The light hadron spectrum from lattice QCD

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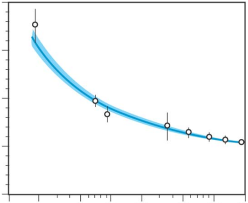
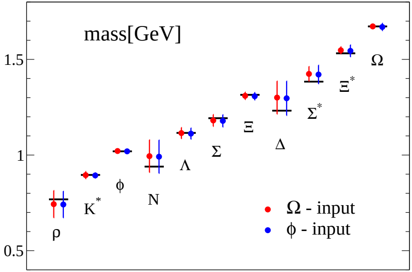
Outline

QCD AS THE SIGNAL

- Asymptotic freedom: good agreement between theory and experiment

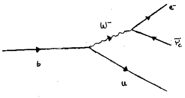




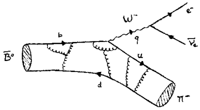
- Good evidence that QCD describes the strong interaction in the non-perturbative domain (e.g. CP-PACS '07, $N_f = 2 + 1$, $210\,\mathrm{MeV} \leq M_\pi \leq 730\,\mathrm{MeV}$, $a \simeq 0.087\,\mathrm{fm}$, $L \lesssim 2.8\,\mathrm{fm}$, $M_\pi L \simeq 2.9$)
- However, systematic errors not yet under control

QCD AS BACKGROUND

At the quark level | As seen in experiment



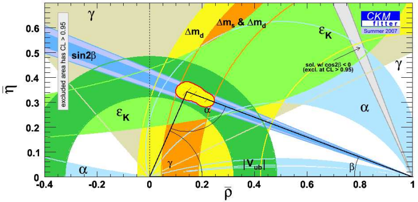
$\sim V_{ub} \longrightarrow$



$\sim V_{ub} \langle \pi^- | \bar{u}\gamma_\mu b | \bar{B}^0 \rangle$

$|V_{ub}|$ from experiment $\Rightarrow$ must evaluate non-perturbative strong interaction corrections

- Must be done in QCD to test quark-flavor mixing and CP violation and possibly reveal new physics
- Must match accuracy of (BaBar, BELLE, CDF, D0, ALEPH, DELPHI, KLOE, NA48, KTEV, LHC-b, etc.)



$\Rightarrow$ High-precision Lattice QCD

WHY THE LIGHT HADRON SPECTRUM?

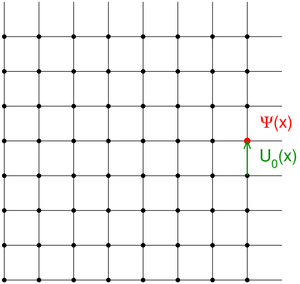
- Goal:
  - Firmly establish (or invalidate?) QCD as the theory of strong interaction in the low energy region
- Method:
  - Post-diction of light hadron spectrum
    - Octet baryons
    - Decuplet baryons
    - Vector mesons
- Challenge:
  - Minimize and control all systematics
    - 2+1 dynamical fermion flavors
    - Physical quark masses
    - Continuum
    - Infinite volume (treatment of resonant states)

LATTICE QCD

Lattice gauge theory $\longrightarrow$ mathematically sound definition of NP QCD:

- UV (and IR) cutoffs and a well defined path integral in Euclidean spacetime:

$$\langle O \rangle = \int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi\, e^{-S_G - \int \bar{\psi} D[M] \psi}\, O[U,\psi,\bar{\psi}]$$
$$= \int \mathcal{D}U\, e^{-S_G} \det(D[M])\, O[U,\psi,\bar{\psi}]_{\mathrm{Wick}}$$



- $e^{-S_G} \det(D[M]) \geq 0$ and finite # of dof's $\rightarrow$ evaluate numerically using stochastic methods

NOT A MODEL: Lattice QCD is QCD when $a \rightarrow 0$, $V \rightarrow \infty$ and stats $\rightarrow \infty$

In practice, limitations . . .

STATISTICAL AND SYSTEMATIC ERROR SOURCES

Limited computer resources $\rightarrow$ a, L and m_q are compromises and statistics finite

Associated errors:

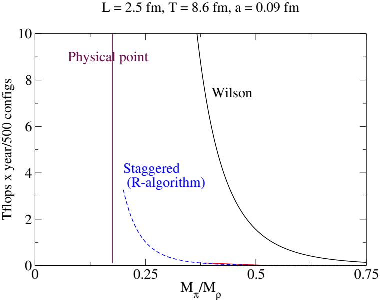
- **Statistical:** $1/\sqrt{N_{conf}}$; eliminate with $N_{conf} \rightarrow \infty$
- **Discretization:** $a\Lambda_{QCD},\ am_q,\ a|\vec{p}|$, with $a^{-1} \sim 2-4\,\mathrm{GeV}$

  $1/m_b < a < 1/m_c \Rightarrow b$ quark cannot be simulated directly
  $\rightarrow$ rely on effective theories (large m_Q expansions of QCD)

  Eliminate with continuum extrapolation $a \rightarrow 0$: need at least three a's
- **Chiral extrapolation:** $m_q \rightarrow m_u,\ m_d$
  Use χPT to give functional form $\rightarrow$ chiral logs $\sim M_\pi^2 \ln(M_\pi^2/\Lambda_\chi)$
  Requires a number of $M_\pi \lesssim 500\,\mathrm{MeV}$
- **Finite volume:** for simple quantities $\sim e^{-M_\pi L}$ and $M_\pi L \gtrsim 4$ usually safe
  Eliminate with $L \rightarrow \infty$ (χPT gives functional form)
- **Renormalization:** LQCD gives bare quantities $\rightarrow$ must renormalize: can be done in PT, best done non-perturbatively

THE BERLIN WALL CA. 2001

Unquenched calculations very demanding: # of d.o.f. $\sim \mathcal{O}(10^9)$ and large overhead for computing $\det(D[M])$ ($\sim 10^9 \times 10^9$ matrix) as $m_q \to m_{u,d}$

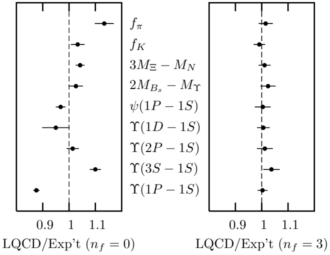


Staggered and Wilson with traditional unquenched algorithms ($\leq$ 2004)

- cost $\sim N_{\text{conf}} V^{5/4} m_q^{-2.5\to 3} a^{-7}$ (Gottlieb '02, Ukawa '02)
- Both formulations have a cost wall
- Wall appears for lighter quarks w/ staggered

$\longrightarrow$ MILC got a head start w/ staggered fermions: $N_f = 2 + 1$ simulations with $M_\pi \gtrsim 250\,\mathrm{MeV}$

- Impressive effort: many quantities studied
- Detailed study of chiral/continuum extrapolation with staggered χPT



(Davies et al '04)

Devil's advocate! $\longrightarrow$ potential problems:

- $\det(D[M])_{N_f=1} \equiv \det(D[M]_{\rm stagg})^{1/4}$ to eliminate spurious "tastes"
  $\Rightarrow$ corresponds to non-local theory (Durr, C.H. 2003-2006; Shamir, Bernard, Golterman, Sharpe, 2004-2008)
  $\Rightarrow$ more difficult to argue that $a \to 0$ is QCD
- at current a, significant lattice artefacts
  $\Rightarrow$ complicated chiral extrapolations w/ SχPT

$\Rightarrow$ it is important that approaches on firmer theoretical ground also be used

Wilson fermions strike back:

- Schwarz-preconditioned Hybrid Monte Carlo (SAP) (Lüscher '03-'04)
- HMC algorithm with multiple time-scale integration and mass preconditioning (Sexton et al '92, Hasenbusch '01, Urbach et al '06)
- Crucial insight: seperate scales (even better: also remove UV "junk")

N_f=2+1 WILSON FERMIONS À LA BMW

(Dürr et al (BMW Coll.) arXiv:0802.2706)

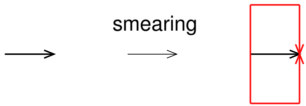
3 essential components:

- Separation of scales in HMC evolution: Mass preconditioning (w/ multiple time scales, mixed precision inverters, Omelyan integrator)
- Effective supression of irrelevant UV modes: Stout link smearing (6-step, $\rho = 0.11$)
- Action improvement: Tree level $\mathcal{O}(a)$ improved Wilson fermion action, tree level $\mathcal{O}(a^2)$ improved gauge action
  - Why not go beyond tree level? Keeping it simple (parameter fine tuning), no real improvement
  - This is a crucial advantage of our approach

Last two ingredients were shown in the quenched case to lead to excellent improvement (Capitani, Durr, C.H., 2006)

- Better chiral behavior
- renormalization constants, improvement coefficients closer to tree level

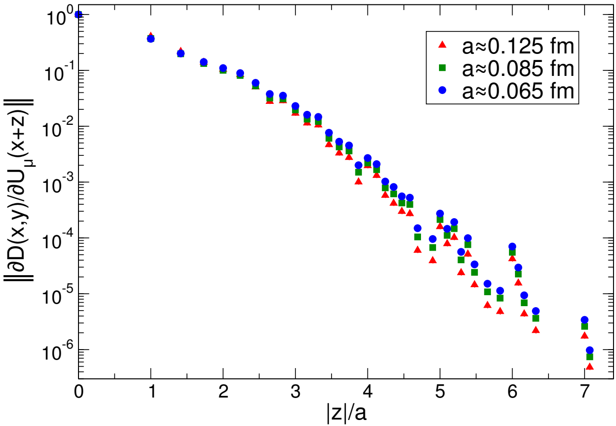
LOCALITY PROPERTIES



- locality in position space:
  $|D(x,y)| < \mathrm{const}\, e^{-\lambda|x-y|}$ with $\lambda = O(a^{-1})$ for all couplings.
  Our case: $D(x,y) = 0$ as soon as $|x-y| > 1$
  (despite 6 smearings).

- locality of gauge field coupling:
  $|\delta D(x,y)/\delta A(z)| < \mathrm{const}\, e^{-\lambda|(x+y)/2-z|}$ with $\lambda = O(a^{-1})$ for all couplings.

GAUGE FIELD COUPLING LOCALITY



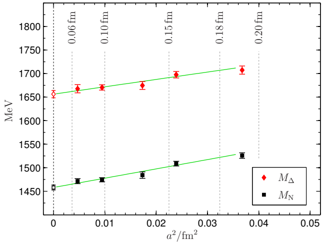
SCALING OF OUR ACTION

(Dürr et al (BMW Coll.) arXiv:0802.2706)

$\Rightarrow$ scaling study: $N_f = 3$ w/ action described above, 5 lattice spacings, $M_\pi L > 4$ fixed and

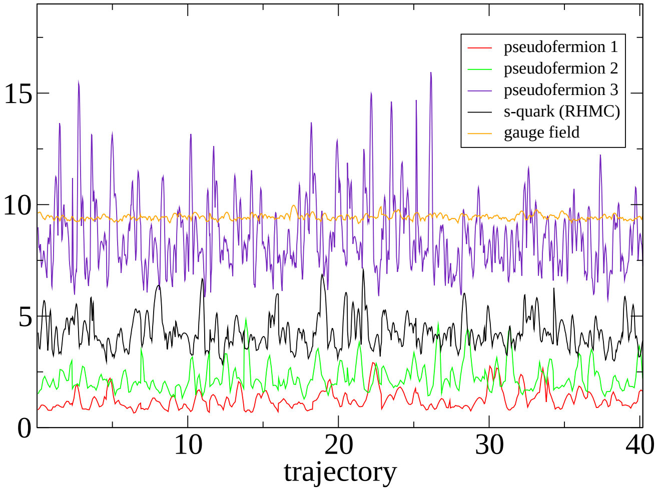
$$M_\pi/M_\rho = \sqrt{2(M_K^{ph})^2 - (M_\pi^{ph})^2}/M_\phi^{ph} \sim 0.67$$

i.e. $m_q \sim m_s$

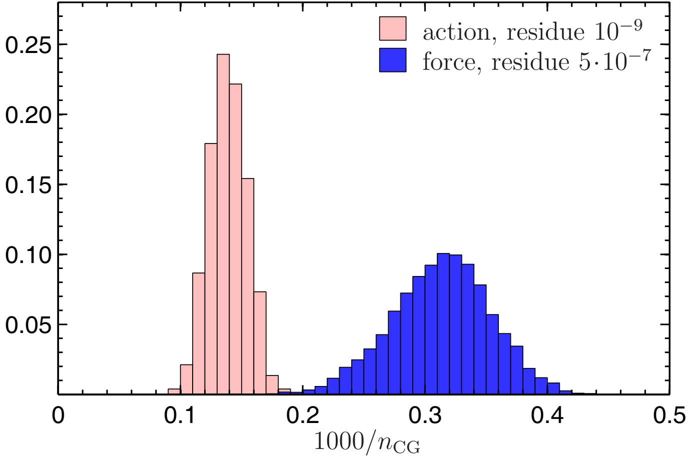


Excellent scaling up to $a \sim 0.2\mathrm{fm}$

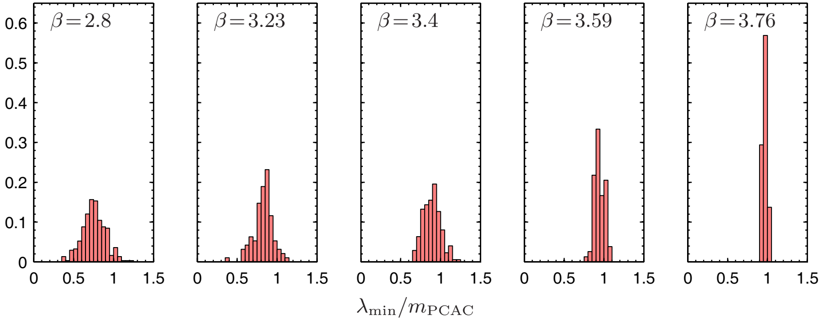
FERMIONIC FORCE HISTORY



INVERSE ITERATION COUNT DISTRIBUTION

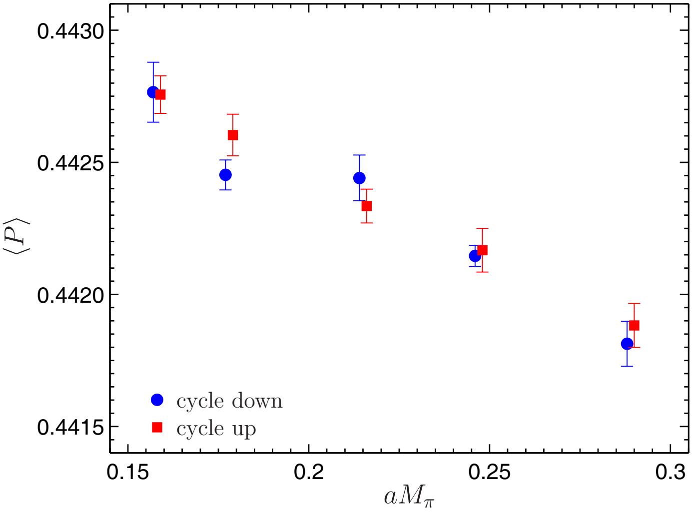


λ_{min}^{-1} DISTRIBUTION



$\mapsto$ Simulations

“THERMAL CYCLE”

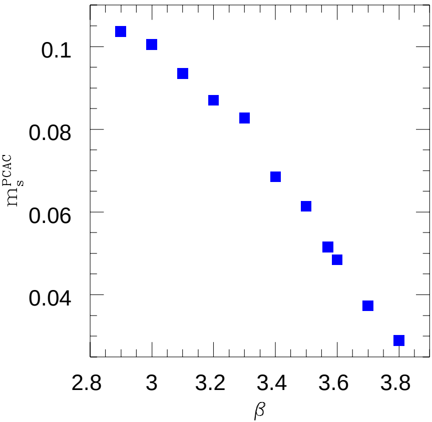


TUNING THE STRANGE QUARK MASS

- We use a $N_f = 3$ simulation to set the strange quark mass
- For each beta, search for the m_q where

$$\frac{m_{ps}}{m_v} = \frac{\sqrt{2m_K^2 - m_\pi^2}}{m_\Phi}$$

- We determined the β dependency in the range ($\beta = 2.9 \ldots 3.8$)

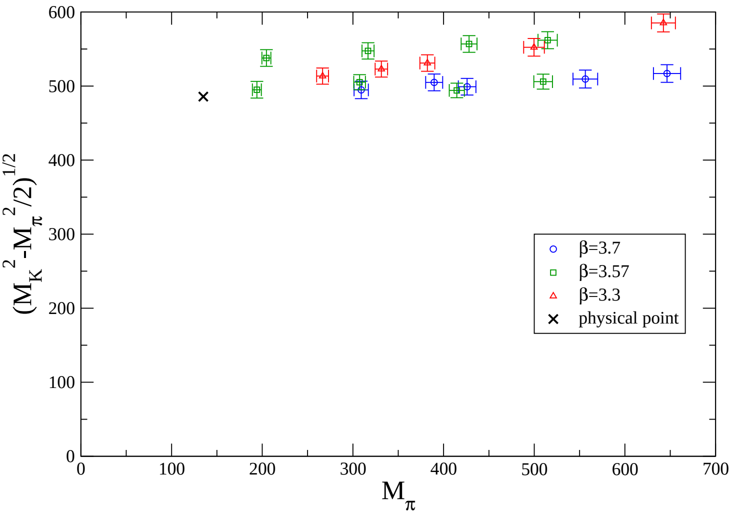


Note: this is a rough papameter tuning; we will properly interpolate to the physical strange quark mass point later!

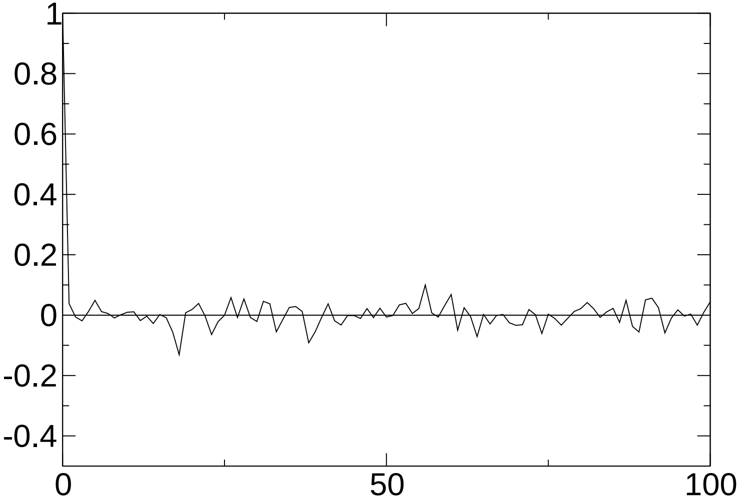
SIMULATION POINTS

β	am_{ud}	M_π [GeV]	am_s	$L^3 \times T$	# traj.
	-0.0960	.55	-0.057	$16^3 \times 32$	10000
	-0.1100	.45	-0.057	$16^3, 32^3 \times 32$	1450,1800
3.3	-0.1200	.36	-0.057	$16^3 \times 64$	4500
	-0.1233	.32	-0.057	$16^3, 24^3, 32^3 \times 64$	5000,2000,1300
	-0.1265	.26	-0.057	$24^3 \times 64$	2100
	-0.0318	.46,.48	0.0, -0.01	$24^3 \times 64$	3300
	-0.0380	.39,.40	0.0, -0.01	$24^3 \times 64$	2900
3.57	-0.0440	.31,.32	0.0, -0.007	$32^3 \times 64$	3000
	-0.0483	.19,.21	0.0, -0.007	$48^3 \times 64$	1500
	-0.007	.58	0.0	$32^3 \times 96$	1100
	-0.013	.50	0.0	$32^3 \times 96$	1450
3.7	-0.020	.40	0.0	$32^3 \times 96$	2050
	-0.022	.36	0.0	$32^3 \times 96$	1350
	-0.025	.29	0.0	$40^3 \times 96$	1450

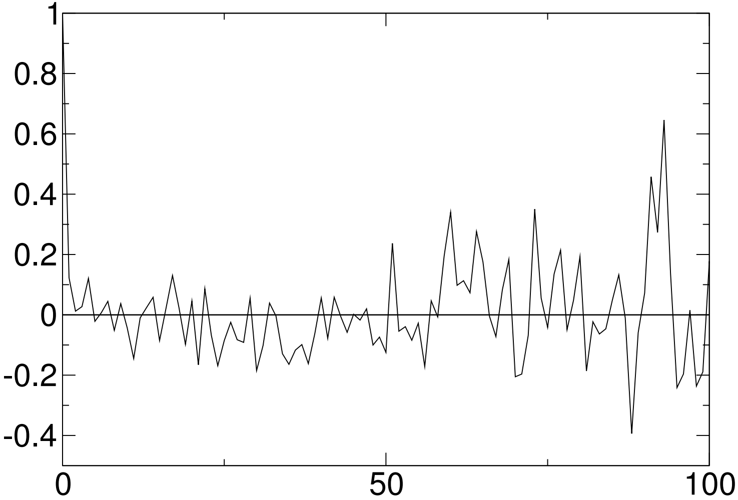
OUR “LANDSCAPE”



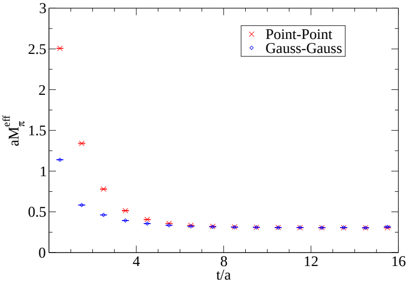
NUCLEON AUTOCORR. ($M_\pi = 550\,\text{MeV}$, $\beta = 3.3$)

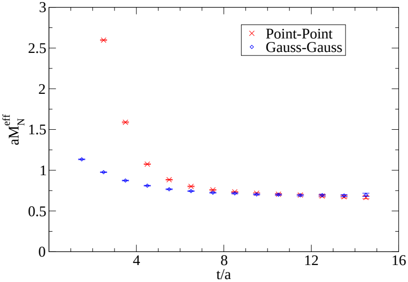


PION AUTOCORR. ($M_\pi = 190\,\mathrm{MeV}$, $\beta = 3.57$)



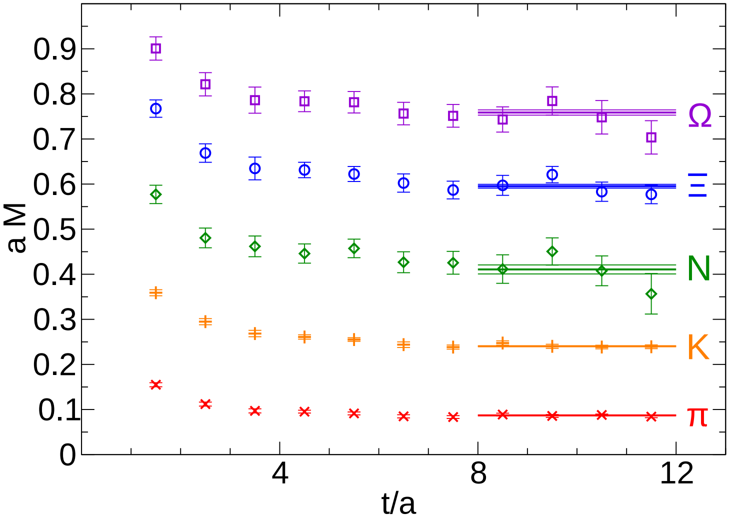
SOURCES





- Gaussian sources $r = 0.32$ fm
- Coulomb gauge
- Gauss-Gauss less contaminated by excited states

EFFECTIVE MASSES AND CORRELATED FITS



SETTING THE LATTICE SPACING VIA HADRON MASS

The particle selected should have a mass

1. that is experimentally well known
2. that is independent of light quark mass $\longrightarrow$ large strange content
3. that can be simulated with small statistical errors $\longrightarrow$ octet better suited than decuplet

All points cannot be fulfilled simultaneously, but

- Ξ: largest strange content of the octet, but still dependent on ud mass
- Ω: member of the decuplet, but largest strange content of particles included in analysis

QUARK MASS DEPENDENCE

Goal:

- Extra-/Interpolate M_X (baryon/vector meson mass) to physical point (M_π, M_K)

Method:

- Use M_Ξ or M_Ω to set the scale
- Variables to parametrize M_π^2 and M_K^2 dependence of M_X:
  - Use bare masses aM_y, $y \in \{X, \pi, K\}$ and a (bootstrapped)
  - Use dimensionless ratios $r_y := \frac{M_y}{M_{\Xi/\Omega}}$ (cancellations)

We use both procedures ➛systematic error

QUARK MASS DEPENDENCE (ctd.)

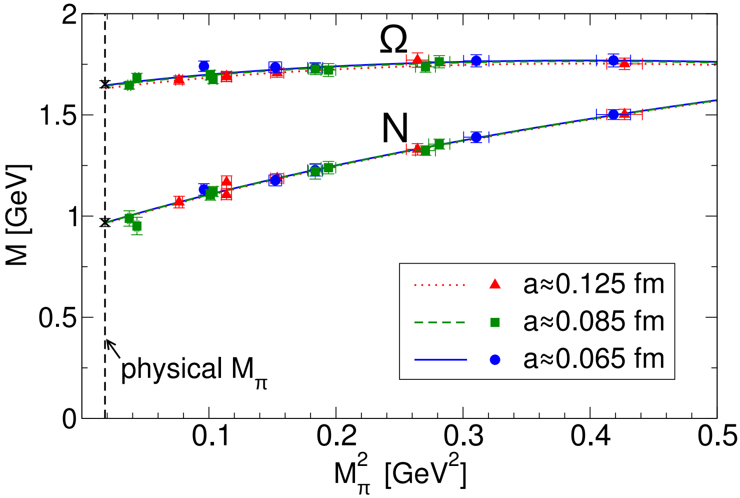
Method (ctd.):

- Parametrization: $M_X = M_x^{(0)} + \alpha M_\pi^2 + \beta M_K^2 +$ higher orders
  - Leading order sufficinet for M_K^2 dependence
  - We include higher order term in M_π^2
    - Next order χPT (around $M_\pi^2 = 0$): $\propto M_\pi^3$
    - Taylor expansion (around $M_\pi^2 \neq 0$): $\propto M_\pi^4$

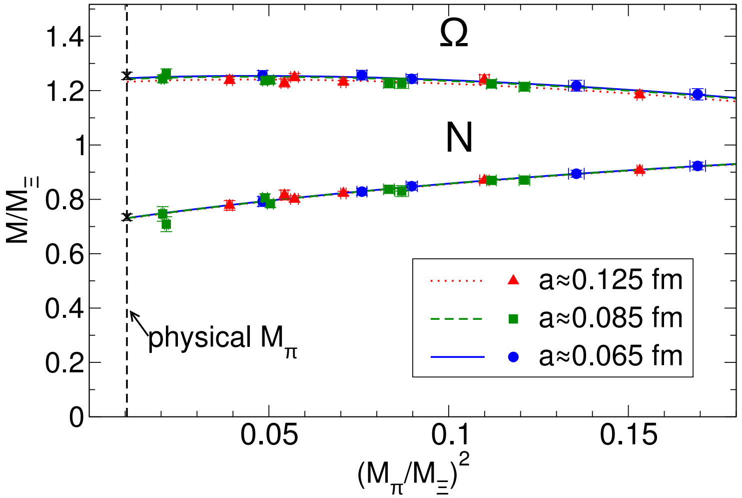
  Both procedures fine ➛systematic error
  No sensitivity to any order beyond these
- Vector mesons: higher orders not significant
- Baryons: higher orders significant
  - Restrict fit range to further estimate systematics:
    - full range, $M_\pi < 550/450$MeV

  We use all 3 ranges ➛systematic error

CHIRAL FIT


Ω
N
M [GeV]
M_π^2 [GeV^2]
physical M_π
a≈0.125 fm
a≈0.085 fm
a≈0.065 fm
0
0.5
1
1.5
2
0.1
0.2
0.3
0.4
0.5

CHIRAL FIT USING RATIOS



CONTINUUM EXTRAPOLATION

Goal:

- Eliminate discretization effects

Method:

- Formally in our action: $O(\alpha_s a)$ and $O(a^2)$
- Discretization effects are tiny
  - Not possible to distinguish between $O(a)$ and $O(a^2)$

➛include both in systematic error

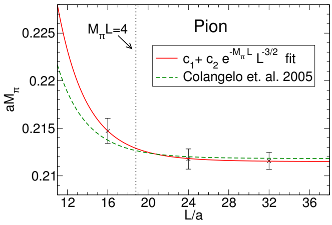
FINITE VOLUME EFFECTS FROM VIRTUAL PIONS

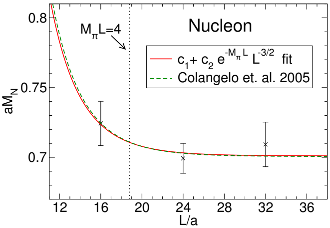
Goal:

- Eliminate virtual pion finite V effects

Method:

- Best practice: use large V
  - We use $M_\pi L \gtrsim 4$ (and one point to study finite V)
  - Effects are tiny and well described by
    $\frac{M_X(L)-M_X}{M_X} = c M_\pi^{1/2} L^{-3/2} e^{M_\pi L}$ (Colangelo et. al., 2005)





FINITE VOLUME EFFECTS IN RESONANCES

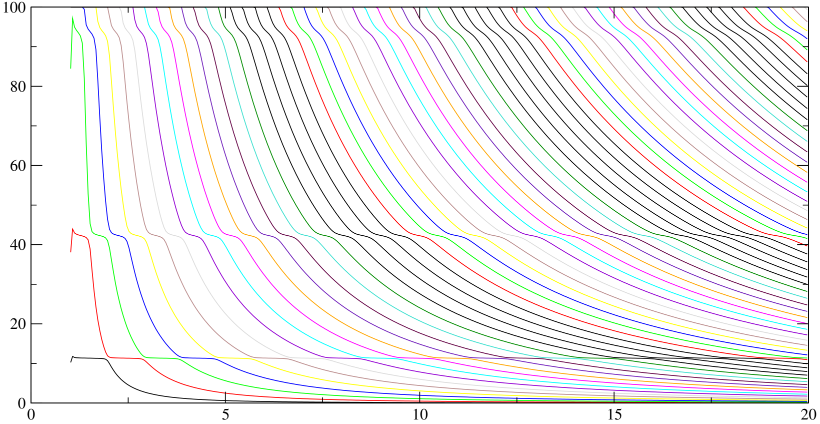
Goal:

- Eliminate spectrum distortions from resonances mixing with scattering states

Method:

- Stay in region where resonance is ground state
  - Otherwise no sensitivity to resonance mass in ground state
- Systematic treatment (Lüscher, 1985-1991)
  - Conceptually satisfactory basis to study resonances
  - Coupling as parameter (related to width)
- Fit for coupling (assumed constant, related to width)
  - No sensitivity on width (compatible within large error)
  - Small but dominant FV correction for light resonances

RESONANCES CTD.



SYSTEMATIC UNCERTAINTIES

Goal:

- Accurately estimate total systematic error

Method:

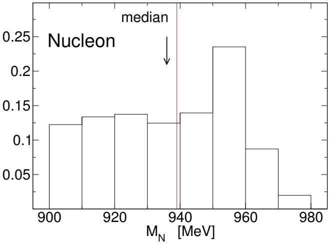
- We account for all the above mentioned effects
- When there are a number of sensible ways to proceed, we take them: Complete analysis for each of
  - 18 fit range combinations
  - ratio/nonratio fits (r_X resp. M_X)
  - $O(a)$ and $O(a^2)$ discretization terms
  - NLO χPT M_π^3 and Taylor M_π^4 chiral fit
  - 3 χ fit ranges for baryons: $M_\pi < 650/550/450$ MeV

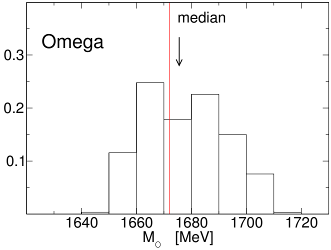
  resulting in 432 (144) predictions for each baryon (vector meson) mass with each 2000 bootstrap samples for each Ξ and Ω scale setting

SYSTEMATIC UNCERTAINTIES II

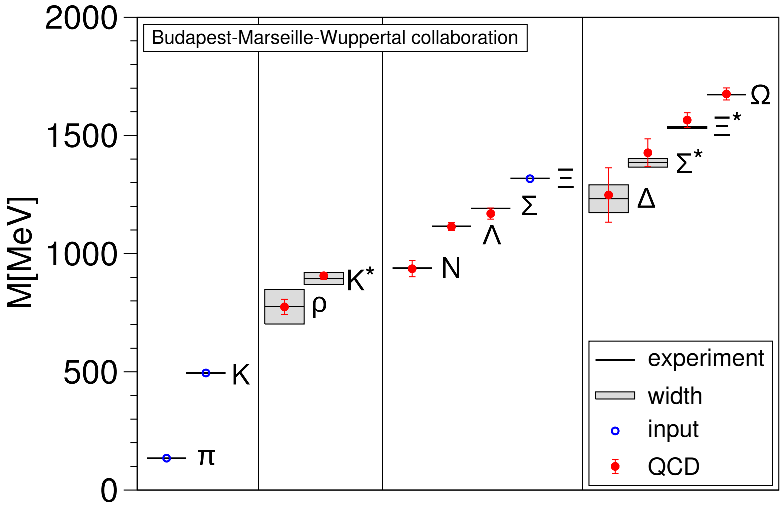
Method (ctd.):

- Weigh each of the 432 (144) central values by fit quality Q
  - Median of this distribution ➛final result
  - Central 68% ➛systematic error
- Statistical error from bootstrap of the medians





THE LIGHT HADRON SPECTRUM



Mass predictions in GeV

	Exp.	Ξ scale	Ω scale
ρ	0.775	0.775(29)(13)	0.778(30)(33)
K^*	0.894	0.906(14)(4)	0.907(15)(8)
N	0.939	0.936(25)(22)	0.953(29)(19)
Λ	1.116	1.114(15)(5)	1.103(23)(10)
Σ	1.191	1.169(18)(15)	1.157(25)(15)
Ξ	1.318		1.317(16)(13)
Δ	1.232	1.248(97)(61)	1.234(82)(81)
Σ^*	1.385	1.427(46)(35)	1.404(38)(27)
Ξ^*	1.533	1.565(26)(15)	1.561(15)(15)
Ω	1.672	1.676(20)(15)	