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EPL, **94** (2011) 18007

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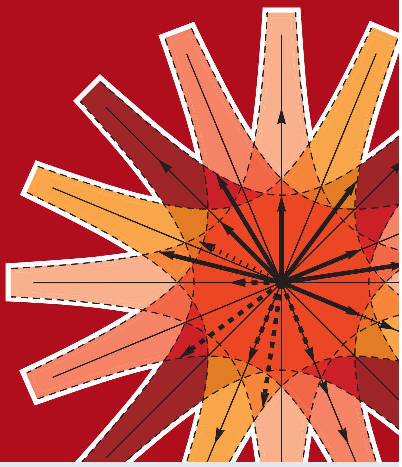

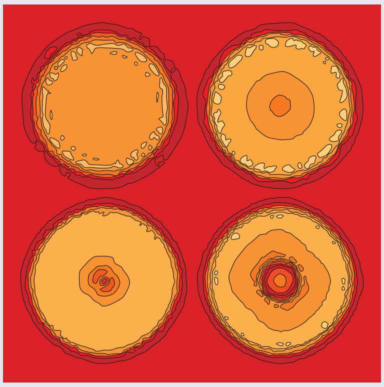

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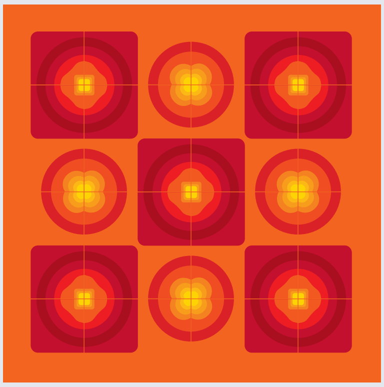

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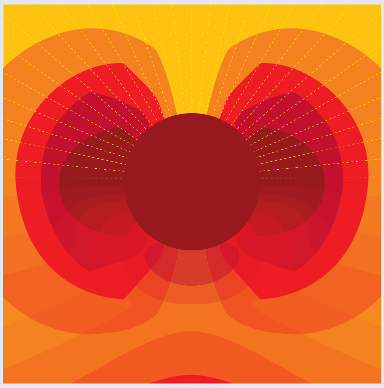

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A Letters Journal Exploring the Frontiers of Physics

April 2011

EPL, **94** (2011) 18007 www.epljournal.org
doi: 10.1209/0295-5075/94/18007

Detrended cross-correlation analysis for non-stationary time series with periodic trends

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received 27 January 2011; accepted in final form 7 March 2011
published online 8 April 2011

PACS 89.20.-a – Interdisciplinary applications of physics
PACS 05.40.-a – Fluctuation phenomena, random processes, noise, and Brownian motion
PACS 02.50.-r – Probability theory, stochastic processes, and statistics

Abstract – Noisy signals in many real-world systems display long-range autocorrelations and long-range cross-correlations. Due to periodic trends, these correlations are difficult to quantify. We demonstrate that one can accurately quantify power-law cross-correlations between different simultaneously recorded time series in the presence of highly non-stationary sinusoidal and polynomial overlying trends by using the new technique of detrended cross-correlation analysis with varying order ℓ of the polynomial. To demonstrate the utility of this new method —which we call DCCA-$\ell(n)$, where n denotes the scale— we apply it to meteorological data.

Data on many real-world systems, ranging from geophysics to physiology, exhibit two important properties, correlations and periodicity. Depending on whether we are studying correlations in a single signal or between a pair of signals, we can use autocorrelation functions or cross-correlation functions to gain insight into the correlation dynamics. But like many other techniques, these methods were devised to identify correlations in data that are stationary and linear. For example, the power spectrum $S(f)$ assumes stationarity in data and is defined as a positive real function of a frequency variable f that describes how the energy of a signal is distributed with frequency. In the case of power-law correlations, using the Fourier transform the Khinchin-Kolmogorov theorem [1] relates the power spectral density $S(f) \sim f^{-\beta}$ of a wide-sense-stationary random process to the corresponding autocorrelation function $C(n) \sim n^{-\gamma}$, where the exponents are related as $\beta = 1 - \gamma$. Despite their widespread popularity and applicability, power spectral density and correlation analysis have limitations when applied to the real-world data associated with physical, biological, hydrological, and social systems. These are commonly non-stationary and they often exhibit periodicity [2,3].

When a time series is non-stationary, the limitations of methods that assume stationarity are clear. Suppose a time series X_t has a large upward trend. Then a large value of X_t is more likely to be followed by a large value of X_{t+1} implying strong autocorrelations, not because autocorrelations are actually present, but because the autocorrelation function is being used for a non-stationary time series —which is inappropriate. Similarly, a US market index time series may strongly cross-correlate with the population of, say, Pakistan, simply because each time series has a characteristic strong upward trend.

Thus, detrending is essential to properly analyze many time series for at least two reasons: i) detrending prevents a time series from being correlated if correlations are not present, and ii) if correlations do exist, detrending reveals a genuine correlation functional dependence—in case of power-law correlations, for example, we expect to obtain a genuine correlation exponent. Most methods using detrending start with the assumption that the functional form of a trend is predetermined [4].

The application of detrending to original data can be either local or global. When done locally, detrended fluctuation analysis (DFA) [4–8] quantifies not periodicity but a single scaling parameter that represents the long-range autocorrelation properties of a signal. DFA has been used in fields ranging from cardiac dynamics [9], bioinformatics [4], and economics [10,11] to meteorology [12,13]. In physiology, the DFA method can be used diagnostically —it can help identify different states of the same system according to their different scaling behaviors,

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e.g., the scaling exponent for heart interbeat intervals in healthy individuals differs from that of unhealthy individuals [9,14]. In its original proposed form [4], the DFA method is based on local linear detrending— for a given box (window) size that is a subset of the entire signal, the method locally subtracts linear polynomials from the original signal in order to obtain locally stationary signals. However, many noisy signals in real-world systems exhibit highly nonlinear trends accompanied by periodic trends [2,3], and the scaling results obtained from the detrending method which locally subtract polynomials of small order from the original signal are clearly not suitable for large box sizes. Reference [5] offers a systematic study of the limitations of the DFA method when data exhibit any of three types of trends: linear, periodic, and power-law. In order to overcome the limitations of DFA in the presence of highly nonlinear trends, different extensions of DFA have been proposed which locally subtract higher-order polynomials from the original signal, called DFA-2, DFA-3, together with the detrended moving average (DMA) method [8,15,16], and multifractal DFA [17,18]. However, detrending procedures based on local subtractions of polynomials of small order are not suitable when quantifying long-range autocorrelations in the presence of periodicity.

The signal outputs of many physical and social systems are often characterized by variables that are not only autocorrelated, but also are cross-correlated [19–27]. However, in contrast to autocorrelations, which are only applicable to a single series, cross-correlations can be determined either between two time series [27–29] or among a large number of time series as in the random matrix theory (RMT) approach where cross-correlations are considered a collective phenomenon [21]. Based on time-lag RMT, ref. [30] reported the magnitudes of power-law cross-correlations in a wide range of phenomena from finance and physiology to genomics. In analogy to DFA, which was proposed for a single time series, detrended cross-correlation fluctuation analysis (DCCA) [28,29] was proposed in order to provide a single scaling parameter representing the long-range cross-correlation properties between two non-stationary signals. Reference [31] proposed a multifractal DCCA in order to quantify power-law cross-correlations in the presence of multifractality. Motivated by DFA, the DCCA method in its original version is based on local detrending —in each of two signals the method locally subtracts a linear polynomial from the original signal in order to obtain a locally stationary signal— therefore, the DCCA method suffers from the same problems as DFA when applied to data that are nonlinear and characterized by periodicity [32].

Hence, both DFA and DCCA methods in their original versions with linear polynomial fits could not subtract every type of non-stationarity, and sinusoidal trends in particular —a linear fit is able to mimic trends for small box sizes, but not for large box sizes [5,32]. However, if detrending is performed with a polynomial of a higher order, all trends can be subtracted. Cross-correlations in the presence of periodicity present more of a challenge than do autocorrelations in the presence of periodicity because in cross-correlations we have two signals, each of which may have a different period. Here we demonstrate that a detrended cross-correlation analysis with varying polynomial order ℓ, and scale n, DCCA-$\ell(n)$, based on detrended covariance, is capable of investigating power-law cross-correlations between different simultaneously recorded time series in the presence of highly non-stationary sinusoidal signals. Our detrending concept with varying ℓ can be successfully applied not only to covariances but also to variances. We demonstrate how DFA and DCCA with varying ℓ are closely related.

A variety of methods for detrending data have been proposed, *e.g.*, using polynomials [4] and the moving average approach [8,15]. The empirical mode decomposition (EMD) approach has the advantage that it does not require that the trend has any predetermined functional dependence [33,34]. An important concern when applying any detrending method is determining whether the method affects the correlation properties. We can test for this by generating a signal characterized by a known correlation exponent and periodicity. If a detrending method calculates trends but also affects the correlation properties of the signal, it obviously cannot be used to quantify the correlation properties. We now show that the DFA method does not affect correlation properties even when the polynomial order values are very large.

We first generate a fractionally autoregressive integrated moving average (ARFIMA) process [28,35–37] with $\rho = 0.2$,

$$y_i = \left[\sum_{n=1}^{\infty} a_n(\rho) y_{i-n}\right] + \eta_i, \qquad (1)$$

where $0 < \rho < 0.5$ is a free parameter, $a_j(\rho)$ are weights defined by $a_j(\rho) \equiv \frac{\Gamma(j-\rho)}{\Gamma(-\rho)\Gamma(1+j)}$ $a_j(\rho) \equiv \Gamma(j-\rho)\Gamma(-\rho)\Gamma(1+j)$, where $\Gamma(j)$ denotes the Gamma-function and η_i denotes an independent and identically distributed Gaussian variable. The parameter ρ is related to the Hurst exponent, $H = 0.5 + \rho$ [35]. Next we employ the DFA method for five different values of polynomial order ℓ. In fig. 1(a) for each of five ℓ value we present the DFA-ℓ square root of detrended variance. We exclude all scales for which $n > N/4$, where N denotes the series length. We show that each DFA-ℓ plot can be presented as $F_{\text{DFA-}\ell}(n) = A_n n^H$. Thus the DFA slope does not depend on ℓ, *i.e.*, the slope is the same no matter which DFA-ℓ we apply. However, different DFA-ℓ curves have different DFA intercepts with corresponding values (see fig. 1(b)). We show also different DFA intercepts for varying ρ, which we later use for our detrending approach with varying polynomial order ℓ, both variance and co-variance. As expected, the DFA intercept decreases with increasing ℓ because polynomials with large values of ℓ can better explain data than polynomials with small values of ℓ. Hence polynomials with large ℓ have smaller residuals and thus a smaller variance of residuals than

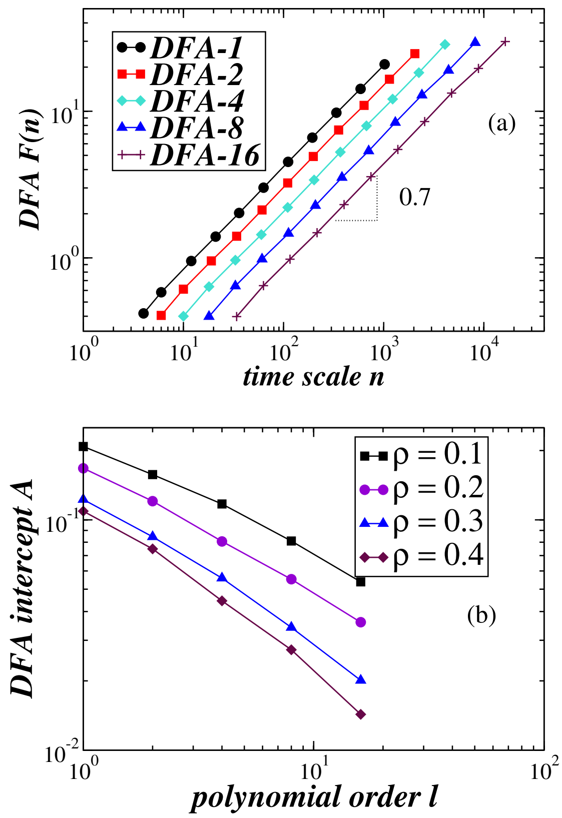

Fig. 1: (Colour on-line) DFA with polynomial of fixed order ℓ. (a) DFA with polynomial of fixed ℓ yields a DFA curve, where the DFA slope does not depend on ℓ. However, the DFA intercept A_n depends on ℓ. (b) A_n *vs.* polynomial order. Shown are values obtained for different values of ρ (DFA α, where $\alpha = 0.4 + \rho$). These values will be used in a definition for both detrended variance of eq. (3) and covariance of eq. (5).

polynomials with smaller ℓ —so polynomials with larger ℓ are below polynomials with lower ℓ in fig. 1(b). Clearly, for a given choice of polynomial order ℓ, the fitting procedure is possible only for lags larger or equal to $\ell + 1$, and thus, the smallest lag used in DFA-ℓ in fig. 1(b) is different for different values of ℓ.

When oscillatory signals are highly non-stationary, we can accurately quantify the correlations for large scales only if large values of ℓ are employed in the detrending procedure. If large values of ℓ are required for large scales, and small values of ℓ are needed for small scales, can we generate a detrending plot for both large and small scales in one unique curve, and not with different curves (cf. fig. 1(a))?

To this end, we introduce a detrending technique to use when a varying polynomial order ℓ (which increases with box size) can be employed for both variance and covariance. First we demonstrate that DFA with varying polynomial order ℓ based on detrended variance, where polynomial order ℓ is changing with scale, is capable of investigating power-law autocorrelations in the presence of highly non-stationary sinusoidal signals. This we accomplish in one unique plot, with scales ranging from smallest to largest.

We briefly introduce the method: Consider a long-range autocorrelated time series $\{y_i\}$ and compute the integrated signal $R_k \equiv \sum_{i=1}^{k} y_i$. Divide the entire time series into $N - n$ overlapping boxes, each containing $n + 1$ values. In each box (that starts at i and ends at $i + n$) calculate the "local trend" using a given polynomial of order ℓ. Note that the order of polynomial is increasing with scale size n. We define the "detrended walk" to be the difference between the original walk and the local trend: $\epsilon_k \equiv R_k - \widetilde{R_{k,i}}$ where the variance of the residuals in each box is calculated as

$$f^2_{\mathrm{DFA}}(n,i) \equiv 1/(n-1) \sum_{k=i}^{i+n} \epsilon_k^2. \tag{2}$$

The detrended variance with varying polynomial order, DFA-$\ell(n)$ is defined by summing over all overlapping $N - n$ boxes of size n,

$$F_{\mathrm{DFA}}(n)^2 \equiv \frac{A_2}{A_n} \sum_{i=1}^{N-n} f^2_{\mathrm{DFA}}(n,i), \tag{3}$$

where the DFA intercept A_n depends on n and is calculated from fig. 1(b)[1]. Note that the values A_n in fig. 1 are obtained for the DFA method using a fixed order of polynomial for each box of size n. In contrast, in eq. (3) for different box sizes we use polynomials with a varying order. A_2 is the intercept corresponding to the smallest scale size, in our case assumed to be $n = 2$. Thus the DFA method based on varying polynomial order differs from the common DFA method based on fixed polynomial order in two ways. In the approach illustrated in eq. (3) i) the polynomial order ℓ is not fixed but is increasing with scale size n, and ii) because of this (and taking into account intercept results shown in fig. 1), the detrended variance must be normalized using the term A_2/A_n.

In fig. 2(a) we show an oscillatory ARFIMA time series with 20000 data points where $\rho = 0.2$ with sinusoidal trend $T = 1000$. Employing the DFA method based on varying polynomial order, DFA-$\ell(n)$ (eq. (3)), in fig. 2(b) we obtain the expected slope $H = 0.7 \pm 0.002 \approx 0.5 + \rho$. Notice that the crossover in the DFA curve around the scale equal to T expected for DFA-1 has disappeared [5]. In fig. 2(b) the DFA-$\ell(n)$ curve is approximately a straight line up to a scale of approximately four periods. In this example, we choose $\ell = 16$ for the largest polynomial order.

We next demonstrate that a similar detrending method with a varying polynomial order ℓ employed for the DCCA, DCCA-$\ell(n)$, can be used to investigate power-law cross-correlations between different simultaneously recorded time series in the presence of highly non-stationary sinusoidal signals.

We define DCCA with varying polynomial order ℓ following the original DCCA method [28]:

1) Consider two long-range cross-correlated time series $\{y_i\}$ and $\{y'_i\}$ of equal length N, compute two integrated signals $R_k \equiv \sum_{i=1}^{k} y_i$ and $R'_k \equiv \sum_{i=1}^{k} y'_i$, where

[1] The algorithm can be found on `http://www.phy.hr/~davorh/dfa.html`

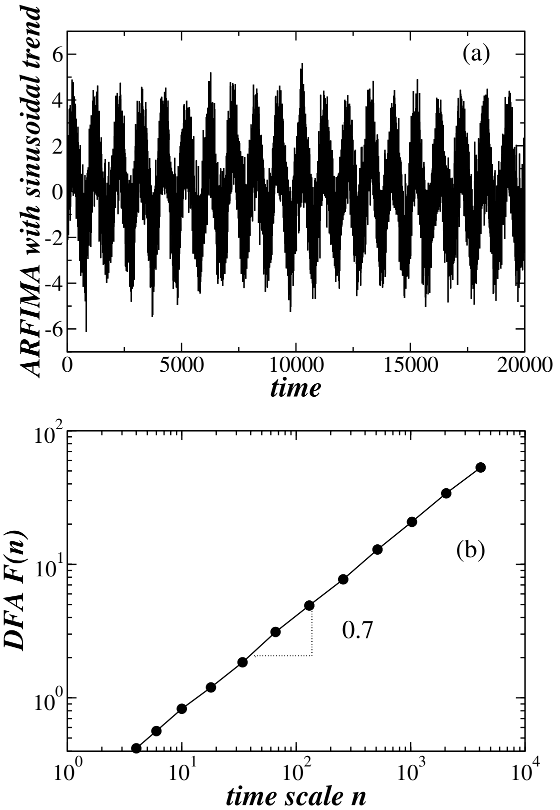

Fig. 2: Detrending with a polynomial of varying order —autocorrelations with sinusoidal trend. RMS of detrended variance with varying order of polynomial ℓ, $F_{\mathrm{DFA}}(n)$. (a) The time series is generated using a ARFIMA process $\{y_i\}$ with $\rho=0.2$ with sinusoidal trend with period $T=1000$. (b) $F_{\mathrm{DFA}}(n)$, where n is the scale. We show the time lags and the corresponding orders of polynomials in parentheses: 4 (1), 6 (2) 10 (4), 18 (8). For all other time lags we use $\ell=16$. Despite strong periodicity, $F_{\mathrm{DFA}}(n)$ with varying order of polynomial follows a straight line up to a scale of four periods.

$k=1,\ldots,N$. Divide the entire time series into $N-n$ overlapping boxes, each containing $n+1$ values.

2) For both time series, $x_n(i)$ and $y_n(i)$, in each box that starts at i and ends at $i+n$, the "local trend." While in the original DCCA-1 the "local trend" is defined to be the ordinate of a linear least-squares fit, in the DCCA with varying polynomial order ℓ, the linear polynomial fit is replaced by a polynomial of higher order ℓ, where ℓ is increasing with box size n. For each time series, we define then the "detrended walk" as the difference between the original walk and the local trend, $\epsilon_k \equiv R_k - \widetilde{R_{k,i}}$ and $\epsilon'_k \equiv R'_k - \widetilde{R'_{k,i}}$.

3) Calculate the covariance of the residuals in each box

$$f^2_{\mathrm{DCCA}}(n,i) \equiv \frac{1}{n-1}\sum_{k=i}^{i+n}\epsilon_k\epsilon'_k. \tag{4}$$

4) In order to uncover multifractality commonly present in real-world data, we define, in analogy with ref. [31], a multifractal DCCA-$\ell(n)$, where the detrended covariance is defined by summing over all overlapping $N-n$ boxes of size n,

$$F_{\mathrm{DCCA}}(n) \equiv \sqrt{\frac{A_2B_2}{A_nB_n}}\left\{\sum_{i=1}^{N-n}[f^2_{\mathrm{DCCA}}(n,i)]^{q/2}\right\}^{1/q}. \tag{5}$$

In analogy with the DFA-$\ell(n)$, we introduce a new term $\sqrt{\frac{A_2B_2}{A_nB_n}}$ in order to normalize covariance since polynomial order ℓ increases with box size n. The difference compared to DFA with varying ℓ, DFA-$\ell(n)$, is that for cross-correlations we have two signals and thus A_n and B_n correspond to each of two signals. Note that $A_n=B_n$ only if both signals have the same DFA exponent. Otherwise $A_n \neq B_n$.

For the moment we set $q=2$ in eq. (4). If cross-correlations decay as a power-law, the corresponding detrended covariances are either always positive or always negative, and the square root of the detrended covariance grows with time scale n as $F_{\mathrm{DCCA}}(n) \propto n^{\lambda_{\mathrm{CC}}}$ [28]. Here λ_{CC} is the cross-correlation exponent. If, however, the detrended covariance oscillates around zero as a function of the time scale n, there are no long-range cross-correlations [28]. When only one random walk is analyzed ($R_k=R'_k$), the *detrended covariance* $F^2_{\mathrm{DCCA}}(n)$ reduces to the *detrended variance* $F^2_{\mathrm{DFA}}(n)$ used in the DFA method [4].

In order to test the applicability of the DCCA method with varying polynomial order, DFA-$\ell(n)$, we generate power-law autocorrelated and cross-correlated time series for which the scaling properties are known. To investigate power-law autocorrelations and power-law cross-correlations and effect of sinusoidal periodicity on cross-correlations, we first define a periodic two-component ARFIMA process [35,36],

$$y_i = \left[\sum_{n=1}^{\infty}a_n(\rho_1)x_{i-n}\right] + A_1\sin\left(\frac{2\pi}{T_1}i\right) + \eta_i, \tag{6}$$

$$y'_i = \left[\sum_{n=1}^{\infty}a_n(\rho_2)y_{i-n}\right] + A_2\sin\left(\frac{2\pi}{T_2}i\right) + \eta_i. \tag{7}$$

Here, η_t is shared between y_i and y'_i in order to enable cross-correlations, $T_1(T_2)$ is the sinusoidal period, and A_1 and A_2 are two sinusoidal amplitudes.

We generate for $\rho=0.2$ the time series x_i (fig. 3(a)) and y_i of eq. (6) and eq. (7), each with 20000 data points. Each time series is power-law autocorrelated, and the DFA exponent is 0.7 [35]. Each signal is oscillatory and thus highly non-stationary where $T_1=1000$ and $T_2=1500$. We apply the DCCA-$\ell(n)$ method of eq. (5) with varying polynomial order. We find, similar to DFA-$\ell(n)$, that the crossover expected in DCCA-1 curve around a scale equal to T [32] now vanishes for the scales shown. In fig. 3(b) the DCCA curve obtained for the DCCA-$\ell(n)$ method of eq. (5) is now a straight line up

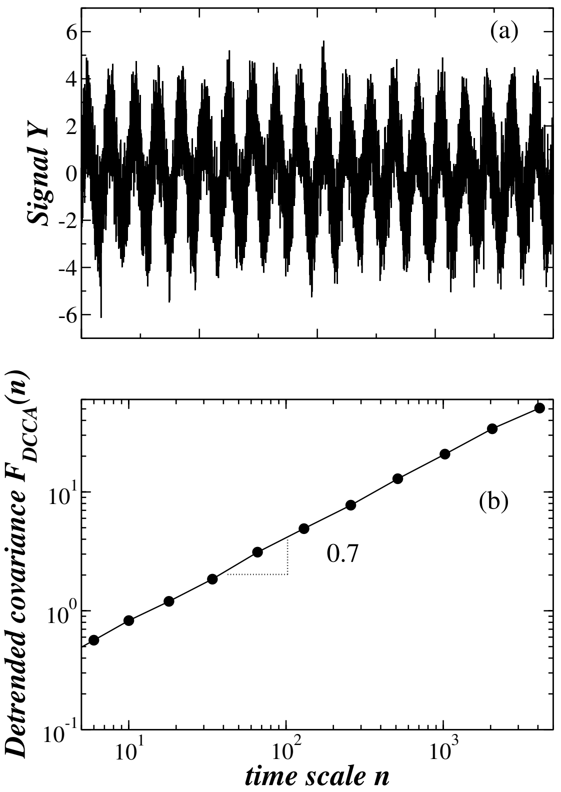

Fig. 3: Detrending with polynomials of varying order —cross-correlations with sinusoidal trend. RMS of detrended covariance with varying order of polynomial ℓ, $F_{\mathrm{DCCA}}(n)$, for two time series generated by two ARFIMA processes: (a) $\{y_i\}$ with $\rho = 0.2$ and $\{y'_i\}$ with $\rho' = 0.2$ (for simplicity we show only $\{y_i\}$). Cross-correlations are generated since we choose the error term to be equal for both time series. (c) RMS of detrended covariance with varying order of polynomial ℓ, $F_{\mathrm{DCCA}}(n)$, where n is a scale. We show time lags and corresponding polynomial orders in parentheses: 4 (1), 6 (2) 10 (4), 18 (8). For all other lags we use $\ell = 16$.

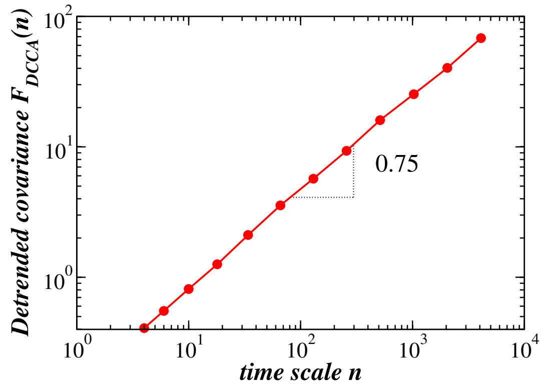

Fig. 4: (Colour on-line) Detrending, with polynomials of varying order —cross-correlations with sinusoidal trend. RMS of detrended covariance $F_{\mathrm{DCCA}}(n)$, with varying polynomial ℓ, for two time series generated by two different ARFIMA processes: (a) $\{y_i\}$ with $\rho_1 = 0.1$ and (b) $\{y'_i\}$ with $\rho' = 0.4$. Cross-correlations are generated since we choose the error term to be equal for both time series. RMS of detrended covariance with varying order of polynomial ℓ, $F_{\mathrm{DCCA}}(n)$, where n is a scale. We show time lags and corresponding polynomial orders in parentheses: 4 (1), 6 (2) 10 (4), 18 (8). For all other time lags we use $\ell = 16$.

to a scale of approximately four periods. In this example, for the largest polynomial order we use $\ell = 16$.

Next we analyze the power-law autocorrelated time series y_i and y'_i of eq. (6) and eq. (7) each with 20000 data points, where $\rho_1 = 0.4$ and $\rho_2 = 0.1$. Each signal is oscillatory where again $T_1 = 1000$ and $T_2 = 1500$. We apply the DCCA method with varying polynomial order, DCCA-$\ell(n)$, of eq. (5), and in fig. 4 find that the DCCA-ℓ curve is a straight line up to a scale of approximately four periods. Note that we use different DFA intercepts of fig. 1(b) since y_i and y'_i are characterized by different ρ values.

Next we analyze air temperature weather data, obtained by the *Mathematica* tool `WeatherData`, in daily intervals between 1 January 1995 and 31 December 2009. We study time series of air temperature at two US airports, LaGuardia Airport (LGA) and John F. Kennedy International Airport (JFK), denoted by $T_{1,i}$ and $T_{2,i}$ respectively (fig. 5(a)). Each time series exhibits apparent periodicity on a yearly basis. In fig. 5(b) we study cross-correlations and find that for time scales up to ≈ 3 years the air temperature data are power-law cross-correlated with exponent ≈ 0.93 (see eq. (5)). Due to the presence of periodicity, in fig. 5(b) the DCCA-$\ell(n)$ curve of eq. (5) exhibits a less pronounced bump for a time scale of ≈ 1 y than it would be using detrending with a linear polynomial. Clearly, air temperature data exhibit not only annual periodicity but also exhibit more complex dynamics. Using the DFA method with a high-order polynomial, DFA-$\ell(n)$ of eq. (3), we also show that $T_{1,i}$ and $T_{2,i}$ each exhibits power-law autocorrelations which is in agreement with a finding that the changes of air temperature data recorded daily are power-law anticorrelated [38]. Note again that DFA with linear polynomial exhibits a more pronounced bump due to presence of periodicity, bending over at ≈ 1 y. How complex is the dynamics of air temperature changes? Reference [36] reports that the changes of air temperature data recorded in 10 min intervals can be approximated by positive power-law autocorrelations. In fig. 5(c) we find that the magnitudes of air temperature differences are power-law cross-correlated with exponent 0.61.

In conclusion, we demonstrate that the crossovers commonly encountered in DCCA and DFA plots —when the detrending polynomial order is low and periodic trends are present— can be subtracted using local detrending with polynomials of large order ℓ. Therefore, to quantify cross-correlations or autocorrelations properly, we must eliminate the trend for both DFA and DCCA by employing detrending with a varying polynomial order ℓ. The more complex the signal trend within a box —*e.g.*, the more periods within a box— the larger is ℓ needed to subtract the trend. In future work we will to employ our method to a diverse range of data, especially in physiology and meteorology where periodicity exists.

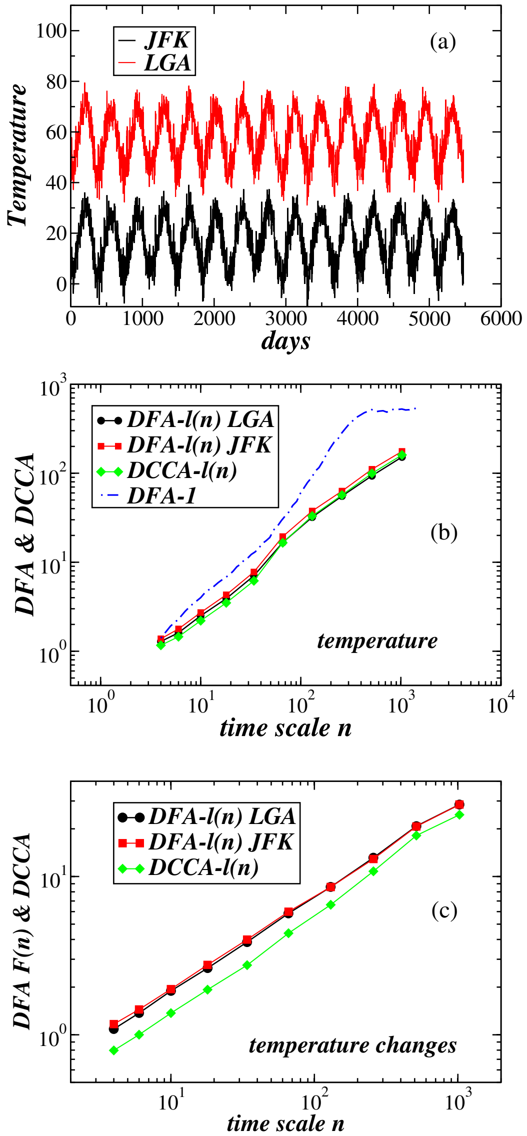

Fig. 5: (Colour on-line) Detrending, with varying polynomial order, for (a) weather data at Kennedy (JFK) and LaGuardia (LGA) airports. (b) DCCA-$\ell(n)$ curve of eq. (5) exhibits a less pronounced bump for a time scale of $\approx 1\,$y than it would be using detrending with a linear polynomial, the DCCA-ℓ. (c) The magnitudes of air temperature differences are power-law cross-correlated with DCCA-$\ell(n)$ exponent 0.61. We show time lags and corresponding polynomial orders in parentheses: 4 (1), 6 (2) 10 (4), 18 (8). For all other time lags we use $\ell = 16$.

Recently, refs. [39,40] proposed that renormalization of trends —that can be considered to be a kind of “detrending”— can be a successful strategy for analyzing financial time series. The authors suggest that the well-known catastrophic bubbles that occur on large time scales —such as the most recent financial crisis— are not outliers but single dramatic representatives caused by the switching between upward and downward trends on time scales varying over nine orders of magnitude from very large ($\approx$ 102 days) down to very small ($\approx 10\,$ms) [39]. We believe it would be very interesting to relate detrending to trending.

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We thank the Ministry of Science of Croatia, and NSF grants PHY-0855453, CHE-0911389, and CHE-0908218 for financial support.

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