Pair production in a model-laser field of finite duration

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Abstract

We calculate electron-positron pair yield in a model laser field of finite duration (t). Within an acceptable range of parameters, the yield is perturbative, and the critical field is scaled down by a factor $1.47\omega_l/m_e$,

1 Introduction

Spontaneous pair (e^+e^-) creation from vacuum induced by an external static, spatially uniform electric field field, often referred as Schwinger mechanism, remains among most intriguing phenomena from both, experimental as well as theoretical point of view. As a highly nonlinear phenomenon, it requires an enormous field strength of the order $E_c = m_e^2 c^3/e\hbar \approx 1.3 \times 10^{18} V/m$. It has been assumed that such a field may be created in an antinode of the standing wave produced by a superposition of two coherent laser beams with the frequency ω_l. The field $\vec{E}(t) = (0, 0, E_m \cos(\omega_l t)), \quad \vec{B}(t) = 0$ is assumed to be "static enough", to enable the use of Schwinger method, under the condition that $\hbar\omega_l << m_e c^2$. The dynamical role of ω_l was examined in refs. [9, 10] but, under the condition that low ω_l limit schould reproduce Schwingers result, they find it to be irrelevant. Nevertheless they anticipated some of the peculiarities (e.g. 2.,3.,5.) listed below.

In this paper we study the dynamics of the system under the influence of the external field $\vec{E}(t)$, assumed to be turned -on and -off by Gaussian envelope with the average duration of laser pulse given the inverse of the width of Fourier transform of Gaussian $\bar{t} \approx \sigma_l^{-1}$. σ_l a new parameter in a game!

The time evolution of number operator is studied within the finite time out of equilibrium field theory. Within the model dynamics it is particularly simple, so that the Dyson-Schwinger equation provides us with a solution in a closed form.

Within the range of parameters defined by inequalities: $n_0 >> 1$, $\omega_l >> n_0^{1/2}\sigma_l$ ($\hbar\omega << m_e c^2$, $m_e c^2 \sigma_l^2 << \hbar\omega_l^3$), the model exibits a peculiar properties: 1. the electron yield is dominated by a single term in perturbation expansion. 2. a critical (dynamical) value of the field $E_d =$ is a few orders lower than Schwinger value $E_d = 1.4 E_c \omega_l/m_e$. 3. emission of (e^-, e^+) is limited to extremely low kinetic energies ($|p_0 - m_e| \le n_0^{1/2}\sigma_l$). In fact particles are emited in bands, each band defined with slightly higher E_d. 4. at low ω_l limit; the phase space schrinks to zero, and Schwinger result need not be reproduced. It emerges from an interplay of higher order terms, ignored in our calculation, but gaining on importance as $\omega_l \to 0$ This limit is in conflict with the condition $\omega_l >> n_0^{1/2}\sigma_l$ and, thus, clearly outside of the upper defined range of laser parameters. 5. the onset of criticality is sudden owing to the power of $^{(}E_m/E_d)^{2n_0}$

In interacting theory we study the time evolution of particle number, which at $t = 0$ has to coincide with the noninteracting one.

1.1 Matrix Propagators and the Transition to R/A Basis

The contour propagators are usually expressed by matrix propagators (indexes 1,2 describing whether the time (x_0 and y_0) corresponds to upper or lower branch of the contour. The two-point propagators are usually Wigner-transformed $(x, y) \to (X = (x+y)/2), s = (x-y)/2$, including Fourrier transform of relative coordinate $s \to p$ to (X, p). In the most of problems owing to the translational invariance propagators are independent of $\vec{X}$. In the limit $X_0 \to \infty$ they correspond to usual propagators and the label X_0 is usually ommited. We write for spin-1/2 fields

$$
\begin{aligned}
S_{11}(p,m) &= [\frac{i}{p^2 - m^2 + 2i\epsilon} - 2\pi\delta(p^2 - m^2) f(\omega_p)](\gamma_0 p_0 - \vec{\gamma}\vec{p} + m), \\
S_{12}(p,m) &= 2\pi\delta(p^2 - m^2)[\Theta(-p_0) - 2f(\omega_p)](\gamma_0 p_0 - \vec{\gamma}\vec{p} + m), \\
S_{21}(p,m) &= 2\pi\delta(p^2 - m^2)[\Theta(p_0) - 2f(\omega_p)](\gamma_0 p_0 - \vec{\gamma}\vec{p} + m), \\
S_{22}(p,m) &= S^*_{11}(p,m), \\
S_{11}(p,m) &- S_{12}(p,m) - S_{21}(p,m) + S_{22}(p,m) = 0.
\end{aligned} \tag{1}
$$

Where we have assumed equal initial distribution functions for particles and antiparticles.

Matrix propagators are further transformed to R, A, K basis:

$$
\begin{aligned}
S_R(p,m) &= -S_{11}(p,m) + S_{21}(p,m) = \frac{-i(\gamma_0 p_0 - \vec{\gamma}\vec{p} + m)}{p^2 - m^2 + 2ip_0\epsilon}, \\
S_A(p,m) &= -S_{11}(p,m) + S_{12}(p,m) = \frac{-i(\gamma_0 p_0 - \vec{\gamma}\vec{p} + m)}{p^2 - m^2 - 2ip_0\epsilon}, \\
S_K(p,m) &= S_{11}(p,m) + S_{22}(p,m) \\
&= 2\pi\delta(p^2 - m^2)(\gamma_0 p_0 - \vec{\gamma}\vec{p} + m)[1 - 2f(\omega_p)].
\end{aligned} \tag{2}
$$

Now we decompose K-propagator to it‘s retarded and advanced part [36]

$$
\begin{aligned}
S_K(p,m) &= -S_{K,R}(p,m) + S_{K,A}(p,m) \\
S_{K,R}(p,m) &= i\frac{\gamma_0\omega_p - (\vec{\gamma}\vec{p} - m)p_0/\omega_p}{(p^2 - m^2 + 2ip_0\epsilon)}(1 - 2n_f(\omega_p)),
\end{aligned}
$$

$$S_{K,A}(p,m) = -i\frac{\gamma_0\omega_p - (\vec{\gamma}\vec{p} - m)p_0/\omega_p}{(p^2 - m^2 - 2ip_0\epsilon)}(1 - 2n_f(\omega_p)). \tag{3}$$

2 Particle Number

Number of particles of momentum $\vec{p}$ found in the element of configuration space d^3xd^3p at the time t

$$< N_{\vec{p}}(t) >= \frac{d\mathcal{N}}{d^3xd^3p}(2\pi)^3, \tag{4}$$

is, by definition, obtained from the evolution of the operator a^+a which can be connected to the equal time limit of the propagator $S_{K,t}$.

$$< 1 - 2N_{+,\vec{p}}(t) >= \frac{\omega_p}{2\pi}\int dp_0 Tr[-S_{K,R,t}(p) + S_{K,A,t}(p)]. \tag{5}$$

The lowest order contribution to the particle number is identical to the initial distribution

$$< 1 - 2N^0_{\vec{p}}(t) >=< 1 - 2N_{\vec{p}}(0) >= 2[1 - 2n_f(\omega_p)], \tag{6}$$

with factor of 2 arising from the number of polarizations. Particle number is expressed through propagators $S_{K,R}$, and $S_{K,A}$ for which we know perturbation expansion.

3 Dyson-Schwinger Equation

Dyson-Schwinger equation is defined by (S is lowest order Green function, and $\mathcal{S}$ is resummed Green function):

$$[S^{-1} - i\Sigma] * \mathcal{S} = 1, \tag{7}$$

where

$$S = \begin{pmatrix} S_R & S_K \\ 0 & S_A \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \Sigma_R & \Sigma_K \\ 0 & \Sigma_A \end{pmatrix}, \quad \mathcal{S} = \begin{pmatrix} \mathcal{S}_R & S_K \\ 0 & \mathcal{S}_A \end{pmatrix} \tag{8}$$

or in components

$$\begin{aligned} \mathcal{S}_R &= S_R + iS_R * \Sigma_R * \mathcal{S}_R \\ \mathcal{S}_A &= S_A + iS_A * \Sigma_A * \mathcal{S}_A \\ \mathcal{S}_K &= S_K + i[S_R * \Sigma_K * \mathcal{S}_A + S_K * \Sigma_A * \mathcal{S}_A + S_R * \Sigma_R * \mathcal{S}_K] \end{aligned} \tag{9}$$

Formal solution is

$$\begin{aligned} \mathcal{S}_R &= [1 - iS_R * \Sigma_R]^{-1} * S_R = S_R * [1 - i\Sigma_R * S_R]^{-1} \\ \mathcal{S}_A &= [1 - iS_A * \Sigma_A]^{-1} * S_A = S_A * [1 - i\Sigma_A * S_A]^{-1} \\ \mathcal{S}_K &= i\mathcal{S}_{\mathcal{R}} * \Sigma_K * \mathcal{S}_A + \mathcal{S}_{\mathcal{R}} * S_R^{-1} * S_K * S_A^{-1} * \mathcal{S}_A] \end{aligned} \tag{10}$$

Note here that $S_R^{-1} * S_K * S_A^{-1} = h(S_R^{-1} - S_A^{-1}) \propto \epsilon$, but $\epsilon \neq 0$ until the end of calculation and the product of $\mathcal{S}_{\mathcal{R}}$ and $\mathcal{S}_A$ (if there is a common singularity of pinching type) may turn it into the δ function .

4 classical fields

Now we consider time dependent classical fields turned on at $t = 0$. In the case of classical fields the self energy is single-point function and satisfies:

$$\Sigma_R(t) = \Sigma_A(t) = \Sigma(t), \quad \Sigma_K = 0. \tag{11}$$

Its Fourier transform

$$\Sigma(\omega) = \int dt \Theta(t) e^{i\omega t} \Sigma(t),$$
$$\Sigma(t) = \frac{1}{2\pi} \int d\omega e^{-i\omega t} \Sigma(\omega), \tag{12}$$

is analytic function of ω above the real axis.

The $*$-product with $\Sigma(t)$ is simple generalization of the product of two two-point functions

$$[S_1 * \Sigma * S_2](x,y) = \Theta(x_0)\Theta(y_0) \int_0^\infty dz_0 \int d^3 z S_1(x,z)\Sigma(z_0)S_2(z,y)$$
$$= \frac{1}{2\pi} \int d\omega \Sigma(\omega)\Theta(x_0)\Theta(y_0) e^{-i\omega z_0} \int_0^\infty dz_0 \int d^3 z S_1(x,z) S_2(z,y) \tag{13}$$

The Wigner transform of the product is simple:

$$[S_1 * \Sigma * S_2]_{X_0}(p_0, \vec{p}) = \int_{-2X_0}^{2X_0} ds_0 d^3 s e^{i(p_0 s_0 - \vec{p}\vec{s})} [S_1 * \Sigma * S_2](X + \frac{s}{2}, X - \frac{s}{2})$$
$$= \frac{1}{2\pi} \int d\omega \Sigma(\omega) \int dp_{0,1} dp_{0,2} P_{X_0}(p_0, \frac{p_{0,1} + p_{0,2}}{2}) \frac{i}{2\pi} \frac{e^{-iX_0(p_{0,1} - p_{0,2} + i\epsilon)}}{p_{0,1} - p_{0,2} - \omega + i\epsilon}$$
$$S_{1,\infty}(p_{0,1}, \vec{p}) S_{2,\infty}(p_{0,2}, \vec{p})$$
$$= \frac{1}{2\pi} \int d\omega \Sigma(\omega) \int dp_0' P_{X_0}(p_0, p_0' + \frac{\omega}{2}) e^{-iX_0\omega} S_{1,\infty}(p_0' + \omega, \vec{p}) S_{2,\infty}(p_0', \vec{p}) \tag{14}$$

The last expression is obtained by integration over ω, and closing the integration path from above. One picked the pole at $p_{0,1} - p_{0,2} - \omega = 0$, i.e. the energy at the vertex is conserved.

Now, one can write Eq. (5):

$$< 1 - 2N_{+,\vec{p}}(t) >= \frac{\omega_p}{2\pi} \int dp_{0,1} Tr[1 - iS_R * \Sigma_R]^{-1} * S_K * [1 - i\Sigma_A * S_A]^{-1}$$

$$= \frac{\omega_p}{2\pi} \int dp_{0,1} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \Pi_{r=0}^{n} [\int \frac{d\omega_r e^{-it\omega_r}}{2\pi}] \Pi_{s=0}^{k} [\int \frac{d\omega_{n+s} e^{-it\omega_{n+s}}}{2\pi}]$$

$$Tr \Pi_{r=0}^{n} [S_R(p_r)\Sigma(\omega_r)][-S_{K,R}(p_{n+1}) + S_{K,A}(p_{n+1})] \Pi_{s=0}^{k} [\Sigma(\omega_{n+s}) S_A(p_{n+s+1})]$$

$$p_{0,r+1} = p_{0,r} - \omega_r, \ \vec{p}_{r+1} = \vec{p}_1 \ r = 1, ..., n+k, \tag{15}$$

Owing to the spinors the products are ordered in a way such that $S_R(p_1)$ appears at the ultimate left, $S_A(p_{n+k+1})$ at the ultimate right, and $S_{K,R(A)}(p_{n+1})$ between the two groups.

5 Pair Production in a "Model-Laser" Field

A stationary field of two laser beams directed to each other is mimicked by a field sinusoidal in time and constant in space. The sinusoidal field with Gaussian envelope is turned on at $t = 0$, and turned off at $t = 2\tau$.

$$\Sigma(t) = -i\gamma^3 V_0 \sin w_l t \Theta(t)\Theta(2\tau - t)e^{-\frac{\sigma^2(t-\tau)^2}{2}} \tag{16}$$

$$\Sigma(\omega,\tau,\sigma) = \Sigma_+(\omega,\tau,\sigma) + \Sigma_-(\omega,\tau,\sigma)$$

$$\Sigma_\pm(\omega,\tau,\sigma) = \pm\frac{1}{2}\gamma^3 V_0 \int dt e^{i(\omega\mp w_l)t}\Theta(t)\Theta(2\tau - t)e^{-\frac{\sigma^2(t-\tau)^2}{2}}$$

$$\lim_{\tau\to\infty}\Sigma_\pm(\omega,\tau,\sigma) = \pm\frac{\pi^{1/2}}{2^{1/2}\sigma}\gamma^3 V_0 e^{i(\omega\mp w_l)\tau - \frac{(\omega\mp w_l)^2}{2\sigma^2}} \tag{17}$$

We calculate average particle number after turning off the interaction ($t > 2\tau$). By assumption, there are no particles in the initial state, i.e. $f(\pm\omega_p) = 0$. Thus nonzero contribution in $1 - 2n_{f\pm}(\omega_p)$ comes from "1" (i.e. from the Dirac see before the redefinition of vacuum).

The general structure of the contribution is following:

The first term in(15) is connected to the number of particles at $t = 0$. As $n_f(\omega)$, by assumption, vanishes, Eq. (6) just cancels $< 1 >$ in $< 1 - 2N >$.

In all terms the first R-propagator, and the last A-propagator, and the K-propagator are on shell: To see it one integrates over ω_{n+k}. Owing to the factor $e^{-it\omega_{n+k}}$ (which dominates over the correstponding $\Sigma(\omega_{n+k})$ behaviour!), one may close the integration path from below and catch the poles of $S_A(p_{n+k+1})$. There are two poles at $\bar{p}_{0,n+k+1} = \lambda_{n+k+1}\omega_p$ ($\lambda_{n+k+1} = \pm 1$). In the next integration, one may choose as independent variables p_{n+k}, ω_1, ... ω_{n+k-1} and integrate over ω_1. Owing to the factor $e^{-it\omega_1}$, which dominates over the correstponding $\Sigma(\omega_1)$ behaviour!, one may close the integration path from above one catches the poles of $S_R(p_{0,1},\vec{p}_1)$. There are two poles $\bar{p}_{0,1} = \lambda_1 m$. Then $\bar{\omega}_{n+k} = p_{n+k} - \lambda_{n+k+1}\omega_p$. $\bar{\omega}_1 = (\lambda_1 - \lambda_{n+k+1})\omega_p - \sum_{r=2}^{n+k-1}\omega_r - p_{n+k}$. Finally, in the further integrations one cannot close the integration path neither from below nor from above. There are always pole terms and principal value terms. The exception is the $-S_{K,R} + S_{K,A}$ combination in which principal value term is cancelled. The poles are at $\bar{p}_{n+1} = \lambda_{n+1}\omega_p$. the combinations of lambdas (i.e.poles) (λ_{n1}, λ_{n+1}, λ_{n+k+1}) are classified as: 1. (1,1,1) and (-1,-1,-1) terms which have nothing to do with a pair creation they are probably related to the renormalisation of S_K propagator. 2. (1,1,-1), (1,-1,-1), (-1,1,1), and (-1,-1,1) are terms expected to be small (negligible). 3.

(1,-1,1) and (-1,1,-1) terms relevant for pair production. As we do not separate particles from antiparticles we have to add both terms and divide by two. As, the contributions are almost exactly equal it is enough to choose (1,-1,1) term. 4. further appearance of the poles of S_R and/or S_A is factorising the segments of the type (1,1), (1,-1), (-1,1), and (-1,1), where (λ_j, λ_k) corresponds to the signs of poles of S_R, S_R or S_A, S_A. These will, as well be ignoredas they just refine the above (ignored) terms. Relevant term is (1,-1,1) with principal value contributions for all the remaining integrations.

As Gaussian cutoff makes many terms negligible, we search for contribution surviving it. For given momentum $\vec{p}$, define n_0 as closest order index $|p_0 - n_0\omega_l| = \min_n |p_o - n\omega_l|$. From (15) we choose the term with $n = k = n_0$.

We calculate the lowest order nontrivial contribution. We assume "fine tuned laser" with the property $\omega_l n_0 = 2m >> 1$, and n_0 odd! The laser line is assumed to be narrow ($\sigma << w_l$ and even $n_0^{1/2}\sigma << w_l$). Average duration of the pulse, connected to the shape of gausssian is $< (t-\tau)^2 >= 1/\sigma^2$ In that case the leading term is given by $n = n_0 = k$.

and particles are produced with small momentum ($||\omega_p - m| << \omega_l$ or $|\vec{p}|^2 << 2m\omega_l$).

We define $\delta = 2|\omega_p - m| = |2\omega_p - n_0\omega_p| \approx \vec{p}^2/m$

$$
\begin{aligned}
&< 1 - 2N_{\vec{p}}(t) >= \frac{m}{2\pi}\int dp_{0,1}\prod_{r=1}^{n_0}[\int\frac{d\omega_r}{2\pi}]\prod_{s=1}^{n_0}[\int\frac{d\omega_{n_0+s}}{2\pi}]e^{-it(p_{0,1}-p_{0,2n_0+1})}\\
&Tr\prod_{r=1}^{n}[S_R(p_r)\Sigma(\omega_r)][-S_{K,R}(p_{n_0+1}) + S_{K,A}(p_{n_0+1})]\prod_{s=1}^{n_0}[\Sigma(\omega_{n_0+s})S_A(p_{n_0+s+1})]\\
&p_{0,r+1} = p_{0,r} - \omega_r,\ \vec{p}_{r+1} = \vec{p}_1 = 0,\ r = 1, ..., 2n_0,
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
&< 1 - 2N_{+,\vec{p}}(t) >= (2\pi)^2 m\prod_{r=2}^{n_0}[\int\frac{d\omega_r}{2\pi}]\prod_{s=2}^{n_0}[\int\frac{d\omega_{n_0+s}}{2\pi}]\\
&Tr\prod_{r=2}^{n}[S_R(p_r)\Sigma(\omega_r)](m\gamma^0 - m)\prod_{s=2}^{n_0}[\Sigma(\omega_{n_0+s})S_A(p_{n_0+s+1})]\\
&p_{0,r+1} = p_{0,r} - \omega_r,\ \vec{p}_{r+1} = \vec{p}_1 = 0,\ r = 1, ..., 2n_0,
\end{aligned}
\tag{19}
$$

To perform the integration over $\prod d\omega_s$ one has to exploit the narrow distribution assumption i.e. in the range where Gaussian function changes from zero to maximum and back to zero the $S_{R(A)}$ is almost constant. Then

$$
\int d\omega_s \Sigma_\pm(\omega_s) S_{R(A)}(p_{0,s} - \omega_s) \approx \int d\omega_s \Sigma_\pm(\omega_s) S_{R(A)}(p_{0,s} \mp \omega_l) \tag{20}
$$

One finds that in the lowest nontrivial order the Σ_+ are on the R side of product ([?]) and all the Σ_- are on the A side. Now one can calculate separately the trace,product of energy denominators and two multiple integrals:

The trace :

$$
\begin{aligned}
& p_{0,1} = p_{0,2n_0+1} = -p_{0,n_0+1} = m, \\
& p_{0,r+1} = m - \frac{2r}{n_0} m, r = 1, n_0, \\
& p_{0,n_0+r+1} = -m + \frac{2r}{n_0} m, r = 1, n_0, \\
& \mathcal{T}r = Tr \prod_{r=0}^{n_0-1} [(\gamma^0 p_{0,r+1} + m)\gamma^3](-m\gamma^0 + m) \prod_{s=1}^{n_0} [\gamma^3(\gamma^0 p_{n_0+s+1} + m)] \\
& = m^{2n_0+1} Tr \prod_{r=0}^{n_0-1} [(1 + \gamma^0 \frac{n_0 - 2r}{n_0})\gamma^3](1 - \gamma^0) \prod_{s=1}^{n_0} [\gamma^3(1 - \gamma^0 \frac{n_0 - 2s}{n_0})]
\end{aligned} \tag{21}
$$

For n_0 even, the above trace contains the factor $(\gamma^0 + 1)(\gamma^0 - 1)$, and therefore vanishes. This is consistent with the vanishing of the amplitude for zero-kinetic-energy-pair production by two photons in ordinary S-matrix QED. For odd n_0 the trace is integer

$$
\begin{aligned}
\mathcal{T}r &= -m^{2n_0+1} Tr(1 + \gamma^0)^3 \prod_{r=1}^{n_0-1} [1 + \gamma^0 (-1)^r \frac{n_0 - 2r}{n_0}]^2 \\
&= -\frac{16 m^{2n_0+1} 2^{2n_0-2} [(n_0 - 2)!!]^4}{n_0^{2n_0-2}},
\end{aligned} \tag{22}
$$

The product of energy denominators is

$$
\prod_{r=1}^{r_0} [(1 - \frac{2r}{n_0})^2 m^2 - m^2]^4 = m^{4n_0-4} [\frac{2^{n_0-1}(n_0 - 1)!}{n_0^{n_0-1}}]^4 \tag{23}
$$

We calculate multiple integral over gaussians, under the condition that $\sum_{r=1}^{n_0} \omega_r = 2\omega_p$. The condition is transformed through δ-fuction

$$
\begin{aligned}
& \int d\omega_1 \delta(\sum_{r=1}^{n_0} \omega_r - 2\omega_p) \\
& = \frac{1}{2\pi} \int dx \int d\omega_1 e^{ix(\sum_{r=1}^{n_0} \omega_r - 2\omega_p)}
\end{aligned} \tag{24}
$$

Now

$$
\begin{aligned}
&\Sigma_+(\omega_1)\prod_{r=2}^{n_0}\int\frac{d\omega_r}{2\pi}\Sigma_+(\omega_r) \\
&=\frac{V_0^{n_0}}{(2\pi)^{n_0}}\int dxe^{-2ix\omega_p}\prod_{r=1}^{n_0}\int d\omega_r\Sigma_+(\omega_r)e^{ix\omega_r} \\
&=-\frac{V_0^{n_0}}{2^{r_0}}\int dxe^{ix(n_0\omega_l-2\omega_p)-n_0\sigma^2(x+\tau)^2/2} \\
&=-\frac{\pi^{1/2}V_0^{n_0}}{2^{r_0-1/2}n_0^{1/2}\sigma}e^{i\tau\Delta-\Delta^2/(2n_0\sigma^2)}
\end{aligned}
\tag{25}
$$

Trace, energy denominator and gaussians multiplied, for n_0 large we use approximate expression $(n-1)!\approx(2\pi)^{1/2}n^{n-1/2}e^{-n}$

$$
[\frac{n_0V_0e}{4m(n_0+1)}]^{2n_0}\frac{2^3m^5e^2}{\pi n_0^3\sigma^2}e^{-\vec{p}^2/(mn_0\sigma^2)} \tag{26}
$$

Expression in braces is a new criticality condition. By substituting $E_m=V_0\omega_l$, with $n_0/(n_0+1)\approx 1$ and $[n_0/(n_0+1)]^{2n_0}\approx e^{-2}$, we find

$$
\frac{E_ce}{4\omega_lm}=1 \tag{27}
$$

that the critical field is lowered by a factor $4\omega_l/(em)$

All the factors put together:

$$
<1-2N_{\vec{p}}(t)>=2-\frac{\pi\omega_l^3}{2\sigma^2}[\frac{E_me}{4\omega_lm}]^{2n_0}e^{-\vec{p}^2/(mn_0\sigma^2)} \tag{28}
$$

One finds that the particles are produced with small momentum ($||\omega_p-m|<<\omega_l$ or $|\vec{p}|^2<<2m\omega_l$). One may integrate over momenta to obtain

$$
\int<N_{\vec{p}}(t)>d^3p=[\frac{E_me}{4\omega_lm}]^{2n_0}2^{3/2}\pi^{5/2}m^3\omega_l^{3/2}\sigma_l \tag{29}
$$

5.1 Contribution of the Next to Leading Order

a) lower order. Lower order is $n_0 - 2$. In this case one defines $\Delta = 2|\omega_p - (n_0 - 2)\omega_p| = \Delta \pm 2\omega_p$. Inserted in Eq. (25) it gives enormous dumping by Gaussian.

b) higher order.

Next order is acchieved by insertion of $\Sigma_{+(-)}$ into a chain of $\Sigma_{-(+)}$ at some position r_1. Then there are extra propagators $S_{R(A)}$ and $\Sigma_{-(+)}$.

The trace factor receives insertion of $(\gamma^0 p_{0,r_1-1} + m)\gamma^3(\gamma^0 p_{0,r_1} + m)\gamma^3$

The denomiator is multiplied by $m^4[(1-2(r_1-1)/n_0)^2 - 1][(1-2r_1/n_0)^2 - 1]$

The integral over Gaussians receives factor (-1) and replacement $n_0 \to n_0 + 2$.

Total contribution is the sum of all posible insertions and apears to be

$CORRECTION = LOWEST\ ORDER \times E_0/[\omega_l m] 2\ln((n_0+1)/2)(n_0+1)$

6 Discussion of the Results

1. We calculate pair production in a model laser field, within the well defined set of laser parameters.

2. The results support somewhat different mechanism of production, with the scaling down of critical field by a factor $1.46\omega_l/m$

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