

Computer One-loop Calculation of Rare Decays

Krešimir Kumerički, University of Zagreb

work in progress, collaboration with I. Picek (Zagreb) and J. O. Eeg (Oslo)

ABSTRACT

We use software packages `FeynArts`, `FormCalc` and `LoopTools` [1–3] to calculate one-loop Standard Model (SM) and Minimal Supersymmetric Standard Model (MSSM) contributions to various rare decays of heavy flavours. Here we sketch the procedure and show some representative results.

1. INTRODUCTION

Even in the Standard Model, complete one-loop calculations can be very tedious and demanding. Here is an example of one of the simpler of such calculations, that of the flavour-changing neutral current (FCNC) Z -boson vertex, done by hand, and in the approximation of small external momenta only:

[... in the poster version only ...]

Due to the proliferation of particles, the situation in popular extensions of the SM is even worse. On the other hand, since these calculations are in most parts straightforward and algorithmic in nature they are good candidates for computer implementations.

2. $t \rightarrow cZ$ EXAMPLE

There already exist several software packages that automatize some stages of these calculations, and in our work we have found combination of `FeynArts`, `FormCalc` and `LoopTools` packages very complete and useful.

`FormCalc` is originally written with $2 \rightarrow 2$ scatterings in mind.

What we have done is extending its functionality to enable calculations of $1 \rightarrow 2$ and $1 \rightarrow 3$ decays. Extension to $1 \rightarrow 2$ processes was relatively easy to do, but that to $1 \rightarrow 3$ ones involved the implementation of quite non-trivial numerical integration over the final three-particle phase space.

Let us now describe this procedure step-by-step on the example of $t \rightarrow cZ$ decay. This decay is induced by the FCNC Z -vertex of the same kind as the one mentioned above, but here the approximation of small external momenta is obviously invalid and hand calculation is even more demanding [4].

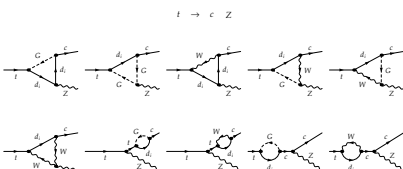
STEP 1: Generation of diagrams

STEP 2: Generation of amplitudes

Both of these steps are performed by `FeynArts` in which Feynman rules for both SM and MSSM are implemented. Instructions

```
tops = CreateTopologies[1, 1 -> 2];
ins = InsertFields[tops, F[3, {3}] ->
      {F[3, {2}], V[1]}];
amp = CreateFeynAmp[ins];
```

produce the following diagrams



Corresponding Feynman amplitudes are written to a file.

STEP 3: Dirac traceology

STEP 4: Algebraic simplification

These steps include squaring the Feynman invariant amplitude, calculation of traces of products of Dirac matrices and algebraic simplification of resulting expressions using kinematical properties of the physical process (such as $k \cdot \epsilon = 0$ etc.). It is done by `FormCalc` which in turn relegates most of the hard work to Vermaseren's FORM.

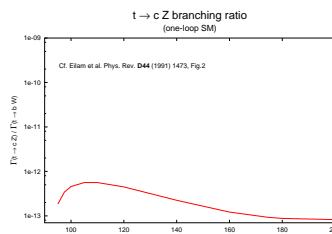
```
amploop = CalcFeynAmp[amp];
hel = HelicityME[All, amploop];
WriteSquaredME[{}, amploop, hel, {}, Abbr[],
               "for"];
```

Instructions above result in the Fortran code of amplitude squared as a function of model parameters and in terms of 2-, 3-, and 4-point one-loop functions (see below). For this particular process amplitude consists of cca. 500 lines of Fortran code in the SM case, and cca. 1200 lines in the MSSM case.

STEP 5: Numerical evaluation

Finally, the amplitude is evaluated numerically and integrated over the phase space with the help of the third part of this software package, `LoopTools`, which includes Fortran subroutines for 2-, 3-, and 4-point one-loop scalar- and tensor-coefficient functions. This then, after compilation and execution of the Fortran code, gives either differential or total decay width as the final result. There is also a possibility of automatic plotting of the dependence of the decay width on some parameter.

E.g. we can readily plot the dependence of the $t \rightarrow cZ$ decay width on the top mass:

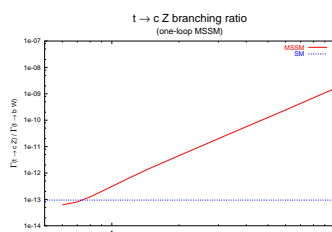


The whole procedure can be *completely* automated by writing a simple batch script, which contains the above computer instructions. Such script takes about one minute to execute on the computer with 800 MHz Athlon processor, giving the final result for the $t \rightarrow cZ$ decay width

$$\Gamma(t \rightarrow c\gamma) = 5.18 \times 10^{-13} \text{ GeV} \quad (\text{SM})$$

in agreement with our hand calculation (and in disagreement with some published results!).

By simple change of the model option in the same script we can produce MSSM result, plotted here as a function of $\tan \beta$ parameter:

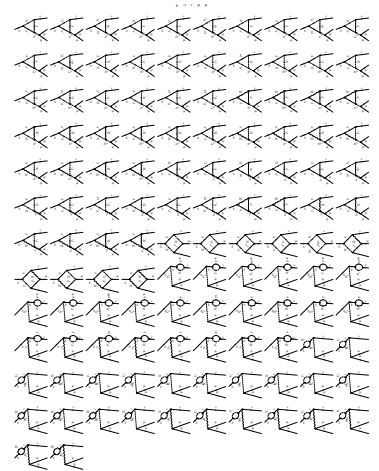


3. $b \rightarrow s\mu^- \mu^+$ IN THE MSSM

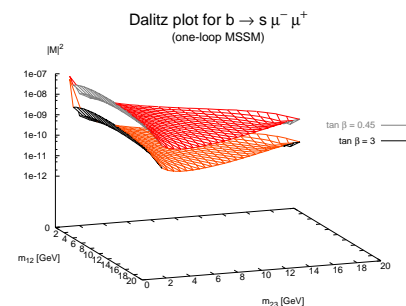
As another example, we consider the process $b \rightarrow s\mu^- \mu^+$ which induces very interesting inclusive pro-

cesses $B \rightarrow X_s \mu^- \mu^+$, as well as the exclusive ones $B \rightarrow K \mu^- \mu^+$ and $B \rightarrow K^* \mu^- \mu^+$. These can be used in the determination of $|V_{ts}|$ CKM matrix element or in the search for the new physics.

In the MSSM there are 122 Feynman diagrams contributing in the 't Hooft-Feynman gauge, and there are 13392 lines of automatically generated Fortran code. For the interested reader, all the diagrams are displayed here:



The problem is that, unlike the $t \rightarrow cZ$ decay, these decays have large non-perturbative contributions. The situation is most severe in the region of large invariant dilepton mass (m_{23} on the plot below) due to the intermediate $\bar{c}c$ states. Because of this, it doesn't make much sense to blindly integrate perturbative amplitude over the whole phase space. Thus, here we give only the Dalitz plots, for two different values of $\tan \beta$, out of which some useful information can in principle be extracted.



With some modifications these methods can be used for calculation of other quantities, such as forward-backward asymmetries or high-energy matching conditions in operator-product expansion calculations. It might even be possible to perform some specific *two-loop* calculations similar to that of the short-distance dispersive contribution to rare $K_L \rightarrow \mu^- \mu^+$ decay [5].

REFERENCES

- [1] J. Küblbeck, M. Böhm and A. Denner, *Comp. Phys. Comm.* **60** (1990) 165, hep-ph/0012260, <http://www.feynarts.de/>.
- [2] R. Mertig, M. Böhm and A. Denner, *Comp. Phys. Comm.* **64** (1991) 345.
- [3] T. Hahn and M. Perez-Victoria, *Comp. Phys. Comm.* **118** (1999) 153, hep-ph/9807565.
- [4] G. Eilam, J. L. Hewett and A. Soni, *Phys. Rev.* **D44** (1991) 1473, hep-ph/9807565.
- [5] J. O. Eeg, K. Kumerički and I. Picek, *Eur. Phys. J.* **C1** (1998) 531-537, hep-ph/9605337

contact: kkumer@phy.hr