# Mellin-Barnes approach to GPD modelling and fitting at NLO and beyond

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# Outline

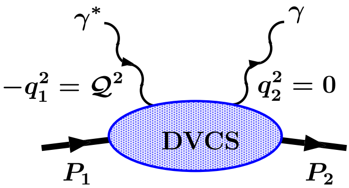
Mellin-Barnes representation of DVCS amplitude

Modelling conformal moments of GPDs

(N)NLO corrections and fitting

# Probing the proton with two photons

- Deeply virtual Compton scattering (DVCS) [Müller '92, et al. '94]




$$P = P_1 + P_2 \,, \qquad \Delta = P_2 - P_1$$

$$q = (q_1 + q_2)/2$$

Generalized Bjorken limit:

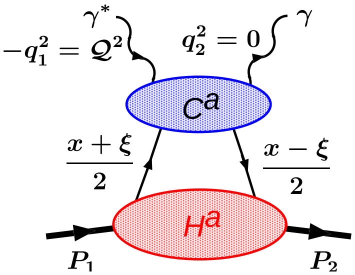
$$-q^2 \simeq \mathcal{Q}^2/2 \to \infty$$

$$\xi = \frac{-q^2}{2P \cdot q} \to \text{const}$$

- To leading twist-two accuracy cross-section can be expressed in terms of Compton form factors (CFF)

$$\mathcal{H}(\xi, \Delta^2, \mathcal{Q}^2), \mathcal{E}(\xi, \Delta^2, \mathcal{Q}^2), \tilde{\mathcal{H}}(\xi, \Delta^2, \mathcal{Q}^2), \tilde{\mathcal{E}}(\xi, \Delta^2, \mathcal{Q}^2), \ldots$$

# Factorization of DVCS $\longrightarrow$ GPDs

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- Compton form factor is a convolution:

$$^a\mathcal{H}(\xi, \Delta^2, \mathcal{Q}^2) = \int \mathrm{d}x \; C^a(x, \xi, \mathcal{Q}^2/\mu^2) \; H^a(x, \eta = \xi, \Delta^2, \mu^2)$$

$a$=NS,S($\Sigma$,$G$)

- $H^a(x, \eta, \Delta^2, \mu^2)$ — Generalized parton distribution (GPD)

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  - ...instead of considering momentum fraction dependence $H(x, \ldots)$
  - ...it is convenient to make a transform into complementary space of conformal moments $j$:

$$H_j^q(\eta, \ldots) \equiv \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} \mathrm{d}x \, \eta^j \, C_j^{3/2}(x/\eta) \, H^q(x, \eta, \ldots)$$

- They are analogous to Mellin moments in DIS: $x^j \to C_j^{3/2}(x)$
- $C_j^{3/2}(x)$ — Gegenbauer polynomials
- $H_j^q(\eta, \ldots)$ are even polynomials with maximal power $\eta^{j+1}$

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4. New possibilities for **GPD modelling**
5. Moments are equal to matrix elements of **local** operators and are thus directly accessible on the **lattice**

# Mellin-Barnes representation of CFFs (I)

- Factorization formula for CFFs ...

$$ {}^{S}\mathcal{H}(\xi, \Delta^2, \mathcal{Q}^2) = \int \mathrm{d}x \, \mathbf{C}(x, \xi, \mathcal{Q}^2/\mu^2) \, \mathbf{H}(x, \xi, \Delta^2, \mu^2) $$

- ... is in moment space written as conformal operator product expansion (COPE)

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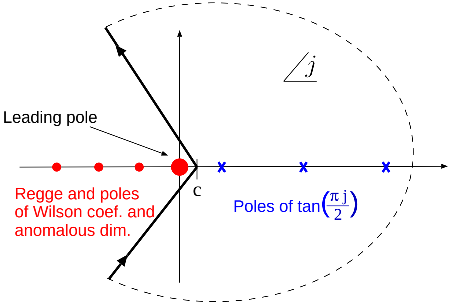
- However, this series converges only for unphysical $\xi > 1$
- To evaluate it for $\xi < 1$ we analytically continue in complex j plane and write the COPE sum as a Mellin-Barnes integral ...

# Mellin-Barnes representation of CFFs (II)

- . . . using Sommerfeld-Watson transformation and dispersion relations:

$$ {}^{\mathrm{S}}\mathcal{H}(\xi, \Delta^2, \mathcal{Q}^2) = 2\sum_{j=0}^{\infty} \xi^{-j-1} \mathbf{C}_j(\mathcal{Q}^2/\mu^2, \alpha_s(\mu))\, \mathbf{H}_j(\eta, \Delta^2, \mu^2) $$

$$ = \frac{1}{2i}\int_{c-i\infty}^{c+i\infty} dj\, \xi^{-j-1}\left[i + \tan\left(\frac{\pi j}{2}\right)\right] \mathbf{C}_j(\mathcal{Q}^2/\mu^2, \alpha_s(\mu))\, \mathbf{H}_j(\xi, \Delta^2, \mu^2) $$



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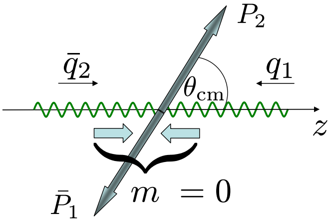


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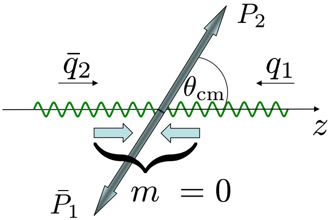


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- ...and dependence on $\theta_{\rm cm}$ in $t$-channel is given by SO(3) partial wave decomposition of $\gamma^*\gamma$ scattering

$$\mathcal{H}(\eta, \ldots) = \mathcal{H}^{(t)}(\cos\theta_{\rm cm} = -\frac{1}{\eta}, \ldots) = \sum_J (2J+1) f_J(\ldots) d^J_{0,\nu}(\cos\theta)$$

- $d^J_{0,\nu}$ — Wigner SO(3) functions (Legendre, Gegenbauer,...)
  $\nu = 0, \pm 1$ — depending on hadron helicities

# Modelling conformal moments of GPDs (II)

- OPE expansion of both $\mathcal{H}$ and $\mathcal{H}^{(t)}$, as well as trivial crossing properies of Wilson coefficients $C_j$, leads to

$$H_j(\eta, t) = \eta^{j+1}\, H_j^{(t)}(\cos\theta = -\frac{1}{\eta}, s^{(t)} = t)$$

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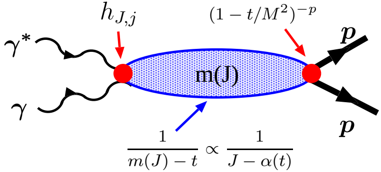


$$H_j(\eta, t) = \sum_J^{j+1} h_{J,j} \frac{1}{J-\alpha(t)} \frac{1}{\left(1-\frac{t}{M^2(J)}\right)^p}\, \eta^{j+1-J} d^J_{0,\nu}$$

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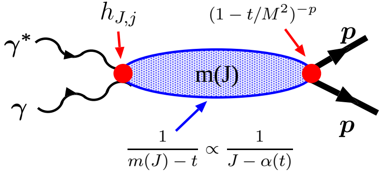


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- Similar to "dual" parametrization [Polyakov, Shuvaev '02]

# Choice of GPD Ansatz

- Taking just a leading partial wave $J = j + 1$ (good enough for HERA kinematics) gives ansatz:

$$\mathbf{H}_j(\xi, \Delta^2, \mu_0^2) = \begin{pmatrix} N'_\Sigma \, F_\Sigma(\Delta^2) \, \mathrm{B}\big(1 + j - \alpha_\Sigma(0), 8\big) \\ N'_G \, F_G(\Delta^2) \, \mathrm{B}\big(1 + j - \alpha_G(0), 6\big) \end{pmatrix}$$

$$\alpha_a(\Delta^2) = \alpha_a(0) + 0.15\Delta^2 \qquad F_a(\Delta^2) = \frac{j + 1 - \alpha(0)}{j + 1 - \alpha(\Delta^2)} \left(1 - \frac{\Delta^2}{M_0^{a2}}\right)^{-p_a}$$

…corresponding in forward case ($\Delta = 0$) to PDFs of form

$$\Sigma(x) = N'_\Sigma \, x^{-\alpha_\Sigma(0)} \, (1 - x)^7 \; ; \qquad G(x) = N'_G \, x^{-\alpha_G(0)} \, (1 - x)^5$$

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- Ansatz parameters: $N_\Sigma$, $\alpha_\Sigma(0)$, $M_0^\Sigma$, $N_G$, $\alpha_G(0)$, $M_0^G$

For small $\xi$ (small $x_{Bj}$) valence quarks are less important $\Rightarrow \Sigma \approx$ sea

# We have applied this framework to:

1. investigate size of NLO and NNLO corrections to non-singlet [Müller '05] and singlet [K.K., Müller, Passek-Kumerički, and Schäfer '06] , [K.K., Müller and Passek-Kumerički '07] Compton form factor $\mathcal{H}$
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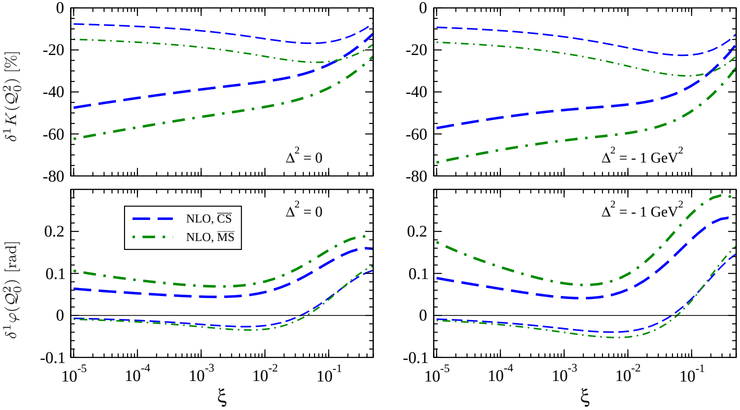
- Why study NNLO corrections?
- Gluons start to contribute at NLO order, and are important. Thus, it is necessary to go to NNLO to assess the convergence of the perturbation series.

# NLO corrections




Thick lines:
"hard" gluon
$N_G = 0.4$
$\alpha_G(0)$=$\alpha_\Sigma(0)$
$+ 0.05$

Thin lines:
"soft" gluon
$N_G = 0.3$
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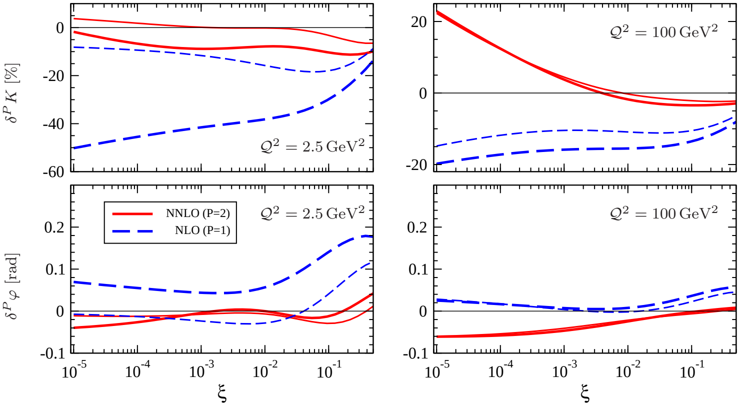
$- 0.02$

$$\delta^P K = \frac{\left|\mathcal{H}^{\mathrm{N}^P\mathrm{LO}}\right|}{\left|\mathcal{H}^{\mathrm{N}^{P-1}\mathrm{LO}}\right|} - 1\,, \qquad \delta^P \varphi = \arg\left(\frac{\mathcal{H}^{\mathrm{N}^P\mathrm{LO}}}{\mathcal{H}^{\mathrm{N}^{P-1}\mathrm{LO}}}\right)$$

# NNLO corrections

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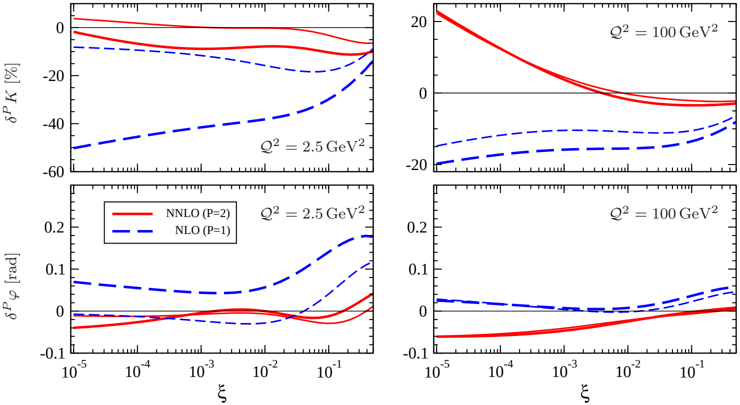


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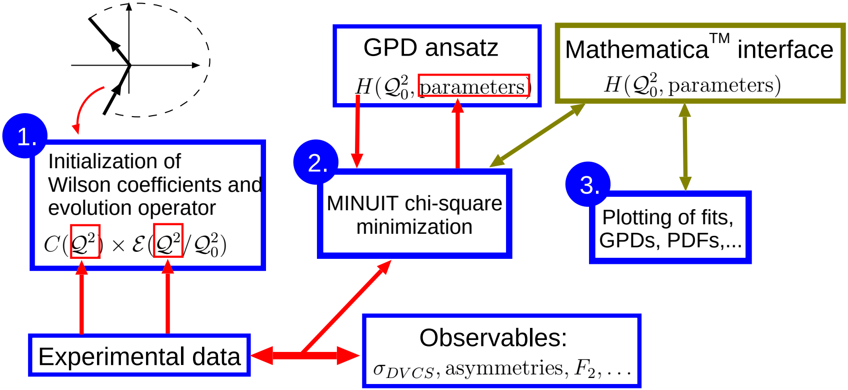
- breakdown at small-$x_{Bj}$, coming from $\alpha_s ln(1/x_{Bj})$ behaviour in evolution operator. Situation maybe worse for meson production [Diehl, Kugler, Ivanov, Szymanowski, Krasnikov]

  $\Rightarrow$ resummation needed

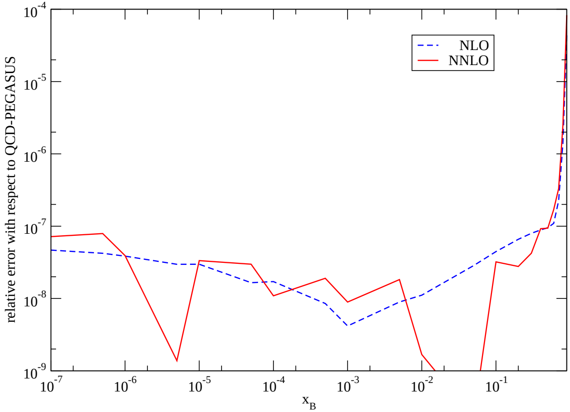
# Fast fitting routine (GeParD)



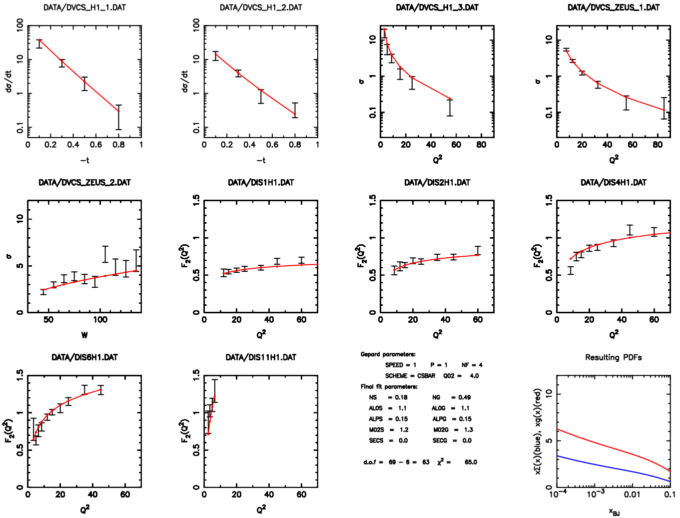

- Observable $= \int \mathrm{d}j \; C_j(\mathcal{Q}^2) \times \mathcal{E}_j(\mathcal{Q}^2, \mathcal{Q}_0^2) \times H_j(\mathcal{Q}_0^2)$

# Check

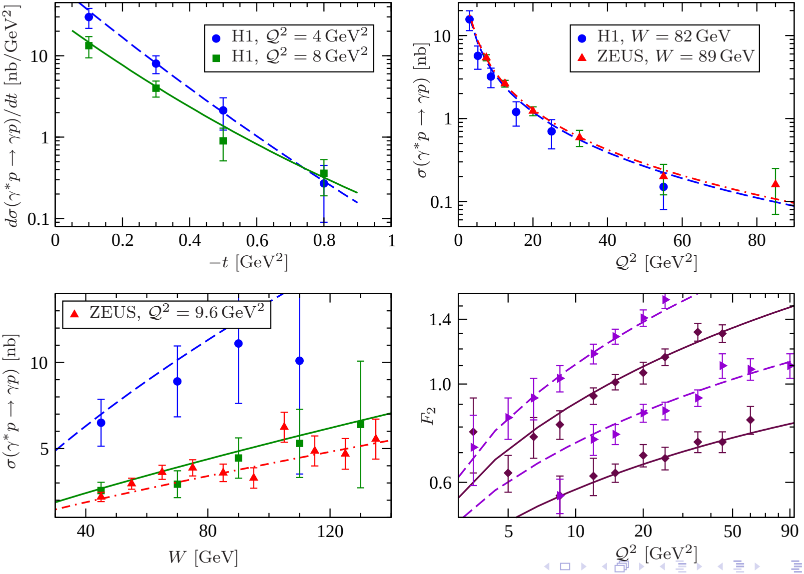
- Check by comparison to QCD-PEGASUS [Vogt '04]
- evolution of Les Houches benchmark PDFs:



# Automatically produced fits



# Example of final fit result



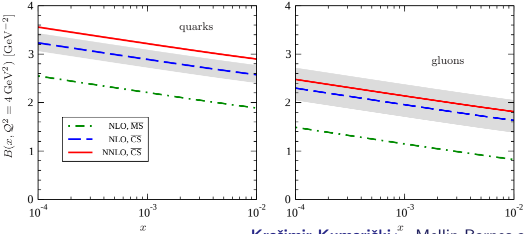
# Parton probability density

- Fourier transform of GPD for $\eta = 0$ can be interpreted as probability density depending on $x$ and transversal distance $b$ [Burkardt '00, '02]

$$H(x, \vec{b}) = \int \frac{d^2\vec{\Delta}}{(2\pi)^2} \, e^{-i\vec{b}\cdot\vec{\Delta}} H(x, \eta = 0, \Delta^2 = -\vec{\Delta}^2) \, ,$$

- Average transversal distance :

$$\langle \vec{b}^2 \rangle (x, \mathcal{Q}^2) = \frac{\int d\vec{b} \, \vec{b}^2 H(x, \vec{b}, \mathcal{Q}^2)}{\int d\vec{b} \, H(x, \vec{b}, \mathcal{Q}^2)} = 4B(x, \mathcal{Q}^2) \, ,$$

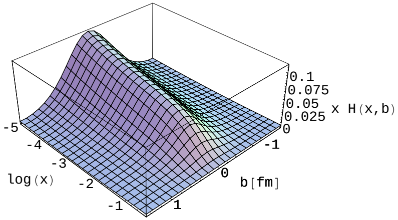


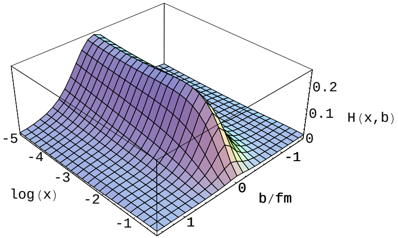

(at $\mathcal{Q}^2 = 4\,\mathrm{GeV}^2$)

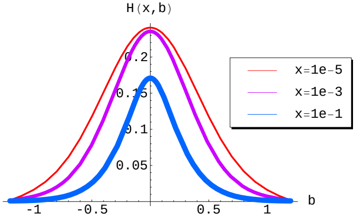
$$\langle \vec{b}^2 \rangle_{\mathrm{gluon}}(\xi = 10^{-3}) = 0.30^{+0.07}_{-0.04}\,\mathrm{fm}^2$$

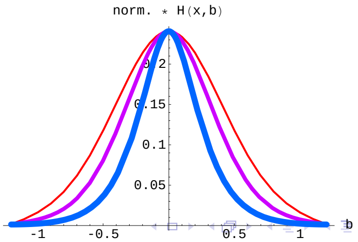
# Three-dimensional image of a proton

Quarks: Gluons:



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The End

# Conformal algebra

- Conformal group restricted to light-cone $\sim O(2,1)$

$$L_+ = -iP_+ \qquad [L_0, L_\mp] = \mp L_\mp$$

$$L_- = \frac{i}{2}K_- \qquad [L_-, L_+] = -2L_0$$

$$L_0 = \frac{i}{2}(D + M_{-+})$$

Casimir:

$$L^2 = L_0^2 - L_0 + L_- L_+$$

conf.spin $j$:

$$[L^2, \mathbb{O}_{n,n+k}] = j(j-1)\mathbb{O}_{n,k}$$

($D$ — dilatations, $K_-$ — special conformal transformation (SCT))

# Operator Product Expansion

$$J_{\rm em}(x)J_{\rm em}(0) \longrightarrow \sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\left(\frac{1}{x^2}\right)^2 x_-^{n+k+1} C_{n,k} O_{n,k}$$

$$O_{n,k} \equiv (i\partial_+)^k \, \bar\psi \, \gamma^+ (i \overleftrightarrow{D}_+)^n \psi$$

$$\overleftrightarrow{D}_+ \equiv \overrightarrow{D}_+ - \overleftarrow{D}_+$$

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$$k = 0 : \qquad O_{n,0} \equiv \qquad \bar{\psi}\, \gamma^+ (i \overleftrightarrow{D}_+)^n \psi$$

$$\overleftrightarrow{D}_+ \equiv \overrightarrow{D}_+ - \overleftarrow{D}_+$$

- $C_{n,0}$ and $\gamma_n$ of $O_{n,0}$ are well known from DIS up to NNLO.

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- (At least) to LO answer is: use conformal operators.

# Conformal operators

$$\mathbb{O}_{n,n+k} = (i\partial^+)^{n+k}\,\bar{\psi}\,\gamma^+\,C_n^{3/2}\left(\frac{\overleftrightarrow{D}^+}{\partial^+}\right)\psi$$

- they have well-defined conformal spin $j = n+2$
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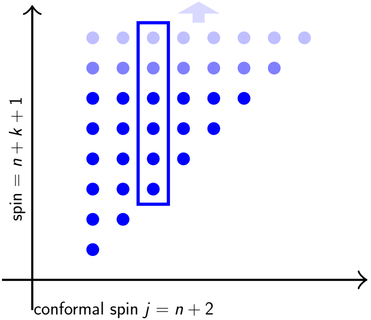
$\Rightarrow$ $\mathbb{O}_{n,n+k}$ start to mix at NLO

# Conformal Towers



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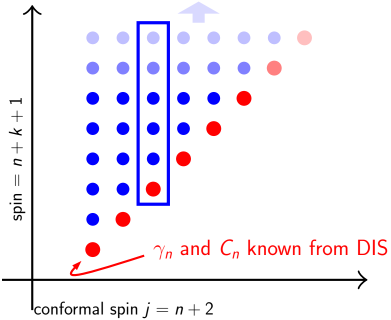

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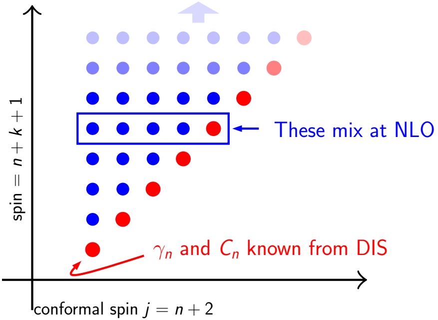

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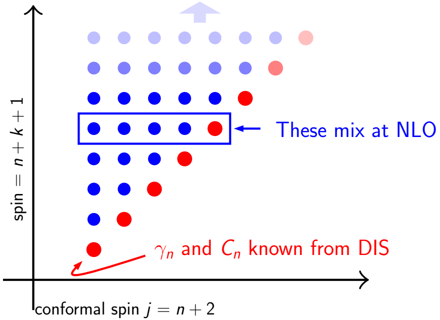


- Diagonalize in artificial $\beta = 0$ theory by changing scheme

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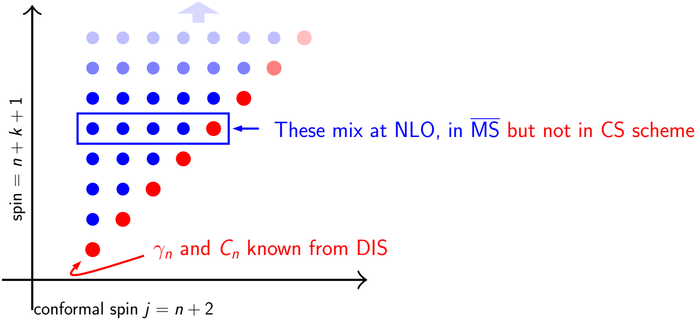


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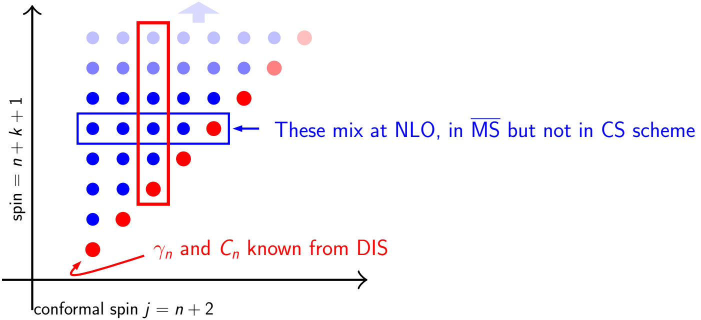


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- $C_{n,k} = (-1)^k \frac{(n+2)_k}{k!(2n+4)_k} C_{n,0} \qquad \Rightarrow$ summing complete tower

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- But how to calculate rotation matrix $B$? This is problem equivalent to calculation of $\gamma_{j,k}$.

# $\beta \neq 0$ (II)

- The $B^{(\beta=0)}$ is constrained by conformal Ward identities ...

$$B_{jk}^{(\beta=0)\mathrm{NLO}} = \delta_{jk} - \frac{\alpha_s}{2\pi}\theta(j > k)\frac{\gamma_{jk}^{\mathrm{SCT,\ LO}}}{a_{jk}}$$

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- Final result:
  $n$-loop DIS (diagonal) result + $(n-1)$-loop SCT anomaly =
  $n$-loop non-diagonal prediction

# NNLO DVCS (I)

- DVCS amplitude in terms of conformal moments:

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{}^{\mathrm{S}}\mathcal{H}(\xi, \Delta^2, \mathcal{Q}^2) = 2\sum_{j=0}^{\infty} \xi^{-j-1} \mathbf{C}_j(\mathcal{Q}^2/\mu^2, \alpha_s(\mu))\, \mathbf{H}_j(\xi = \eta, \Delta^2, \mu^2)
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H_j^q(\eta, \ldots) = \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} \mathrm{d}x\, \eta^{j-1} C_j^{3/2}(x/\eta) H^q(x, \eta, \ldots)
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- Here, Wilson coefficients $C_j$ read ...

$\Rightarrow$

# NNLO DVCS (II)

$$C_j(Q^2/\mu^2, Q^2/\mu^{*2}, \alpha_s(\mu)) = \sum_{k=j}^{\infty} C_k(1, \alpha_s(Q))\, \mathcal{P} \exp\left\{ \int_Q^{\mu} \frac{d\mu'}{\mu'} \left[ \gamma_j(\alpha_s(\mu'))\delta_{kj} + \frac{\beta}{g} \Delta_{kj}(\alpha_s(\mu'), \mu'/\mu^*) \right] \right\}$$

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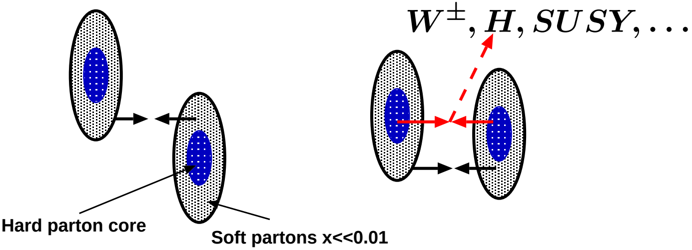
- Finally, evolution of conformal moments is given by . . . $\Rightarrow$

# NNLO DVCS (III)

$$\mu \frac{d}{d\mu} H_j(\cdots, \mu^2) = -\gamma_j(\alpha_s(\mu))\, H_j\,(\cdots, \mu^2) - \frac{\beta(\alpha_s(\mu))}{g(\mu)} \sum_{k=0}^{j-2} \eta^{j-k} \Delta_{jk}\left(\alpha_s(\mu), \frac{\mu}{\mu^*}\right) H_k(\cdots, \mu^2)$$

- $\Delta_{jk}$ — unknown correction, starts at NNLO, and can be suppressed by choice of initial condition — neglected
- $\gamma_j$ from [Vogt, Moch and Vermaseren '04]

# Relevance of GPDs for collider physics




- heavy particle production $\Rightarrow$ collision is more central
  $\Rightarrow$ larger probability for multiple parton collisions
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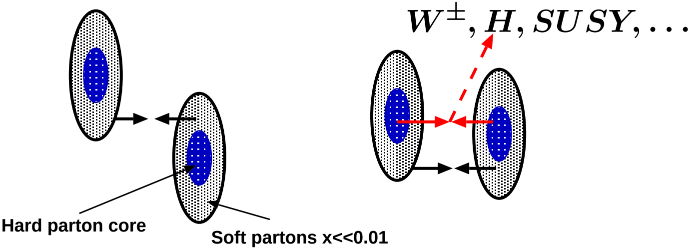


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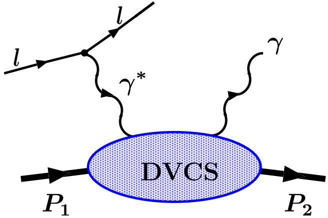
- [Frankfurt, Strikman and Weiss '04]
- No influence on total inclusive cross sections, but event structure is sensitive to transversal parton distributions.
- Relevant for LHC?

# Proton holography by electroproduction of photons

- Measured in leptoproduction of a real photon:



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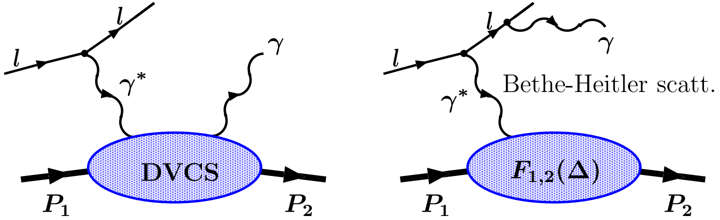


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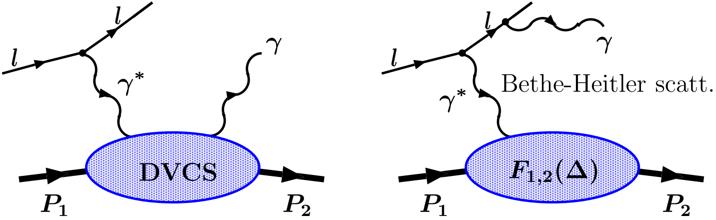


- There is a background process but it can be used to our advantage:

$$\sigma \propto |\mathcal{T}_{\rm DVCS}|^2 + |\mathcal{T}_{\rm BH}|^2 + \mathcal{T}^*_{\rm DVCS}\mathcal{T}_{\rm BH} + \mathcal{T}_{\rm DVCS}\mathcal{T}^*_{\rm BH}$$

- Using $\mathcal{T}_{\rm BH}$ as a referent "source" enables measurement of the phase of $\mathcal{T}_{\rm DVCS}$ → proton "holography" [Belitsky and Müller '02]