Newtonov zakon gravitacije

- Sila između dva tijela: $\vec{F} = \frac{-GMm}{r^2}\hat{r}$
- Sunce miruje u ishodištu
- Radijvektor planeta $\vec{r}$
- Jednadžba gibanja $m\ddot{\vec{r}} = \frac{-GMm}{r^2}\hat{r}$
- Centralna sila → očuvan moment impulsa

$$\vec{L} = m\,\vec{r}\times\dot{\vec{r}}$$

$$\dot{\vec{L}} = m(\dot{\vec{r}}\times\dot{\vec{r}} + \vec{r}\times\ddot{\vec{r}}) = \frac{-GMm}{r^2}\,\vec{r}\times\hat{r} = 0$$

- Gibanje u ravnini okomitoj na $\vec{L}$ $\quad \vec{r}\,\vec{L} = 0$

Orbite

- Zakoni očuvanja energije i momenta impulsa:

$$\frac{1}{2}m\dot{r}^2+\frac{1}{2}mr^2\dot{\phi}^2-\frac{GMm}{r}=E$$

$$mr^2\dot{\phi}=L$$

- Jednadžba putanje:

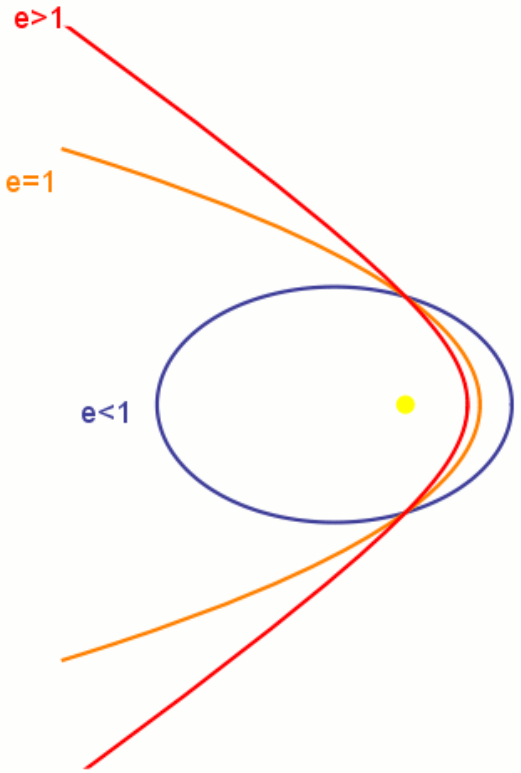
$$\frac{1}{2}\left(\frac{dr}{d\phi}\right)^2+\frac{1}{2}r^2-\frac{GM}{L^2/m^2}r^3=\frac{E/m}{L^2/m^2}r^4$$

- Rješenje:

$$r=\frac{p}{1+\epsilon\cos(\phi-\phi_0)}$$

$$p=\frac{(L/m)^2}{GM}$$

$$\epsilon=\sqrt{1+\frac{2E(L/m)^2}{m(GM)^2}}$$



Keplerovi zakoni

- Za E<0 jednadžba orbite predstavlja elipsu sa Suncem u žarištu i poluosima:

$$a = \frac{p}{1-\epsilon^2}$$

$$b = \frac{p}{\sqrt{1-\epsilon^2}}$$

- Iz očuvanja momenta impulsa:

$$\frac{dA}{dt} = \frac{1}{2}|\vec{r}\times\dot{\vec{r}}| = \frac{L}{2m} = const.$$

- Nakon jednog perioda, radijvektor prebriše cijelu površinu:

$$T\frac{dA}{dt} = \pi a b$$

$$\Rightarrow \frac{a^3}{T^2} = \frac{GM}{4\pi^2}$$