Generalized parton distributions, and accessing them via deeply virtual Compton scattering beyond NLO

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Collaboration with:

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Outline

Introduction to Generalized Parton Distributions (GPDs)

- Proton Structure
- Definition and Properties of GPDs

Conformal Approach to DVCS Beyond NLO

- Deeply Virtual Compton Scattering (DVCS)
- Conformal Approach
- NNLO DVCS

Results

- Choice of GPD Ansatz
- Size of Radiative Corrections
- Scale Dependence
- Fitting GPDs to Data

Summary

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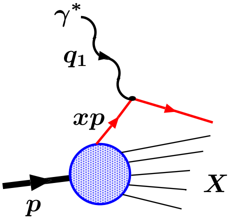
Parton Distribution Functions

- Deeply inelastic scattering, $-q_1^2 \to \infty$, $x_{BJ} \equiv \frac{-q_1^2}{2p \cdot q_1} \to \mathrm{const}$



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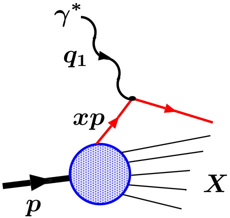

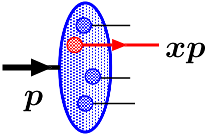

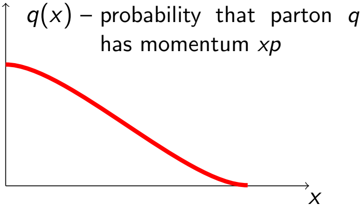

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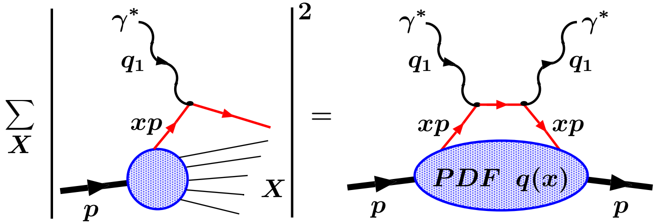

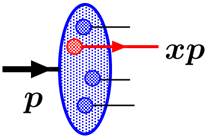

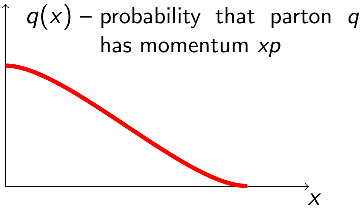

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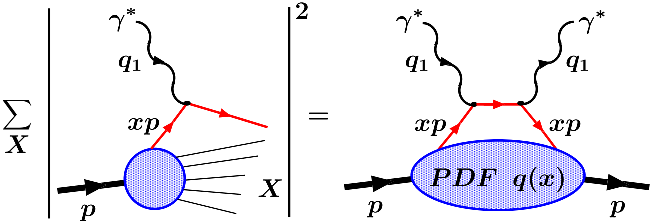
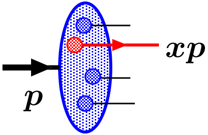
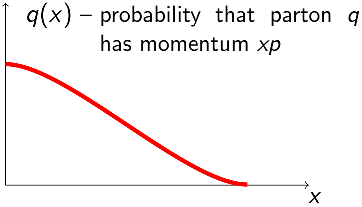
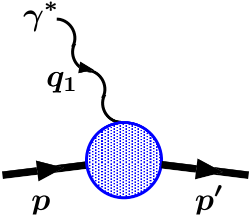






- no information on spatial distribution of partons

Electromagnetic Form Factors



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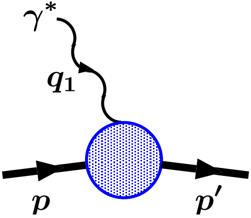

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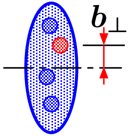

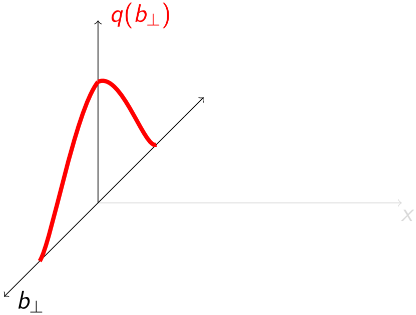

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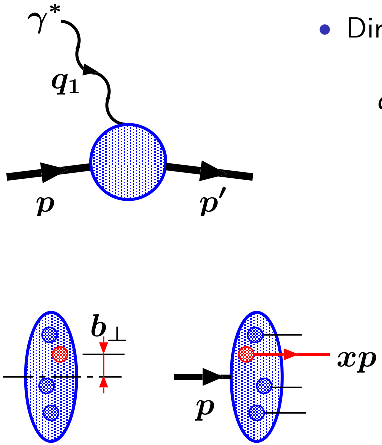

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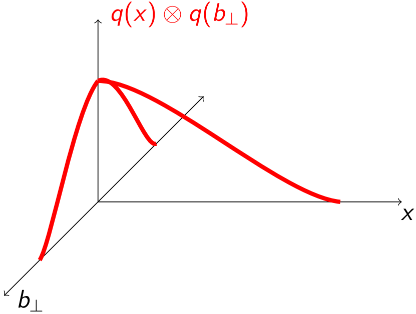

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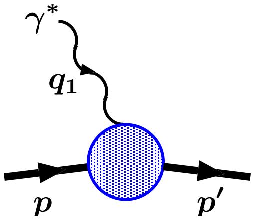

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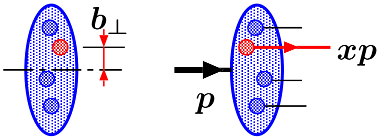

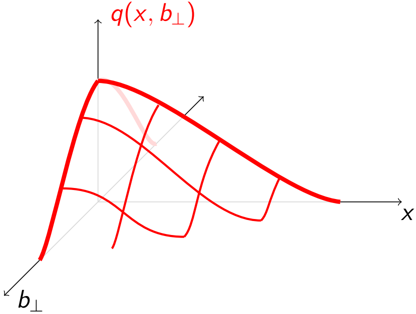

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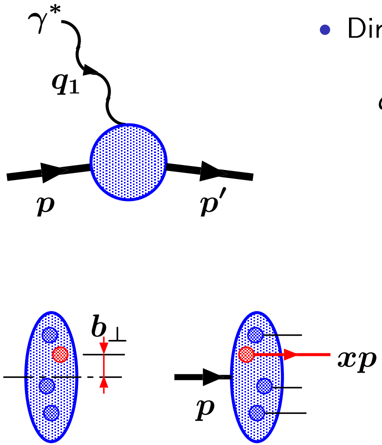

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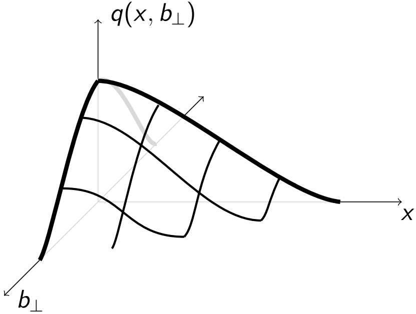

- GPD: $H^q(x, 0, t = \Delta^2) = \int \mathrm{d}b_\perp\, e^{i\Delta\cdot b_\perp} q(x, b_\perp)$

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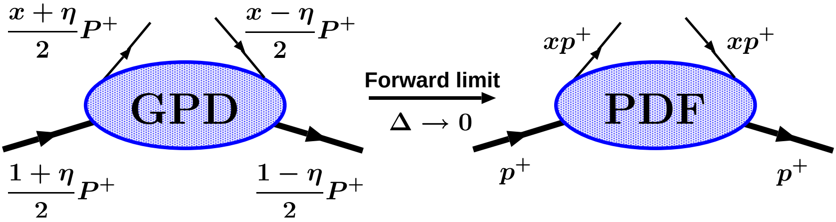

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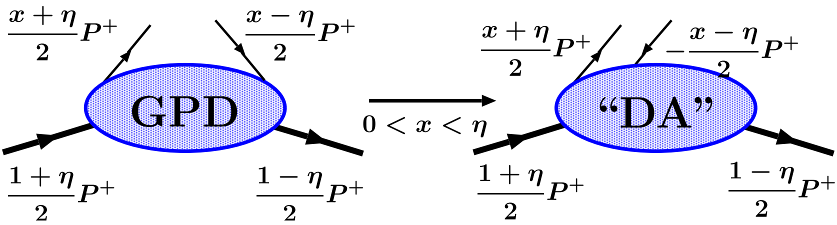

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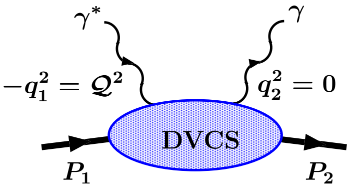
$$\frac{1}{2}\int_{-1}^{1} dx\, x \left[H^q(x,\eta,\Delta^2) + E^q(x,\eta,\Delta^2) \right] = J^q(\Delta^2)$$

[Ji '97]

Deeply Virtual Compton Scattering (DVCS)



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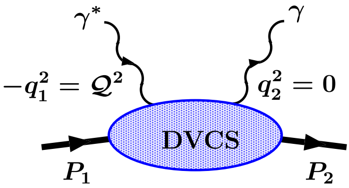

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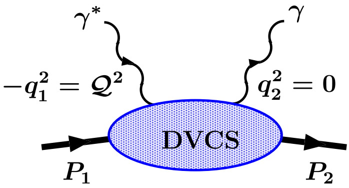

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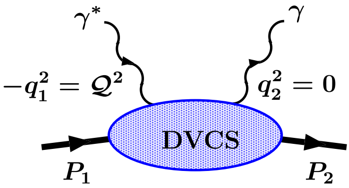

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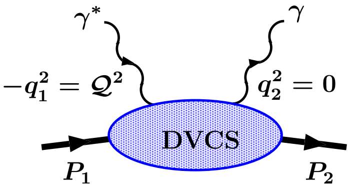

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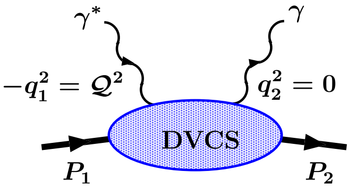

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$$J_{\rm em}(x)J_{\rm em}(0) \longrightarrow \sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\left(\frac{1}{x^2}\right)^2 x_-^{n+k+1} C_{n,k}\, O_{n,k}$$

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- ...so instead of $O_{n,k}$ choose their linear combinations which diagonalize LO evolution operator

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- conformal symmetry broken at the loop level (renormalization introduces mass scale, dimensional transmutation) $\Rightarrow$
  - running of the coupling constant
  - anomalous dimensions of operators $\gamma_{jk} = \delta_{jk}\gamma_j + \gamma_{jk}^{\mathrm{ND}}$

Conformal operators

$$\mathbb{O}_{n,n+k} = (i\partial^+)^{n+k}\, \bar{\psi}\, \gamma^+\, C_n^{3/2}\left(\frac{\overleftrightarrow{D}^+}{\partial^+}\right) \psi$$

- they have well-defined conformal spin $j = n + 2$ (see talk by D. Müller)
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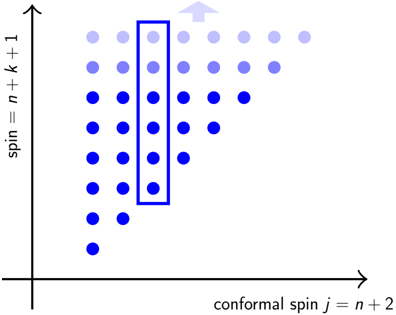
$\Rightarrow$ $\mathbb{O}_{n,n+k}$ start to mix at NLO

Conformal Towers



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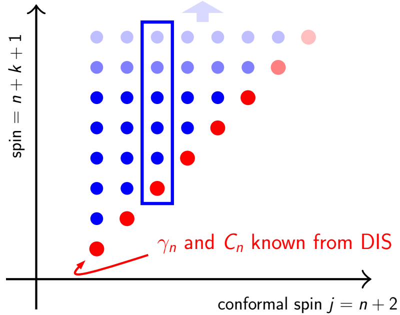

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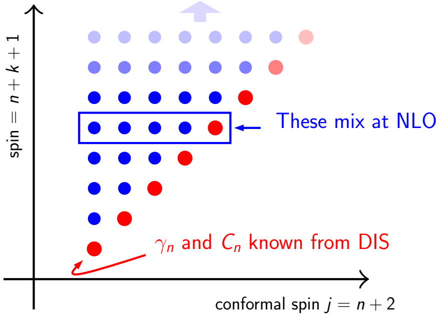

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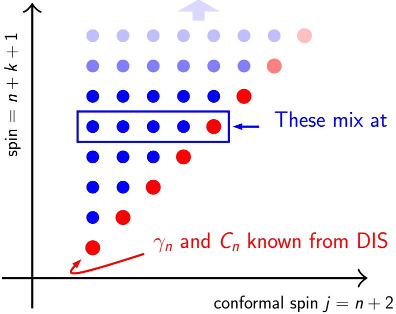

- Diagonalize in artificial $\beta = 0$ theory by changing scheme

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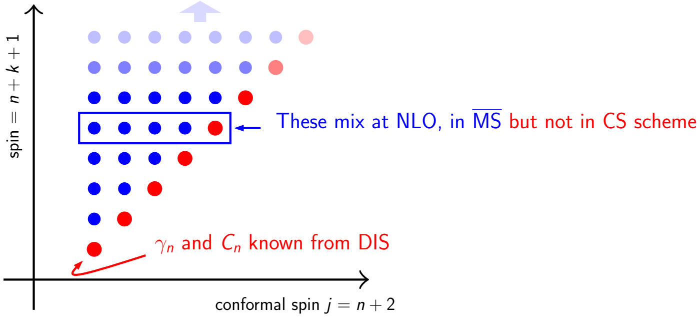

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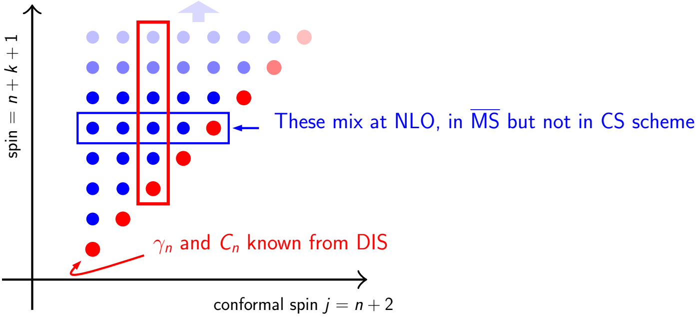

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- $C_{n,k} = (-1)^k \frac{(n+2)_k}{k!(2n+4)_k} C_{n,0}$ $\Rightarrow$ summing complete tower

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$$B_{jk}^{(\beta=0)\mathrm{NLO}} = \delta_{jk} - \frac{\alpha_s}{2\pi}\theta(j>k)\frac{\gamma_{jk}^{c(0)}}{a_{jk}}$$

(a_{jk} — known matrix)
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- Knowledge of B^{NLO} enables reconstruction of $\overline{\mathrm{MS}}$ $\gamma_{\mathrm{ND}}^{(1)}$

NNLO DVCS

- DVCS amplitude in terms of conformal moments:

$$ {}^{\mathrm{S}}\mathcal{H}(\xi, \Delta^2, \mathcal{Q}^2) = 2 \sum_{j=0}^{\infty} \xi^{-j-1} \mathbf{C}_j(\mathcal{Q}^2/\mu^2, \alpha_s(\mu)) \, \mathbf{H}_j(\xi = \eta, \Delta^2, \mu^2) $$

$$ H_j^q(\eta, \ldots) = \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} \mathrm{d}x \, \eta^{j-1} C_j^{3/2}(x/\eta) H^q(x, \eta, \ldots) $$

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$$ {}^{\mathrm{S}}\mathcal{H} = \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dj \, \xi^{-j-1} \left[i + \tan\left(\frac{\pi j}{2} \right) \right] \mathbf{C}_j \mathbf{H}_j $$

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- Here, Wilson coefficients C_j read ...

$\Rightarrow$

NNLO DVCS II

$$C_j(Q^2/\mu^2, Q^2/\mu^{*2}, \alpha_s(\mu)) = \sum_{k=j}^{\infty} C_k(1, \alpha_s(Q))\, \mathcal{P} \exp\left\{ \int_Q^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_j(\alpha_s(\mu'))\delta_{kj} + \frac{\beta}{g} \Delta_{kj}(\alpha_s(\mu'), \mu'/\mu^*) \right] \right\}$$

with

$$C_j(1, \alpha_s(Q)) = \frac{2^{1+j+\gamma_j(\alpha_s)/2}\Gamma\left(\frac{5}{2} + j + \gamma_j(\alpha_s)/2\right)}{\Gamma(3/2)\Gamma\left(3 + j + \gamma_j(\alpha_s)/2\right)}\, c_j^{\overline{\mathrm{MS}},\mathrm{DIS}}(\alpha_s)$$

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- Finally, evolution of conformal moments is given by . . . $\Rightarrow$

NNLO DVCS III

$$\mu \frac{d}{d\mu} H_j(\cdots, \mu^2) = -\gamma_j(\alpha_s(\mu))\, H_j\,(\cdots, \mu^2) - \frac{\beta(\alpha_s(\mu))}{g(\mu)} \sum_{k=0}^{j-2} \eta^{j-k} \Delta_{jk}\left(\alpha_s(\mu), \frac{\mu}{\mu^*}\right) H_k(\cdots, \mu^2)$$

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- We have used these expressions to
  1. investigate size of NNLO corrections to non-singlet [Müller '06] and singlet [K.K., Müller, Passek-Kumerički and Schäfer '06] Compton form factors
  2. perform fits to DVCS (and DIS) data and extract information about GPDs [Müller et al., in preparation]

Results on NNLO DVCS

- We use simple Regge-inspired ansatz for GPDs ...

$$\mathbf{H}_j(\xi, \Delta^2, \mathcal{Q}_0^2) = \begin{pmatrix} N'_\Sigma \, F_\Sigma(\Delta^2) \, \mathrm{B}\big(1 + j - \alpha_\Sigma(\Delta^2), 8\big) \\ N'_G \, F_G(\Delta^2) \, \mathrm{B}\big(1 + j - \alpha_G(\Delta^2), 6\big) \end{pmatrix}$$

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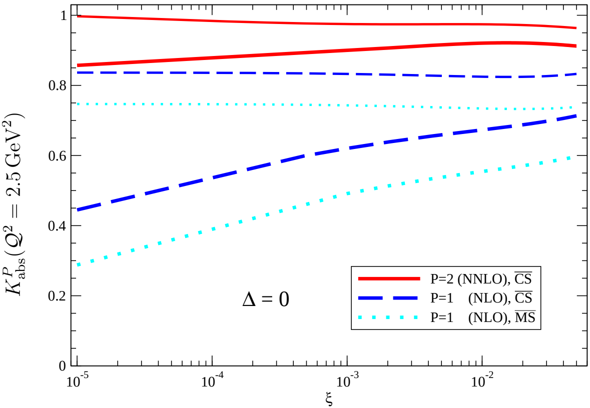
- small ξ (small x) $\Rightarrow$ neglect valence quarks contribution
- We calculate K-factors

$$K^P_{\mathrm{abs}} = \frac{\left| {}^{\mathrm{S}}\mathcal{H}^{\mathrm{N}^P\mathrm{LO}} \right|}{\left| {}^{\mathrm{S}}\mathcal{H}^{\mathrm{N}^{P-1}\mathrm{LO}} \right|} \; ; \qquad K^P_{\mathrm{arg}} = \frac{\arg\left({}^{\mathrm{S}}\mathcal{H}^{\mathrm{N}^P\mathrm{LO}} \right)}{\arg\left({}^{\mathrm{S}}\mathcal{H}^{\mathrm{N}^{P-1}\mathrm{LO}} \right)} \, .$$

Size of Radiative Corrections - Modulus

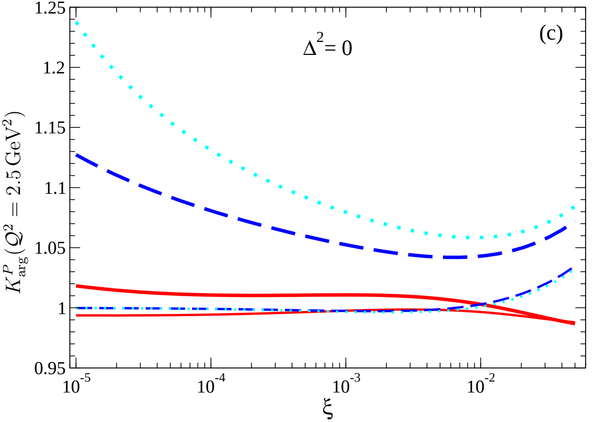


Thick lines:
"hard" gluon
$N_G = 0.4$
$\alpha_G(0)$=$\alpha_\Sigma(0)$+0.1

Thin lines:
"soft" gluon
$N_G = 0.3$
$\alpha_G(0)$=$\alpha_\Sigma(0)$

- NLO: up to 40–70% ($\overline{\rm MS}$); up to 30–55% ($\overline{\rm CS}$) ["hard"]
- NNLO: 8–14% ("hard"); 1-4% ("soft") [$\overline{\rm CS}$]

Size of Radiative Corrections - phase



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Thin lines:
"soft" gluon
$N_G = 0.3$
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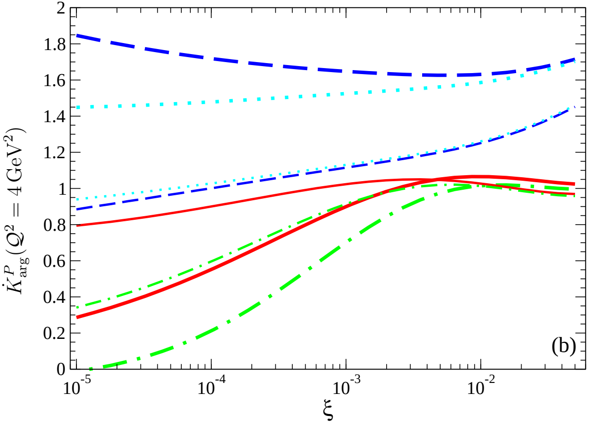
- NLO: up to 24% ($\overline{\rm MS}$); up to 13% ($\overline{\rm CS}$) ["hard"]
- NNLO and "soft" NLO — less than 5%

Scale Dependence

Same K-factors, but with $\mathcal{H} \to \mathrm{d}\mathcal{H}/\mathrm{d}\ln\mathcal{Q}^2$



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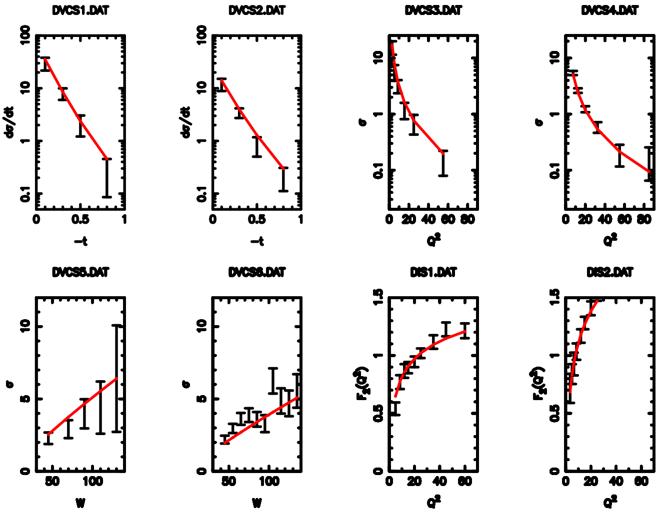
- NLO: even 100%
- NNLO: somewhat smaller, but acceptable only for larger ξ

GPD Fits



- $N_\Sigma = 0.143$, $\alpha_\Sigma(0) = 1.10$, $m_\Sigma = 1.26$, $N_G = 0.549$, $\alpha_G(0) = 0.915$, $m_G = 1.66$, $\mathcal{Q}_0^2 = 2.5\ \mathrm{GeV}^2$
- $\chi^2/(\text{number of degrees of freedom}) = 53.8/64$

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The End

Conformal algebra

- Conformal group restricted to light-cone $\sim O(2,1)$

$$L_+ = -iP_+ \qquad [L_0, L_\mp] = \mp L_\mp$$

$$L_- = \frac{i}{2} K_- \qquad [L_-, L_+] = -2L_0$$

$$L_0 = \frac{i}{2}(D + M_{-+})$$

Casimir:

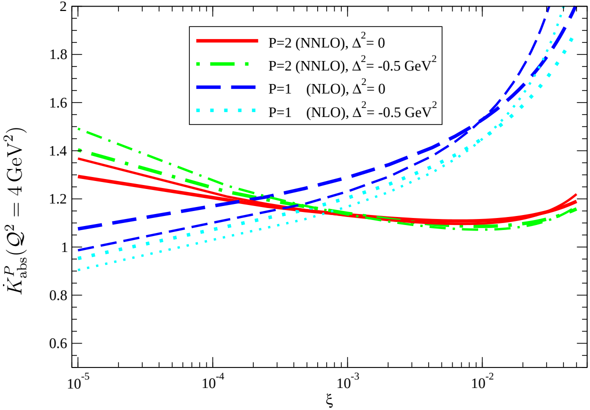
$$L^2 = L_0^2 - L_0 + L_- L_+$$

conf.spin j:

$$[L^2, \mathbb{O}_{n,n+k}] = j(j-1)\mathbb{O}_{n,k}$$

(D — dilatations, K_- — special conformal transformation (SCT))

Scale Dependence - Modulus



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- NLO: even 100%
- NNLO: smaller (largest for "hard" gluons)