_γ and \mathcal{L}_σ , respectively. This choice leads us closer to the empirical C_{K_2} amplitude, but in fact somewhat underestimates the share of the on-shell amplitude. In both tables we choose the harmonic-potential strength $\frac{4}{3}V_0 = (289 \text{ MeV})^3$, which reproduces the experimental pion and kaon masses for the standard, well-established ratio $m_s/m_{u,d}=25$ [19] (and for $m_{u,d} \approx 2$ MeV) and is also close to the V_0 values used by other authors, *e.g.* [17]. Since the quantities pertinent for estimating the relevance of off-shellness (C_{K_2} and $C_{K_2}^\sigma$, as clarified below) depend on V_0 in the same way, their *relative* importance stays the same for any other value of V_0 . Thus, for our purposes it suffices to tabulate the dependence on m_q .

For easier comparison with the more familiar, famous $\pi^0 \rightarrow \gamma\gamma$ decay, let us first present the results in table 1 in the $SU(3)$ limit. Such a symmetry limit of equal quark masses ($m_u = m_d = m_s$) exhibits (i) identical solutions of SDE and BSE for π^0 and \bar{K}^0 , (ii) equal bound-state masses, $M_\pi = M_K$ and (iii) includes the chiral limit, $m_{u,d,s} = 0$.

It is important to note that the axial-anomaly contribution to the $\pi^0, \bar{K}^0 \rightarrow \gamma\gamma$ amplitudes must be quark mass independent. Table 1 shows that the amplitudes C_π, C_{K_2} as calculated in our bound-state model, satisfy this requirement very well even relatively far from the strict chiral limit, up to more than $m_q = (\frac{4}{3}V_0)^{1/3} \times 0.1 \approx 30$ MeV. This important property also holds for any other choice of the potential parameter V_0 . This testifies that the bound-state approach considered correctly reproduces the mass-independent anomaly behaviour. Such qualitatively correct behaviour of the amplitude is for us more instructive than its absolute size.

Another important qualitative feature is that for any potential strength V_0 , the meson masses M_π and M_K calculated in the present bound-state model behave as the masses of (pseudo-)Goldstone bosons must behave because of PCAC, namely $M_{\pi,K} \propto \sqrt{m_q}$. However, both tables show that, for the parameters which fit π and K masses well, the decay constants F_π, F_K , the radiative decay amplitude C_π , and especially C_{K_2} , come out systematically too small. Such suppression seems to be a systematic feature of this and similar brands of “pairing” bound-state models, *e.g.* [15, 17, 6, 14, 18]. One general mechanism contributing to such suppression in the Bethe–Salpeter approach relatively to the naive quark-model value has been offered by Ko-

niuk and collaborators [20] on account of the dissolution of the $q\bar{q}$ Fock-space component into multi-pair Fock-space components in the strongly bound systems.

Of course, with another choice of V_0 and m_q , one can achieve much better agreement for meson decays if one accepts meson masses which are several times too high, as shown in [6] on the example of the pion. However, as already remarked above, the problems of accounting quantitatively for the hadronic structure and decays within the bound-state approach are out of scope of this paper. Our study here focuses on the qualitative issue whether there are significant off-shell effects in weak decays of hadrons, or not.

How do we establish the presence of significant off-shell effects? If they were zero or negligible, one could drop \mathcal{L}_F (4), as is often done, and instead of the complete effective $s \rightarrow d\gamma$ lagrangian (2) and (3) use just the off-diagonal magnetic-moment term \mathcal{L}_σ (5), the part which survives when quarks are put on shell. Nevertheless, as \mathcal{L}_σ vanishes in the chiral limit ($m_{u,d} = m_s = 0$), the nonvanishing amplitude in this limit clearly demonstrates the importance of off-shell effects. In the nomenclature of refs. [1, 2] followed in eqs. (16) and (17) in this paper, off-shellness manifests itself by non-cancellation of F_{LD} and F_K (stemming from fig. 1 and fig. 2, respectively).

How do things change for non-vanishing current quark masses, $m_q \neq 0$? Then $\mathcal{L}_\sigma \neq 0$, and the measure of the importance of the off-shellness is provided by the difference between $|C_{K_2}|$ and $|C_{K_2}^\sigma|$, resulting from lagrangians (2) and (5), respectively. Table 1 shows that not too far from the chiral limit the situation remains essentially the same as in this limit: $|C_{K_2}^\sigma|$ rises approximately linearly with $m_{u,d} = m_s$, but remains quite small in comparison with the complete $|C_{K_2}|$, which is constant in an excellent approximation, as emphasized above.

Since $|C_{K_2}^\sigma|$ is artificially small for the current quark masses, which are more appropriate for the pion than for the kaon, let us continue studying the off-shell effect for finite and non-degenerate quark masses in table 2. We choose the aforementioned $m_s/m_{u,d} = 25$ value, but we have in fact found the results to be quite stable for the allowed $m_s/m_{u,d} \in (20, 30)$ interval. (At present, however, there are numerical limitations in finding the solutions of bound-state equations for m_q significantly larger than 100 MeV, unless the model scale $\frac{4}{3}V_0$ is correspondingly increased over its chosen value.) Here we again find that $|C_{K_2}^\sigma|$ constitutes less than 10% of the total contribution, so that the off-shell effect is overwhelming. So, the simplification which has often been used and which keeps only \mathcal{L}_σ from the very beginning, thereby neglecting off-shell contributions, is not always justified, and certainly not for pseudoscalar mesons.

As an aside, note the onset of the significant non-anomalous part of $|C_{K_2}|$ in the third row of table 2. There, $|C_{K_2}|$ is increased by 30% over the first row,

which is still fully anomalous, being almost the same as the mass-independent $|C_{K_2}|$ of the “small-mass” table 1. This is as expected, since for the realistic kaon, with the rather heavyish strange quark, the non-anomalous part of the direct two-photon amplitude should be of the same order of magnitude as the anomalous part, in counterdistinction to π^0 .

To our knowledge, in the literature there is no direct (non-pole) χ PT term of order $\mathcal{O}(p^4)$ responsible for $K_L \rightarrow \gamma\gamma$. Thus, similarly as in ref. [2], our dominating amplitude provides an entirely new direct-decay piece of order $\mathcal{O}(p^4)$. The suppressed matrix element is of higher order, $\mathcal{O}(p^6)$, and belongs to the same class as the ambiguous reducible pole diagrams. We stick to the *direct* amplitudes, interesting in their own right – in the study of the CP violation, which actually triggered [1] the study of the off-shellness. The similar results of two very different approaches (the chiral quark model evaluation [1, 2] and the present one) give us confidence that we have achieved a qualitative understanding of the matrix elements at hand. A more quantitative result will probably be obtained if more refined BS approaches are applied. (*E. g.*, ref. [21] with the ansatz gluon propagator.) However, we are sure that our qualitative conclusions about the importance of off-shell effects will not change because of such an improved description of the hadronic structure. Namely, it seems impossible to envision such hadronic “dressing” which would enhance the \mathcal{L}_σ contribution above the off-shell contribution calculated in the present model to be an order of magnitude larger.

Let us stress that in the meantime “the anatomy” of the off-shellness has been demonstrated in detail on the example of the heavy-light B meson [22] and a comparative study of $K, B \rightarrow \gamma\gamma$ decay [23]. The off-shell part of the $B \rightarrow \gamma\gamma$ amplitude that can not be transformed away has been displayed on Fig. 2 of ref. [22]. However, in order to calculate the $B \rightarrow \gamma\gamma$ amplitude we had to invoke some bound-state model, different from the chiral quark model applicable for $K_L \rightarrow \gamma\gamma$ decay. As one might expect, the off-shell effect for $B \rightarrow 2\gamma$ turns out to be suppressed by the (*binding energy*)/ m_b , but is still numerically interesting. Therefore one would welcome further investigation of this effect in the approaches able to account for the nonperturbative QCD effects also in the heavy light systems.

An obvious candidate would be the QCD-sum-rule approach (*e.g.* ref. [24]). The necessary ingredient in such an approach is the study of the mixing of the appropriate operators under the renormalization [25]. For the purposes of such future considerations, it might therefore be worth recalling the (at first sight surprising) result of ref [22], that the operator in \mathcal{L}_F (4) has zero anomalous dimension, whereas the corresponding magnetic type operator in \mathcal{L}_σ (5) is well known (Grinstein *et. al.* in ref. [7]) to have a nonvanishing anomalous dimension. The observation that the photonic part

of the covariant derivative in \mathcal{L}_F (4) leads to an operator proportional to the current, can be a quick way to conjecture (in agreement with [26]) that the anomalous dimensions of the respective operator should be zero. This has been confirmed by an explicit calculation leading to eq. (5) in ref. [22]. This fact can facilitate the consideration of the mixing of the operator in eq. (4) under renormalization with other operators of the same dimension.

We believe that the QCD-sum-rule study of the operator in (4) might shed further light on the bound-state/off-shell effects, and deserves more investigation. In turn, the consideration of the off-shell effects seems to be unavoidable in any attempt of a precise evaluation of the kaon direct-decay amplitudes.

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Tables

$m_d = m_s$	$M_\pi = M_K$	$F_\pi = F_K$	$ C_\pi $	$ C_{K_2} _{(I)}$	$ C_{K_2}^\sigma _{(I)}$
0	0	33	1.8	1.7	0.0
2.0	140	34	1.8	1.7	0.009
28.9	479	47	2.1	1.7	0.14
$C_\pi^{exp} = 4.3 \times 10^{-4}$, $C_{K_2}^{exp} = 59 \times 10^{-12}$					

Table 1: Pion and kaon masses $M_{\pi,K}$, decay constants $F_{\pi,K}$ (in MeV), the absolute value of the $\pi^0 \rightarrow \gamma\gamma$ decay amplitude $|C_\pi|$ (in 10^{-4} MeV $^{-1}$) and the $K_2 \rightarrow \gamma\gamma$ decay amplitudes $|C_{K_2}|$ and $|C_{K_2}^\sigma|$ (in 10^{-12} MeV $^{-1}$) are given for various values of the $SU(3)$ -symmetric current quark masses ($m_s = m_{u,d}$). Throughout the table, $4/3 V_0 = (289\text{MeV})^3$ and $\hat{B}_u = 0.2$.

m_d	m_s	M_K	F_K	$ C_{K_2} _{(I)}$	$ C_{K_2}^\sigma _{(I)}$	$ C_{K_2} _{(II)}$	$ C_{K_2}^\sigma _{(II)}$
2.4	61	500	47	1.8	0.16	5.9	0.25
3.5	87	577	53	2.0	0.25	6.7	0.39
4.6	116	646	60	2.5	0.38	8.1	0.61
$C_{K_2}^{exp} = 59 \times 10^{-12}$							

Table 2: The kaon mass M_K , the decay constant F_K (in MeV), and the absolute values of the $K_2 \rightarrow \gamma\gamma$ decay amplitudes $|C_{K_2}|$ and $|C_{K_2}^\sigma|$ (in 10^{-12} MeV $^{-1}$) are given for various values of the current quark masses $m_{u,d}$ and m_s (respecting the ratio $m_s/m_{u,d} = 25$). Two cases are shown: (I) with $\hat{B}_u = 0.2$, and (II) with $\hat{B}_u = 0.66$ in the calculation of $|C_{K_2}|$, but $\hat{B}_u = 0.32$ in the calculation of $|C_{K_2}^\sigma|$. Throughout the table, $4/3 V_0 = (289\text{MeV})^3$.

Figure captions

Figure 1: Decay of the kaon bilocal due to the one-photon flavour-changing vertex $V_{1\gamma}$. Dressed quark propagators emanating out of the bound-state vertex and circling around the loop have dynamically generated mass.

Figure 2: Decay of the kaon bound state due to the two-photon flavour-changing vertex $V_{2\gamma}$. As in figure 1, the internal lines are dressed quark propagators.

HEP-PH/9501350 figures :

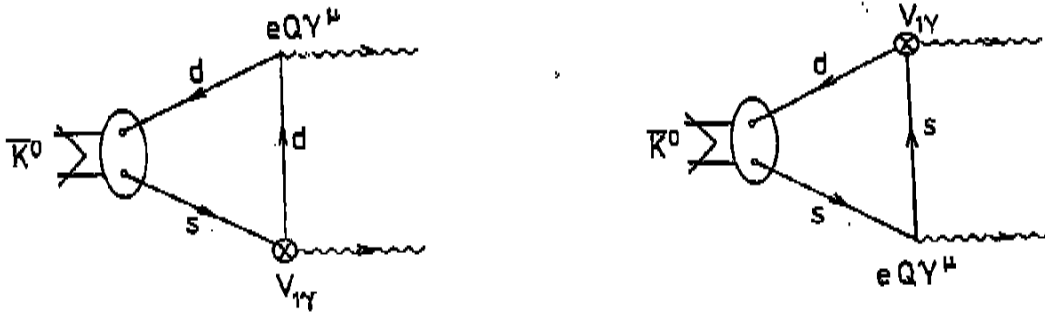


Fig. 1

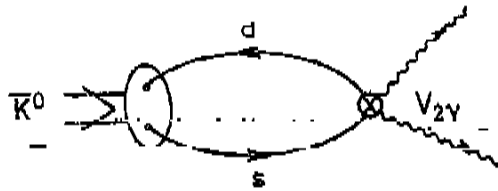


Fig. 2