On the Influence of the Infrared on the Chiral Transition Temperature

Martina Joergler Andreas Krassnigg



3rd Croatian - Hungarian - Austrian Workshop Sept. 2, 2008



People involved:

R. Alkofer, G. Eichmann, B.-J. Schaefer (University of Graz) D. Horvatić, D. Klabučar (University of Zagreb)

- In association with Doctoral College Graz 'Hadrons in Vacuum, Nuclei and Stars'
- Supported by

Austrian Science Fund

Österreichische Forschungsgemeinschaft



Der Wissenschaftsfonds.





Outline

Introduction QCD Gap Equation

Zero Temperature

(Some) Interaction Models Quark Propagator and Meson Masses

Finite Temperature

Recipe for Finite Temperature Calculations Interaction Models at Finite T

Results

 $\text{Limit } T \to 0$

Chiral Phase Transition

Comparison of Transition Temperatures

Outlook: Critical Exponents



Introduction		

 Use Dyson-Schwinger equations (DSE) and Bethe-Salpeter equation (BSE) to calculate physical observables from QCD



Introduction		

- Use Dyson-Schwinger equations (DSE) and Bethe-Salpeter equation (BSE) to calculate physical observables from QCD
- DSE: Obtain full Green's functions



Introduction		

- Use Dyson-Schwinger equations (DSE) and Bethe-Salpeter equation (BSE) to calculate physical observables from QCD
- DSE: Obtain full Green's functions
- BSE: Description of bound states (e.g. meson masses)



Introduction		

- Use Dyson-Schwinger equations (DSE) and Bethe-Salpeter equation (BSE) to calculate physical observables from QCD
- DSE: Obtain full Green's functions
- BSE: Description of bound states (e.g. meson masses)
- Focus here: DSE for quark propagator at zero and finite temperature (T)



Introduction		
•0		





Introduction		
•0		

Structure:



 Full gluon propagator and quark-gluon vertex are (until now) unknown

In principle: obtain them from their DSE



Introduction		
•0		

Structure:



 Full gluon propagator and quark-gluon vertex are (until now) unknown

In principle: obtain them from their DSE

▶ For numerical study (e.g. mesons): Use truncation



Introduction		
00		

Work in Rainbow-truncation:





Introduction		
00		

Work in Rainbow-truncation:



► Full vertex times full gluon propagator \rightarrow bare vertex times bare propagator times some function $\gamma_{\mu} G_{\mu\nu}(p_{\alpha}) D(p^2)$ [Landau gauge]



Introduction		
00		

Work in Rainbow-truncation:



- ► Full vertex times full gluon propagator \rightarrow bare vertex times bare propagator times some function $\gamma_{\mu} G_{\mu\nu}(p_{\alpha}) D(p^2)$ [Landau gauge]
- Model for interaction: Choice of $\mathcal{D}(p^2)$



Zero Temperature		
• 0000 000		

$$\blacktriangleright \mathcal{D} = \frac{D}{2} (2\pi)^4 \delta^4(p_\mu)$$



Zero Temperature		
• 0000 000		

- $\blacktriangleright \mathcal{D} = \frac{D}{2} (2\pi)^4 \delta^4(p_\mu)$
- Leads to algebraic equations



- $\blacktriangleright \mathcal{D} = \frac{D}{2} (2\pi)^4 \delta^4(p_\mu)$
- Leads to algebraic equations
- Can be solved analytically



- $\blacktriangleright \mathcal{D} = \frac{D}{2} (2\pi)^4 \delta^4(p_\mu)$
- Leads to algebraic equations
- Can be solved analytically
- Strength parameter D



- MN Model [Munczek, Nemirovsky 1983]:
 - $\triangleright \mathcal{D} = \frac{D}{2} (2\pi)^4 \delta^4(p_\mu)$
 - Leads to algebraic equations
 - Can be solved analytically
 - Strength parameter D
 - In gap equation: Quark mass as additional parameter!



Finite Temperature

Results 000000 Outlook: Critical Exponents

(Some) Interaction Models

Gaussian Model [Alkofer, Watson, Weigel 2002]:

$$\mathcal{D}(p^2) = rac{D}{2} \; rac{(2\pi)^2}{\omega^6} p^2 e^{-p^2/\omega^2}$$



Results 000000

(Some) Interaction Models

Gaussian Model [Alkofer, Watson, Weigel 2002]:

$$\mathcal{D}(p^2) = rac{D}{2} \; rac{(2\pi)^2}{\omega^6} p^2 e^{-p^2/\omega^2}$$

Strength parameter D



Finite Temperature

Results 000000 Outlook: Critical Exponents

(Some) Interaction Models

Gaussian Model [Alkofer, Watson, Weigel 2002]:

$$\mathcal{D}(p^2) = rac{D}{2} \; rac{(2\pi)^2}{\omega^6} p^2 e^{-p^2/\omega^2}$$

- Strength parameter D
- Width of Gaussian \overline{\o



Finite Temperati

Results 000000 Outlook: Critical Exponents

(Some) Interaction Models

Gaussian Model [Alkofer, Watson, Weigel 2002]:

$$\mathcal{D}(p^2) = rac{D}{2} \; rac{(2\pi)^2}{\omega^6} p^2 e^{-p^2/\omega^2}$$

- Strength parameter D
- Width of Gaussian \overline{\o
- Original definition: $\frac{D}{\omega^2}$ instead of $\frac{D}{\omega^6}$



IntroductionZero Temperature
000000Finite Temperature
0000Results
000000Outlook: Critical Exponents
000000

(Some) Interaction Models

MR - Model [Maris, Roberts 1997]:

$$\mathcal{D}(p^2) = \frac{D}{2} \left((2\pi)^4 \delta^4(p_\mu) + \frac{(2\pi)^2}{\omega^6} p^2 e^{-p^2/\omega^2} \right) + \mathcal{F}_{UV}(p^2)$$



Zero Temperature		
0000000		

MR - Model [Maris, Roberts 1997]:

$$\mathcal{D}(p^2) = \frac{D}{2} \left((2\pi)^4 \delta^4(p_\mu) + \frac{(2\pi)^2}{\omega^6} p^2 e^{-p^2/\omega^2} \right) + \mathcal{F}_{UV}(p^2)$$

• Infrared: $\delta^4(p)$ and Gaussian term

Zero Temperature		
0000000		

MR - Model [Maris, Roberts 1997]:

$$\mathcal{D}(p^2) = \frac{D}{2} \left((2\pi)^4 \delta^4(p_\mu) + \frac{(2\pi)^2}{\omega^6} p^2 e^{-p^2/\omega^2} \right) + \mathcal{F}_{UV}(p^2)$$

- Infrared: $\delta^4(p)$ and Gaussian term
- Parameters: D, ω



Zero Temperature		
0000000		

MR - Model [Maris, Roberts 1997]:

$$\mathcal{D}(p^2) = \frac{D}{2} \left((2\pi)^4 \delta^4(p_{\mu}) + \frac{(2\pi)^2}{\omega^6} p^2 e^{-p^2/\omega^2} \right) + \mathcal{F}_{UV}(p^2)$$

- Infrared: $\delta^4(p)$ and Gaussian term
- Parameters: D, ω
- F_{UV}(p²): Proportional to α(p²)/p² for large p² (preserves one-loop RG behaviour of QCD)



troduction Zero Temperature Finite Temperatur

Results

Outlook: Critical Exponents

(Some) Interaction Models

MT - Model [Maris, Tandy 1999]:

$$\mathcal{D}(p^{2}) = \frac{D}{2} \frac{(2\pi)^{2}}{\omega^{6}} p^{2} e^{-p^{2}/\omega^{2}} + \mathcal{F}_{UV}(p^{2})$$



MT - Model [Maris, Tandy 1999]:

$$\mathcal{D}(p^2) = rac{D}{2} \; rac{(2\pi)^2}{\omega^6} p^2 e^{-p^2/\omega^2} + \mathcal{F}_{UV}(p^2)$$

• MR - Model without $\delta^4(p_\mu)$ - term



Outlook: Critical Exponents

(Some) Interaction Models

MT - Model [Maris, Tandy 1999]:

$$\mathcal{D}(p^2) = rac{D}{2} \; rac{(2\pi)^2}{\omega^6} p^2 e^{-p^2/\omega^2} + \mathcal{F}_{UV}(p^2)$$

• MR - Model without
$$\delta^4(p_\mu)$$
 - term

Parameters: ω, D



Zero Temperature 0000●000		

 All models give descriptions of physical states (e.g. meson masses)





- All models give descriptions of physical states (e.g. meson masses)
- Common property: 'Strength' of the interaction in the infrared





- All models give descriptions of physical states (e.g. meson masses)
- Common property: 'Strength' of the interaction in the infrared
- ► No dependence on pointwise behaviour $(\delta^4(p_\mu)$ yes or no)





- All models give descriptions of physical states (e.g. meson masses)
- Common property: 'Strength' of the interaction in the infrared
- ▶ No dependence on pointwise behaviour $(\delta^4(p_\mu)$ yes or no)

Define:

$$I.S. = \int \frac{d^4p}{(2\pi)^4} \mathcal{D}^{IR}$$

(leave out $\mathcal{F}_{UV}(p^2)$)





- All models give descriptions of physical states (e.g. meson masses)
- Common property: 'Strength' of the interaction in the infrared
- No dependence on pointwise behaviour $(\delta^4(p_\mu)$ yes or no)

Define:

$$I.S. = \int rac{d^4p}{(2\pi)^4} \mathcal{D}^{IR}$$

(leave out $\mathcal{F}_{UV}(p^2)$)

MR - Model: D





- All models give descriptions of physical states (e.g. meson masses)
- Common property: 'Strength' of the interaction in the infrared
- No dependence on pointwise behaviour $(\delta^4(p_\mu)$ yes or no)

Define:

$$I.S. = \int \frac{d^4p}{(2\pi)^4} \mathcal{D}^{IR}$$

(leave out $\mathcal{F}_{UV}(p^2)$)

- MR Model: D
- Other models (MN, MT, Gaussian): ^D/₂



Zero Temperature ○○○○●○○		

Quark Propagator ...

Insert one of the models into the Gap Equation to get the quark propagator for any current quark mass m_q


Zero Temperature		

Quark Propagator ...

Insert one of the models into the Gap Equation to get the quark propagator for any current quark mass m_q

$$S^{-1}(p^2) = i \gamma \cdot p A(p^2) + B(p^2)$$



Zero Temperature		
00000000		

Quark Propagator ...

Insert one of the models into the Gap Equation to get the quark propagator for any current quark mass m_q

$$S^{-1}(p^2) = i \gamma \cdot p A(p^2) + B(p^2)$$

• Calculate Quark mass functions: $M(p^2) = B(p^2)/A(p^2)$





Quark Propagator ...

Insert one of the models into the Gap Equation to get the quark propagator for any current quark mass m_a

$$S^{-1}(p^2) = i \gamma \cdot p A(p^2) + B(p^2)$$

FILIE

• Calculate Quark mass functions: $M(p^2) = B(p^2)/A(p^2)$



Zero Temperature		
00000000		

To test models, calculate observables



Zero Temperature		
00000000		

- To test models, calculate observables
- Here: Insert quark propagator in the BSE and calculate masses of Mesons



- To test models, calculate observables
- Here: Insert quark propagator in the BSE and calculate masses of Mesons
- ► All models give descriptions of meson-ground states → Used to fix the parameters



- To test models, calculate observables
- Here: Insert quark propagator in the BSE and calculate masses of Mesons
- ► All models give descriptions of meson-ground states → Used to fix the parameters
- E.g., choose parameters such that $m_{\pi} = 140 \text{MeV}$



Zero Temperature ○○○○○○●		

• Example: MT - Model: ω - dependence of meson masses (while keeping $D \cdot \omega$ constant)





• Example: MT - Model: ω - dependence of meson masses (while keeping $D \cdot \omega$ constant)



[Krassnigg] Ground states are (almost) not sensitive to ω (pointwise behaviour in the IR).



Extend formalism to finite temperature



- Extend formalism to finite temperature
- Introduce Matsubara frequencies $\omega_k = (2k+1)\pi T$, $k \in \mathbb{Z}$



- Extend formalism to finite temperature
- Introduce Matsubara frequencies $\omega_k = (2k+1)\pi T$, $k~\in~\mathbb{Z}$
- and corresponding sum $\sum_{k=-\infty}^{+\infty}$ (Matsubara sum)

- Extend formalism to finite temperature
- Introduce Matsubara frequencies $\omega_k = (2k+1)\pi T$, $k~\in~\mathbb{Z}$
- and corresponding sum $\sum_{k=-\infty}^{+\infty}$ (Matsubara sum)
- Modify momenta: $p_{\mu} \rightarrow (\mathbf{p}, \omega_k)$



- Extend formalism to finite temperature
- Introduce Matsubara frequencies $\omega_k = (2k+1)\pi T$, $k~\in~\mathbb{Z}$
- and corresponding sum $\sum_{k=-\infty}^{+\infty}$ (Matsubara sum)
- Modify momenta: $p_{\mu} \rightarrow (\mathbf{p}, \omega_k)$
- \blacktriangleright \rightarrow breaks Lorentz symmetry of QCD

- Extend formalism to finite temperature
- Introduce Matsubara frequencies $\omega_k = (2k+1)\pi T$, $k~\in~\mathbb{Z}$
- and corresponding sum $\sum_{k=-\infty}^{+\infty}$ (Matsubara sum)
- Modify momenta: $p_{\mu} \rightarrow (\mathbf{p}, \omega_k)$
- \blacktriangleright \rightarrow breaks Lorentz symmetry of QCD
- Inverse Quark propagator:

$$S^{-1} = i\gamma \cdot \mathbf{p} A(\mathbf{p}, \omega_k) + i\gamma_4 \omega_k C(\mathbf{p}, \omega_k) + B(\mathbf{p}, \omega_k)$$



 To build models: Define functions that multiply the gluon propagator



- To build models: Define functions that multiply the gluon propagator
- $D_{\mu\nu}(\mathbf{p},\Omega_k) = \mathcal{D}(\mathbf{p},\Omega_k;m_g)P_{\mu\nu}^L + \mathcal{D}(\mathbf{p},\Omega_k;0)P_{\mu\nu}^T$



- To build models: Define functions that multiply the gluon propagator
- $D_{\mu\nu}(\mathbf{p},\Omega_k) = \mathcal{D}(\mathbf{p},\Omega_k;m_g)P_{\mu\nu}^L + \mathcal{D}(\mathbf{p},\Omega_k;0)P_{\mu\nu}^T$
- $\Omega_k = 2\pi kT$ Boson Matsubara frequency

- To build models: Define functions that multiply the gluon propagator
- $D_{\mu\nu}(\mathbf{p},\Omega_k) = \mathcal{D}(\mathbf{p},\Omega_k;m_g)P_{\mu\nu}^L + \mathcal{D}(\mathbf{p},\Omega_k;0)P_{\mu\nu}^T$
- $\Omega_k = 2\pi kT$ Boson Matsubara frequency
- ► $P_{\mu\nu}^T$ 3-d transverse Projector, $P_{\mu\nu}^T + P_{\mu\nu}^L = \delta_{\mu\nu} \frac{p_{\mu}p_{\nu}}{p_{\alpha}p_{\alpha}}$

- To build models: Define functions that multiply the gluon propagator
- $D_{\mu\nu}(\mathbf{p},\Omega_k) = \mathcal{D}(\mathbf{p},\Omega_k;m_g)P_{\mu\nu}^L + \mathcal{D}(\mathbf{p},\Omega_k;0)P_{\mu\nu}^T$
- $\Omega_k = 2\pi kT$ Boson Matsubara frequency
- ► $P_{\mu\nu}^T$ 3-d transverse Projector, $P_{\mu\nu}^T + P_{\mu\nu}^L = \delta_{\mu\nu} \frac{p_{\mu}p_{\nu}}{p_{\alpha}p_{\alpha}}$
- *m_g* Debye screening mass



	Finite Temperature		
	0000		

• Defined as extensions to T = 0



	Finite Temperature	
	0000	

- Defined as extensions to T = 0
- Functions: $\mathcal{D}(\mathbf{p}, \Omega_k; m_g)$



- Defined as extensions to T = 0
- Functions: $\mathcal{D}(\mathbf{p}, \Omega_k; m_g)$
- Substitutions in the formulas:

$$egin{aligned} & p_lpha p_lpha o \mathbf{p}^2 + \omega_k^2 + m_g^2 \ & \int rac{d^4 p}{(2\pi)^4} o T \sum_{k=-\infty}^{+\infty} \int rac{d^3 p}{(2\pi)^3} \end{aligned}$$



	Finite Temperature	
	0000	

Apply the recipe to all discussed models



- Apply the recipe to all discussed models
 - ► MN^[T]: [Blaschke, Roberts, Schmidt 1998]



- Apply the recipe to all discussed models
 - ▶ MN^[T]: [Blaschke, Roberts, Schmidt 1998]
 - ► MR^[T]: [Höll, Maris, Roberts 1999]



- Apply the recipe to all discussed models
 - ▶ MN^[T]: [Blaschke, Roberts, Schmidt 1998]
 - ▶ MR^[T]: [Höll, Maris, Roberts 1999]
 - ► MT^[T]



- Apply the recipe to all discussed models
 - ► MN^[T]: [Blaschke, Roberts, Schmidt 1998]
 - ▶ MR^[T]: [Höll, Maris, Roberts 1999]
 - ► MT^[T]
 - ▶ Gaussian^[T]

- Apply the recipe to all discussed models
 - ► MN^[T]: [Blaschke, Roberts, Schmidt 1998]
 - ▶ MR^[T]: [Höll, Maris, Roberts 1999]
 - ▶ MT^[T]
 - ▶ Gaussian^[T]

Parameters: Strength parameter D, 'infrared width' ω



- Apply the recipe to all discussed models
 - ► MN^[T]: [Blaschke, Roberts, Schmidt 1998]
 - ▶ MR^[T]: [Höll, Maris, Roberts 1999]
 - ▶ MT^[T]
 - ▶ Gaussian^[T]
- Parameters: Strength parameter D, 'infrared width' ω
- Taken from T = 0 studies



		Results ●00000	
Limit T	$\rightarrow 0$		

• Consistency check: Recover the T = 0 results



	Results ●00000	

Limit $T \rightarrow 0$

- Consistency check: Recover the T = 0 results
- The Lorentz symmetry has to be restored



	Results ●00000	

$$Limit \ T \to 0$$

- Consistency check: Recover the T = 0 results
- The Lorentz symmetry has to be restored
- $A(\mathbf{p}, \omega_k)$ and $C(\mathbf{p}, \omega_k)$ become identical as $T \to 0$



			Results 0●0000	
Limit T	$\rightarrow 0$			
► N	$\mathrm{TT}^{[\mathrm{T}]}$ - Model, D	$= 0.465, \ \omega = 0.4$		





	Results	
	00000	

Chiral Phase Transition

• Order parameter: $B(0, \pi T)$




Chiral Phase Transition

- Order parameter: $B(0, \pi T)$
- Example: $\mathrm{MT}^{\mathrm{[T]}}$ Model, D = 0.465, $\omega = 0.4$





Compare T_c for all discussed models:

¹[Blaschke, Roberts, Schmidt 1998]
²[Höll, Maris, Roberts 1999]
³[Maris, Tandy 1999]
⁴[Alkofer, Watson, Weigel 2002]



I Zero Temperature Finite Tem

Results ○○○●○○ Outlook: Critical Exponents

Comparison of Transition Temperatures

• Compare T_c for all discussed models:

Model	<i>I.S.</i>	T_c [MeV]	ω	D
$MN^{[T]}$	0.2809	169 ¹		0.5618
$MR^{[T]}$	0.78	120 ²	0.3	0.78
	0.78	152 ²	0.4	0.78
$MT^{[T]}$	0.62	82	0.3	1.24 ³
	0.465	94	0.4	0.93 ³
	0.372	96	0.5	0.744 ³
$Gaussian^{[T]}$	0.76835	83	0.3	1.5367
	0.57625	95	0.4	1.1525 ⁴
	0.461	97	0.5	0.922 ⁴

¹[Blaschke, Roberts, Schmidt 1998]
²[Höll, Maris, Roberts 1999]
³[Maris, Tandy 1999]
⁴[Alkofer, Watson, Weigel 2002]





There is no simple relation between T_c and integrated strength!





► T_c depends strongly on the pointwise infrared behaviour of the model.



- ► T_c depends strongly on the pointwise infrared behaviour of the model.
- The integrated strength is no model independent measure for T_c.



- ► T_c depends strongly on the pointwise infrared behaviour of the model.
- The integrated strength is no model independent measure for T_c.
- Possibility to test models!



Results 000000 Outlook: Critical Exponents

Outlook: Critical Exponents

Investigation of critical exponents





Outlook: Critical Exponents

- Investigation of critical exponents
- Models including δ-term: Mean field behaviour β = 0.5 [Höll, Maris, Roberts 1999]





Outlook: Critical Exponents

- Investigation of critical exponents
- Models including δ-term: Mean field behaviour β = 0.5 [Höll, Maris, Roberts 1999]
- Preliminary: Critical exponents for non- δ -Models, e.g. $MT^{[T]}$:







Outlook: Critical Exponents

- Investigation of critical exponents
- Models including δ-term: Mean field behaviour β = 0.5 [Höll, Maris, Roberts 1999]
- Preliminary: Critical exponents for non- δ -Models, e.g. $MT^{[T]}$:

