J.P.Morgan/Reuters

RiskMetrics™—Technical Document

- J.P. Morgan and Reuters have teamed up to enhance RiskMetrics™. Morgan will continue to be responsible for enhancing the methods outlined in this document, while Reuters will control the production and distribution of the RiskMetrics™ data sets.
- Expanded sections on methodology outline enhanced analytical solutions for dealing with nonlinear options risks and introduce methods on how to account for non-normal distributions.
- Enclosed diskette contains many examples used in this document. It allows readers to experiment with our risk measurement techniques.
- All publications and daily data sets are available free of charge on J.P. Morgan's Web page on the Internet at **http://www.jpmorgan.com/RiskManagement/RiskMetrics/RiskMetrics.html**. This page is accessible directly or through third party services such as CompuServe®, America Online™, or Prodigy®.

Fourth Edition, 1996

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This *Technical Document* provides a detailed description of RiskMetrics™, a set of techniques and data to measure market risks in portfolios of fixed income instruments, equities, foreign exchange, commodities, and their derivatives issued in over 30 countries. This edition has been expanded significantly from the previous release issued in May 1995.

We make this methodology and the corresponding RiskMetrics™ data sets available for three reasons:

1. We are interested in promoting greater transparency of market risks. Transparency is the key to effective risk management.

2. Our aim has been to establish a benchmark for market risk measurement. The absence of a common point of reference for market risks makes it difficult to compare different approaches to and measures of market risks. Risks are comparable only when they are measured with the same yardstick.

3. We intend to provide our clients with sound advice, including advice on managing their market risks. We describe the RiskMetrics™ methodology as an aid to clients in understanding and evaluating that advice.

Both J.P. Morgan and Reuters are committed to further the development of RiskMetrics™ as a fully transparent set of risk measurement methods. We look forward to continued feedback on how to maintain the quality that has made RiskMetrics™ the benchmark for measuring market risk.

RiskMetrics™ is based on, but differs significantly from, the risk measurement methodology developed by J.P. Morgan for the measurement, management, and control of market risks in its trading, arbitrage, and own investment account activities. **We remind our readers that no amount of sophisticated analytics will replace experience and professional judgment in managing risks**. RiskMetrics™ is nothing more than a high-quality tool for the professional risk manager involved in the financial markets and is not a guarantee of specific results.

RiskMetrics™—Technical Document
Fourth Edition (December 1996)

This book

This is the reference document for RiskMetrics™. It covers all aspects of RiskMetrics and supersedes all previous editions of the *Technical Document*. It is meant to serve as a reference to the methodology of statistical estimation of market risk, as well as detailed documentation of the analytics that generate the data sets that are published daily on our Internet Web sites.

This document reviews

1. The conceptual framework underlying the methodologies for estimating market risks.
2. The statistics of financial market returns.
3. How to model financial instrument exposures to a variety of market risk factors.
4. The data sets of statistical measures that we estimate and distribute daily over the Internet and shortly, the Reuters Web.

Measurement and management of market risks continues to be as much a craft as it is a science. It has evolved rapidly over the last 15 years and has continued to evolve since we launched RiskMetrics in October 1994. Dozens of professionals at J.P. Morgan have contributed to the development of this market risk management technology and the latest document contains entries or contributions from a significant number of our market risk professionals.

We have received numerous constructive comments and criticisms from professionals at Central Banks and regulatory bodies in many countries, from our competitors at other financial institutions, from a large number specialists in academia and last, but not least, from our clients. Without their feedback, help, and encouragement to pursue our strategy of open disclosure of methodology and free access to data, we would not have been as successful in advancing this technology as much as we have over the last two years.

What is RiskMetrics?

RiskMetrics is a set of tools that enable participants in the financial markets to estimate their exposure to market risk under what has been called the “Value-at-Risk framework”. RiskMetrics has three basic components:

- A set of market risk measurement methodologies outlined in this document.
- Data sets of volatility and correlation data used in the computation of market risk.
- Software systems developed by J.P.Morgan, subsidiaries of Reuters, and third party vendors that implement the methodologies described herein.

With the help of this document and the associated line of products, users should be in a position to estimate market risks in portfolios of foreign exchange, fixed income, equity and commodity products.

J.P. Morgan and Reuters team up on RiskMetrics

In June 1996, J.P. Morgan signed an agreement with Reuters to cooperate on the building of a new and more powerful version of RiskMetrics. Since the launch of RiskMetrics in October 1994, we have received numerous requests to add new products, instruments, and markets to the daily volatility and correlation data sets. We have also perceived the need in the market for a more flexible VaR data tool than the standard matrices that are currently distributed over the Internet. The new partnership with Reuters, which will be based on the precept that both firms will focus on their respective strengths, will help us achieve these objectives.

Methodology

J.P. Morgan will continue to develop the RiskMetrics set of VaR methodologies and publish them in the quarterly *RiskMetrics Monitor* and in the annual *RiskMetrics—Technical Document*.

RiskMetrics data sets

Reuters will take over the responsibility for data sourcing as well as production and delivery of the risk data sets. The current RiskMetrics data sets will continue to be available on the Internet free of charge and will be further improved as a benchmark tool designed to broaden the understanding of the principles of market risk measurement.

When J.P. Morgan first launched RiskMetrics in October 1994, the objective was to go for broad market coverage initially, and follow up with more granularity in terms of the markets and instruments covered. This over time, would reduce the need for proxies and would provide additional data to measure more accurately the risk associated with non-linear instruments.

The partnership will address these new markets and products and will also introduce a new customizable service, which will be available over the Reuters Web service. The customizable RiskMetrics approach will give risk managers the ability to scale data to meet the needs of their individual trading profiles. Its capabilities will range from providing customized covariance matrices needed to run VaR calculations, to supplying data for historical simulation and stress-testing scenarios.

More details on these plans will be discussed in later editions of the *RiskMetrics Monitor.*

Systems

Both J.P. Morgan and Reuters, through its Sailfish subsidiary, have developed client-site RiskMetrics VaR applications. These products, together with the expanding suite of third party applications will continue to provide RiskMetrics implementations.

What is new in this fourth edition?

In terms of content, the Fourth Edition of the *Technical Document* incorporates the changes and refinements to the methodology that were initially outlined in the 1995–1996 editions of the *RiskMetrics Monitor*:

- **Expanded framework:** We have worked extensively on refining the analytical framework for analyzing options risk without having to perform relatively time consuming simulations and have outlined the basis for an improved methodology which incorporates better information on the tails of distributions related to financial asset price returns; we've also developed a data synchronization algorithm to refine our volatility and correlation estimates for products which do not trade in the same time zone;

- **New markets:** We expanded the daily data sets to include estimated volatilities and correlations of additional foreign exchange, fixed income and equity markets, particularly in South East Asia and Latin America.

- **Fine-tuned methodology:** We have modified the approach in a number of ways. First, we've changed our definition of price volatility which is now based on a total return concept; we've also revised some of the algorithms used in our mapping routines and are in the process of redefining the techniques used in estimating equity portfolio risk.

- **RiskMetrics products:** While we have continued to expand the list of third parties providing RiskMetrics products and support, this is no longer included with this document. Given the rapid pace of change in the availability of risk management software products, readers are advised to consult our Internet web site for the latest available list of products. This list, which now includes FourFifteen™, J.P. Morgan's own VaR calculator and report generating software, continues to grow, attesting to the broad acceptance RiskMetrics has achieved.

- **New tools to use the RiskMetrics data sets:** We have published an Excel add-in function which enables users to import volatilities and correlations directly into a spreadsheet. This tool is available from our Internet web site.

The structure of the document has changed only slightly. As before, its size warrants the following note: One need not read and understand the entire document in order to benefit from RiskMetrics. The document is organized in parts that address subjects of particular interest to many readers.

Part I: Risk Measurement Framework

This part is for the general practitioner. It provides a practical framework on how to think about market risks, how to apply that thinking in practice, and how to interpret the results. It reviews the different approaches to risk estimation, shows how the calculations work on simple examples and discusses how the results can be used in limit management, performance evaluation, and capital allocation.

Part II: Statistics of Financial Market Returns

This part requires an understanding and interest in statistical analysis. It reviews the assumptions behind the statistics used to describe financial market returns and how distributions of future returns can be estimated.

Part III: Risk Modeling of Financial Instruments

This part is required reading for implementation of a market risk measurement system. It reviews how positions in any asset class can be described in a standardized fashion (foreign exchange, interest rates, equities, and commodities). Special attention is given to derivatives positions. The purpose is to demystify derivatives in order to show that their market risks can be measured in the same fashion as their underlying.

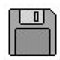
Part IV: RiskMetrics Data Sets

This part should be of interest to users of the RiskMetrics data sets. First it describes the sources of all daily price and rate data. It then discusses the attributes of each volatility and correlation series in the RiskMetrics data sets. And last, it provides detailed format descriptions required to decipher the data sets that can be downloaded from public or commercial sources.

Appendices

This part reviews some of the more technical issues surrounding methodology and regulatory requirements for market risk capital in banks and demonstrates the use of RiskMetrics with the example diskette provided with this document. Finally, Appendix H shows you how to access the RiskMetrics data sets from the Internet.

RiskMetrics examples diskette



This diskette is located inside the back cover. It contains an Excel workbook that includes some of the examples shown in this document. Such examples are identified by the icon shown here.

Future plans

We expect to update this *Technical Document* annually as we adapt our market risk standards to further improve the techniques and data to meet the changing needs of our clients.

RiskMetrics is a now an integral part of J.P. Morgan's Risk Management Services group which provides advisory services to a wide variety of the firm's clients. We continue to welcome any suggestions to enhance the methodology and adapt it further to the needs of the market. All suggestions, requests and inquiries should be directed to the authors of this publication or to your local RiskMetrics contacts listed on the back cover.

Acknowledgments

The authors would like to thank the numerous individuals who participated in the writing and editing of this document, particularly Chris Finger and Chris Athaide from J.P. Morgan's risk management research group, and Elizabeth Frederick and John Matero from our risk advisory practice. Finally, this document could not have been produced without the contributions of our consulting editor, Tatiana Kolubayev. We apologize for any omissions to this list.

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Backtesting

Chapter 11. Performance assessment

Chapter 11. Performance assessment

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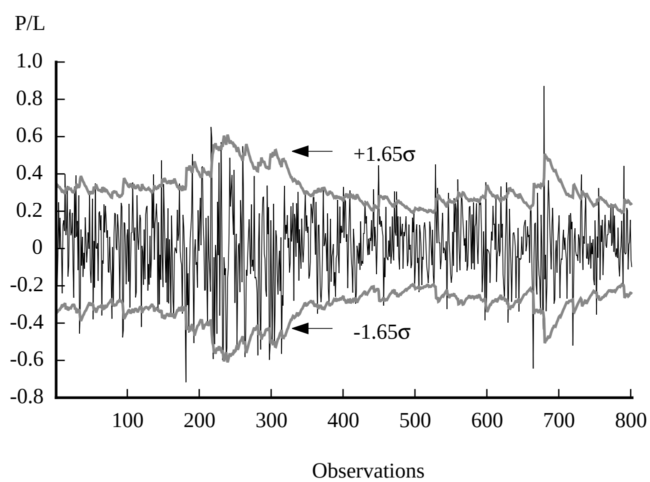
In this chapter we present a process for assessing the accuracy of the RiskMetrics model. We would like to make clear that the purpose of this section is not to offer a review of the quantitative measures for VaR model comparison. There is a growing literature on such measures and we refer the reader to Crnkovic and Drachman (1996) for the latest developments in that area. Instead, we present simple calculations that may prove useful for determining the appropriateness of the RiskMetrics model.

11.1 Sample portfolio

We describe an approach for assessing the RiskMetrics model by analyzing a portfolio consisting of 215 cashflows that include foreign exchange (22), money market deposits (22), zero coupon government bonds (121), equities (12) and commodities (33). Using daily prices for the period April 4, 1990 through March 26, 1996 (a total of 1001 observations), we construct 1-day VaR forecasts over the most recent 801 days of the sample period. We then compare these forecasts to their respective realized profit/loss (P/L) which are represented by 1-day returns.

Chart 11.1 shows the typical presentation of 1-day RiskMetrics VaR forecasts (90% two-tail confidence interval) along with the daily P/L of the portfolio.

Chart 11.1
One-day Profit/Loss and VaR estimates
VaR bands are given by +/− 1.65σ



In Chart 11.1 the black line represents the portfolio return $r_{p,t}$ constructed from the 215 individual returns at time *t*. The time *t* portfolio return is defined as follows:

[11.1] $$r_{p,t} = \sum_{i=1}^{215} \left(\frac{1}{215}\right) r_{i,t}$$

where $r_{i,t}$ represents the log return of the *i*th underlying cashflow. The Value-at-Risk bands are based on the portfolio's standard deviation. The formula for the portfolio's standard deviation, $\sigma_{p,t|t-1}$ is:

$$[11.2]\qquad \sigma_{P,\,t|t-1} = \sqrt{\sum_{i=1}^{215}\left(\frac{1}{215}\right)^2\sigma^2_{i,\,t|t-1} + 2\sum_{i=1}^{215}\sum_{j>i}\left(\frac{1}{215}\right)^2\rho_{ij,\,t|t-1}\sigma_{i,\,t|t-1}\sigma_{j,\,t|t-1}}$$

where $\sigma^2_{i,\,t|t-1}$ is the variance of the *i*th return series made for time *t* and $\rho_{ij,\,t|t-1}$ is the correlation between the *i*th and *j*th returns for time *t*.

11.2 Assessing the RiskMetrics model

The first measure of model performance is a simple count the number of times that the VaR estimates “underpredict” future losses (gains). Recall that in RiskMetrics each day it is assumed that there is a 5% chance that the observed loss exceeds the VaR forecast.[1] For the sake of generality, let’s define a random variable $X(t)$ on any day *t* such that $X(t) = 1$ if a particular day’s observed loss is greater than its corresponding VaR forecast and $X(t)=0$ otherwise. We can write the distribution of $X(t)$ as follows

$$[11.3]\qquad f(X(t)\,|\,0.05) = \begin{cases} 0.05^{X(t)}\,(1-0.05)^{1-X(t)} & \text{X(t)=0,1} \\ 0 & \text{otherwise} \end{cases}$$

Now, suppose we observe $X(t)$ for a total of *T* days, $t = 1,2,...,T$, and we assume that the $X(t)$’s are independent over time. In other words, whether a VaR forecast is violated on a particular day is independent of what happened on other days. The random variable $X(t)$ is said to follow a Bernoulli distribution whose expected value is 0.05.The total number of VaR violations over the time period *T* is given by

$$[11.4]\qquad X_T = \sum_{t=1}^{T} X(t)$$

The expected value of X_T, i.e., the expected number of VaR violations over *T* days, is *T* times 0.05. For example, if we observe $T = 20$ days of VaR forecasts, then the expected number of VaR violations is 20 x 0.05 = 1; hence one would expect to observe one VaR violation every 20 days. What is convenient about modelling VaR violations according to Eq. [11.3] is that the probability of observing a VaR violation over *T* days is same as the probability of observing a VaR violation at any point in time, *t*. Therefore, we are able to use VaR forecasts constructed over time to assess the appropriateness of the RiskMetrics model for this portfolio of 215 cashflows.

Table 11.1 reports the observed percent of VaR violations for the upper and lower tails of our sample portfolio. For each day the lower and upper VaR limits are defined as $-1.65\sigma_{t|t-1}$ and $1.65\sigma_{t|t-1}$, respectively.

Table 11.1
Realized percentages of VaR violations
True probability of VaR violations = 5%

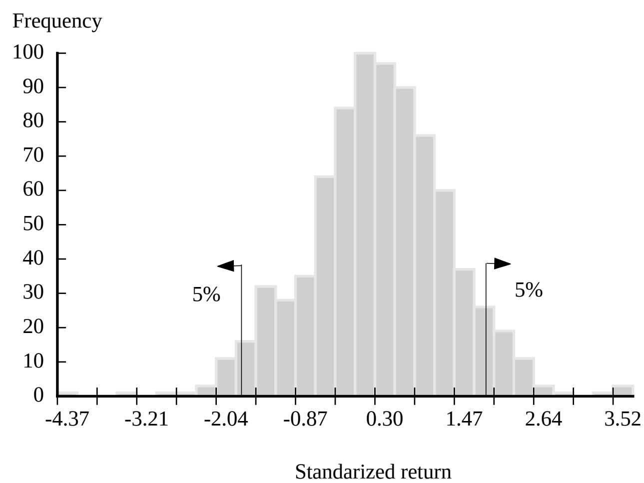
Prob (Loss < $-1.65\sigma_{t|t-1}$)	Prob (Profit > $1.65\sigma_{t|t-1}$)
5.74%	5.87%

A more straightforward approach to derive the preceding results is to apply the maintained assumptions of the RiskMetrics model. Recall that it is assumed that the return distribution of simple portfolios (i.e., those without nonlinear risk) is conditionally normal. In other words, the real-

[1] The focus of this section is on losses. However, the following methodology can also apply to gains.

ized return (P/L) divided by the standard deviation forecast used to construct the VaR estimate is assumed to be normally distributed with mean 0 and variance 1. Chart 11.2 presents a histogram of standardized portfolio returns. We place arrow bars to signify the area where we expect to observe 5% of the observations.

Chart 11.2
Histogram of standardized returns $(r_t/\sigma_{t|t-1})$
Probability that $(r_t/\sigma_{t|t-1})$ *< (>)–1.65 (1.65) = 5%*



A priori, the RiskMetrics model predicts that 5% of the standardized returns fall below (above) –1.65 (1.65). In addition to this prediction, it is possible to derive the expected value (average) of a return **given** that return violates a VaR forecast. For the lower tail, this expected value is defined as follows:

$$[11.5] \qquad E\left[(r_t/\sigma_{t|t-1}) \,\middle|\, (r_t/\sigma_{t|t-1}) < -1.65 \right] = -\left(\frac{\phi(-1.65)}{\Phi(-1.65)} \right) = -2.63$$

where

$\phi(-1.65)$ = the standard normal density function evaluated at -1.65
$\Phi(-1.65)$ = the standard normal distribution function evaluated at -1.65

It follows from the symmetry of the normal density function that the expected value for upper-tail returns is $E\left[(r_t/\sigma_{t|t-1}) \,\middle|\, (r_t/\sigma_{t|t-1}) > 1.65\sigma_{t|t-1} \right] = 2.63$.

Table 11.2 reports these realized expected values for our sample portfolio.

Table 11.2
Realized “tail return” averages
Conditional mean tail forecasts of standardized returns

$E[(r_t/\sigma_{t\|t-1}) \,\|\, ((r_t/\sigma_{t\|t-1}) < -1.65)] = -2.63$	$E[(r_t/\sigma_{t\|t-1}) \,\|\, (r_t/\sigma_{t\|t-1}) > 1.65] = 2.63$
–1.741	1.828

To get a better understanding of the size of the returns that violate the VaR forecasts, Charts 11.3 and 11.4 plot the observed standardized returns (black circles) that fall in the lower (< −1.65) and upper (> 1.65) tails of the standard normal distribution. The horizontal line in each chart represents the average value predicted by the conditional normal distribution.

Chart 11.3
Standardized lower-tail returns
$r_t / \sigma_{t|t-1} < -1.65$

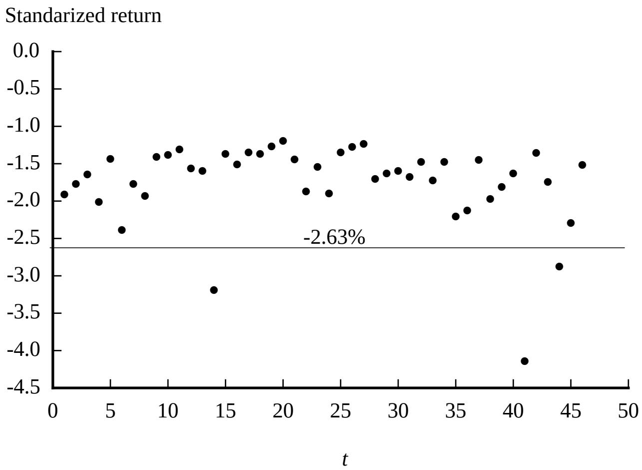
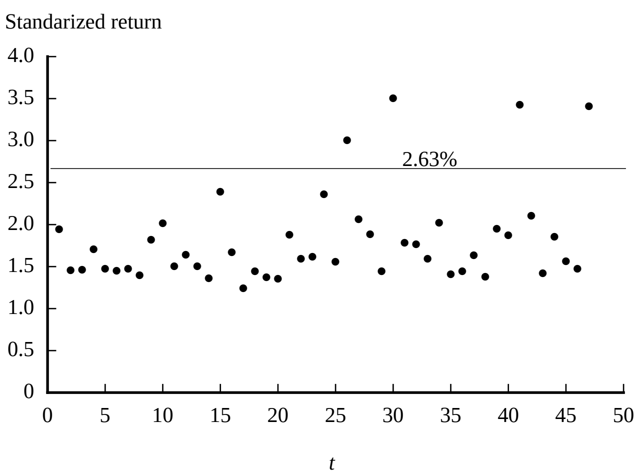


Chart 11.4
Standardized upper-tail returns
$r_t / \sigma_{t|t-1} > 1.65$



Both charts show that the returns that violate the VaR forecasts rarely exceed the expected value predicted by the normal distribution. In fact, we observe about 3 violations out of (approximately) 46/47 tail returns for the upper/lower tails. This is approximately 6.5% of the observations that fall in a particular tail. Note that the normal probability model prediction is 8.5%.[2]

[2] We derive this number from Prob ($X < -2.63$ | $X < -1.65$) = Prob ($X < -2.63$) / Prob ($X < -1.65$).

11.3 Summary

In this chapter we presented a brief process by which risk managers may assess the performance of the RiskMetrics model. We applied these statistics to a sample portfolio that consists of 215 cashflows covering foreign exchange, fixed income, commodities and equities. Specifically, 1-day VaR forecasts were constructed for an 801-day sample period and for each day the forecast was measured against the portfolio's realized P/L. It was found that overall the RiskMetrics model performs reasonably well.

Appendices

Appendix A. Tests of conditional normality

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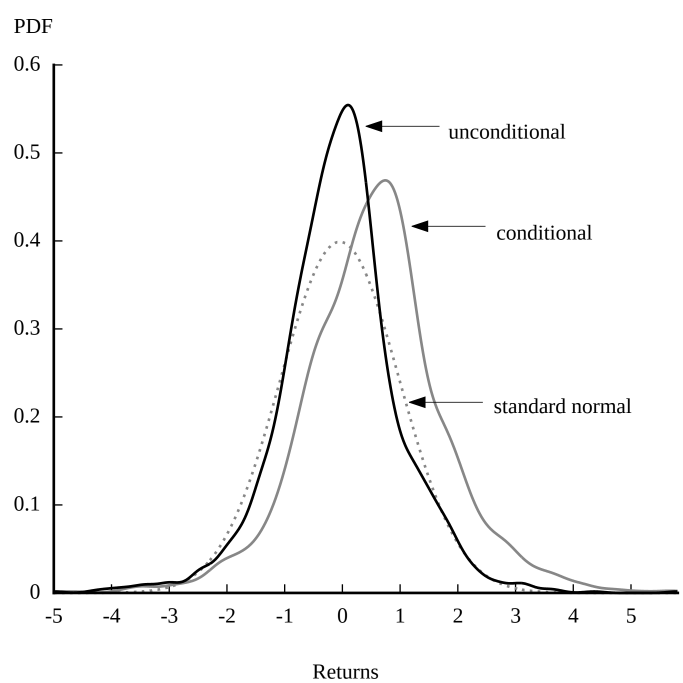
A fundamental assumption in RiskMetrics is that the underlying returns on financial prices are distributed according to the conditional normal distribution. The main implication of this assumption is that while the return distribution at each point in time is normally distributed, the return distribution taken over the entire sample period is **not necessarily** normal. Alternatively expressed, the standardized distribution rather than the observed return is assumed to be normal.

Chart A.1 shows the nontrivial consequence of the conditional normality assumption. The unconditional distribution represents an estimate of the histogram of USD/DEM log price changes that are standardized by the standard deviation taken over the entire sample (i.e., they are standardized by the unconditional standard deviation). As mentioned above, relative to the normal distribution with a constant mean and variance, this series has the typical thin waist, fat tail features. The unconditional distribution represents the distribution of standardized returns which are constructed by dividing each historical return by its corresponding standard deviation forecast[1], i.e., divide every return, r_t, by its standard deviation forecast, $\sigma_{t|t-1}$ (i.e., conditional standard deviation).

Chart A.1
Standard normal distribution and histogram of returns on USD/DEM



The difference between these two lines underscores the importance of distinguishing between conditional and unconditional normality.

1 The exact construction of this forecast is presented in Chapter 5.

A.1 Numerical methods

We now present some computational tools used to test for normality. We begin by showing how to obtain sample estimates of the two parameters that describe the normal distribution. For a set of returns, r_t, where $t = 1,2\ldots,T$, we obtain estimates of the unconditional mean, $\bar{r}$, and standard deviation, $\hat{\sigma}$, via the following estimators:

[A.1] $$\bar{r} = \frac{1}{T}\sum_{t=1}^{T} r_t$$

[A.2] $$\hat{\sigma} = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(r_t - \bar{r})^2}$$

Table A.1 presents sample estimates of the mean and standard deviation for the change series presented in Table 4.1.

Table A.1
Sample mean and standard deviation estimates for USD/DEM FX

Parameter estimates	Absolute price change	Relative price change	Log price change
$\bar{r}$, mean (%)	–0.060	–0.089	–0.090
$\hat{\sigma}$, standard deviation, (%)	0.28	0.42	0.42

Several popular tests for normality focus on measuring **skewness** and **kurtosis**. Skewness characterizes the asymmetry of a distribution around its mean. Positive skewness indicates an asymmetric tail extending toward positive values (right skewed). Negative skewness implies asymmetry toward negative values (left skewed). A simple measure of skewness, the coefficient of skewness, $\hat{\gamma}$, is given by

[A.3] $$\hat{\gamma} = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_t - \bar{r}}{\hat{\sigma}}\right)^3$$

Computed values of skewness away from 0 point towards non-normality. Kurtosis characterizes the relative peakedness or flatness of a given distribution compared to a normal distribution. The standardized measure of kurtosis, the coefficient of kurtosis, $\hat{\kappa}$, is given by

[A.4] $$\hat{\kappa} = \left\{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_t - \bar{r}}{\hat{\sigma}}\right)^4\right\}$$

The kurtosis for the normal distribution is 3. Often, instead of kurtosis, researchers talk about excess kurtosis which is defined as kurtosis minus 3 so that in a normal distribution excess kurtosis is zero. Distributions with an excess kurtosis value greater than 0 are frequently referred to as having fat tails.

One popular test for normality that is based on skewness and kurtosis is presented in Kiefer and Salmon (1983). Shapiro and Wilk (1965) and Bera and Jarcque (1980) offer more computationally intensive tests. To give some idea about the values of the mean, standard deviation, skewness and kurtosis coefficients that are observed in practice, Table A.2 on page 230 presents estimates of these statistics as well as two other measures—tail probability and tail values, to 48 foreign exchange series. For each of the 48 time series we used 86 historical weekly prices for the period July 1, 1994 through March 1, 1996. (Note that many of the time series presented in Table A.2 are not part of the RiskMetrics data set). Each return used in the analysis is standardized by its corresponding 1-week standard deviation forecast. Interpretations of each of the estimated statistics are provided in the table footnotes.

When large data samples are available, specific statistics can be constructed to test whether a given sample is skewed or has excess kurtosis. This allows for formal hypothesis testing. The large sample skewness and kurtosis measures and their distributions are given below:

[A.5] Skewness measure $$\sqrt{T}\gamma \equiv \sqrt{T}\frac{\frac{1}{T}\sum_{t=1}^{T}(r_t - \bar{r})^3}{\left[\frac{1}{T}\sum_{t=1}^{T}(r_t - \bar{r})^2\right]^{\frac{3}{2}}} \sim N(0, 6)$$

[A.6] Kurtosis measure $$\sqrt{T}\kappa \equiv \sqrt{T}\left\{\frac{\frac{1}{T}\sum_{t=1}^{T}(r_t - \bar{r})^4}{\left[\frac{1}{T}\sum_{t=1}^{T}(r_t - \bar{r})^2\right]^2} - 3\right\} \sim N(0, 24)$$

Table A.2
Testing for univariate conditional normality[1]
normalized return series; 85 total observations

	Skewness[2]	Kurtosis[3]	Mean[4]	Std. Dev.[5]	Tail Probability (%)[8] < −1.65	Tail Probability (%)[8] > 1.65	Tail value[9] < −1.65	Tail value[9] > 1.65
Normal	**0.000**	**0.000**	**-**	**1.000**	**5.000**	**5.000**	**−2.067**	**2.067**
OECD								
Australia	0.314	3.397	0.120	0.943	2.900	5.700	−2.586	2.306
Austria	0.369	0.673	−0.085	1.037	8.600	5.700	−1.975	2.499
Belgium	0.157	2.961	−0.089	0.866	8.600	2.900	−1.859	2.493
Denmark	0.650	4.399	−0.077	0.903	11.400	2.900	−1.915	2.576
France	0.068	3.557	−0.063	0.969	8.600	2.900	−2.140	2.852
Germany	0.096	4.453	−0.085	0.872	5.700	2.900	−1.821	2.703
Greece	0.098	2.259	−0.154	0.943	11.400	2.900	−1.971	2.658
Holland	0.067	4.567	−0.086	0.865	5.700	2.900	−1.834	2.671
Italy	0.480	0.019	0.101	0.763	0	2.900	0	1.853
New Zealand	1.746	7.829	0.068	1.075	2.900	2.900	−2.739	3.633
Portugal	1.747	0.533	−0.062	0.889	11.400	2.900	−1.909	2.188
Spain	6.995	1.680	−0.044	0.957	8.600	2.900	−2.293	1.845
Turkey	30.566	118.749	−0.761	1.162	11.400	0	−2.944	0
UK	7.035	2.762	−0.137	0.955	8.600	2.900	−2.516	1.811
Switzerland	0.009	0.001	−0.001	0.995	2.900	5.700	−2.415	2.110
Latin Amer. Econ. System								
Brazil	0.880	1.549	−0.224	0.282	0	0	0	0
Chile	1.049	0.512	−0.291	0.904	8.600	0	−2.057	0
Colombia	2.010	4.231	−0.536	1.289	11.400	2.900	−3.305	2.958
Costa Rica	0.093	33.360	−0.865	0.425	5.700	0	−2.011	0
Dominican Rep	0.026	41.011	0.050	1.183	5.700	5.700	−3.053	3.013
El Salvador	2.708	49.717	0.014	0.504	0	2.900	0	1.776
Equador	0.002	50.097	0.085	1.162	5.700	5.700	−3.053	3.013
Guatemala	0.026	1.946	−0.280	1.036	8.600	5.700	−2.365	2.237
Honduras	42.420	77.277	−0.575	1.415	14.300	0	−3.529	0
Jamaica	81.596	451.212	−0.301	1.137	2.900	2.900	−6.163	1.869
Mexico	13.71	30.237	−0.158	0.597	2.900	0	−2.500	0
Nicaragua	0.051	2.847	−0.508	0.117	0	0	0	0
Peru	122.807	672.453	−0.278	1.365	5.700	0	−5.069	0
Trinidad	0.813	0.339	0.146	1.063	8.600	11.400	−2.171	1.915
Uruguay	0.724	0.106	−0.625	0.371	0	0	0	0

Table A.2 (continued)
Testing for univariate conditional normality[1]
normalized return series; 85 total observations

	Skewness[2]	Kurtosis[3]	Mean[4]	Std. Dev.[5]	Tail Probability (%)[8] < –1.65	Tail Probability (%)[8] > 1.65	Tail value[9] < –1.65	Tail value[9] > 1.65
ASEAN								
Malaysia	1.495	0.265	–0.318	0.926	8.600	0	–2.366	0
Philippines	1.654	0.494	–0.082	0.393	0	0	0	0
Thailand	0.077	0.069	–0.269	0.936	8.600	2.900	–2.184	1.955
Fiji	4.073	6.471	–0.129	0.868	2.900	2.900	–3.102	1.737
Hong Kong	5.360	29.084	0.032	1.001	5.700	5.700	–2.233	2.726
Reunion Island	0.068	3.558	–0.063	0.969	8.600	2.900	–2.140	2.853
Southern African Dev. Comm.								
Malawi	0.157	9.454	–0.001	0.250	0	0	0	0
South Africa	34.464	58.844	–0.333	1.555	8.600	0	–4.480	0
Zambia	22.686	39.073	–0.007	0.011	0	0	0	0
Zimbabwe	20.831	29.234	–0.487	0.762	5.700	0	–2.682	0
Ivory Coast	0.068	3.564	–0.064	0.970	8.600	2.900	–2.144	2.857
Uganda	40.815	80.115	–0.203	1.399	8.600	2.900	–4.092	1.953
Others								
China	80.314	567.012	0.107	1.521	2.900	2.900	–3.616	8.092
Czech Repub	0.167	12.516	–0.108	0.824	5.700	2.900	–2.088	2.619
Hungary	1.961	0.006	–0.342	0.741	5.700	0	–2.135	0
India	5.633	3.622	–0.462	1.336	17.100	5.700	–2.715	1.980
Romania	89.973	452.501	–1.249	1.721	14.300	0	–4.078	0
Russia	0.248	2.819	–0.120	0.369	0	0	0	0

1 Countries are grouped by major economic groupings as defined in *Political Handbook of the World: 1995–1996*. New York: CSA Publishing, State University of New York, 1996. Countries not formally part of an economic group are listed in their respective geographic areas.

2 If returns are conditionally normal, the skewness value is zero.

3 If returns are conditionally normal, the excess kurtosis value is zero.

4 Sample mean of the return series.

5 Sample standard deviation of the normalized return series.

8 Tail probabilities give the observed probabilities of normalized returns falling below −1.65 and above +1.65. Under conditional normality, these values are 5%.

9 Tail values give the observed average value of normalized returns falling below −1.65 and above +1.65. Under conditional normality, these values are −2.067 and +2.067, respectively.

A.2 Graphical methods

Q-Q (quantile-quantile) charts offer a visual assessment of the deviations from normality. Recall that the qth quantile is the number that exceeds q percent of the observations. A Q-Q chart plots the quantiles of the standardized distribution of observed returns (observed quantiles) against the quantiles of the standard normal distribution (normal quantiles). Consider the sample of observed returns, $r_t, t = 1, \ldots, T$. Denote the jth observed quantile by q_j so that for all T observed quantiles we have

[A.7] $$\text{Probability}\ (\tilde{r}_t < q_j) \cong p_j$$

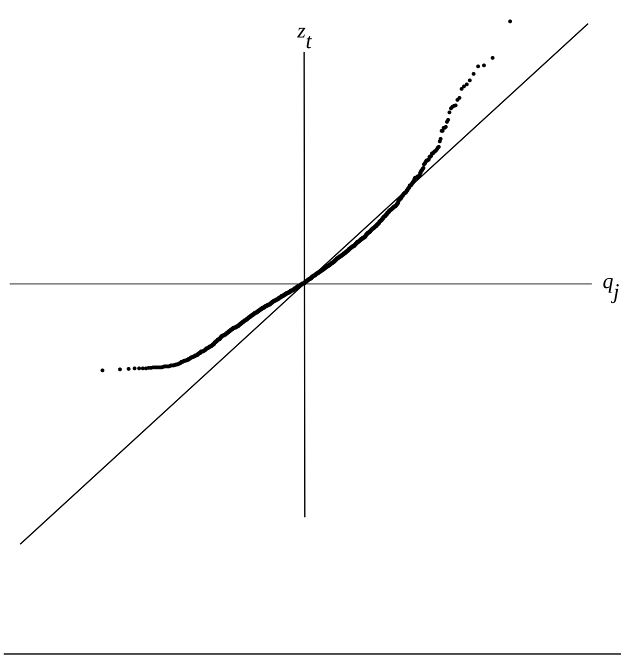
where $p_j = \dfrac{j - 0.5}{T}$

Denote the jth standard normal quantile by z_j for $j = 1\ldots,T$. For example, if $T = 100$, then $z_5 = -1.645$. In practice, the five steps to compute the Q-Q plot are given below:[2]

1. Standardize the daily returns by their corresponding standard deviation forecast, i.e., compute $\tilde{r}_t$ from r_t for $t = 1,\ldots,T$.
2. Order $\tilde{r}_t$ and compute their percentiles q_j, $j = 1,\ldots,T$.
3. Calculate the probabilities p_j corresponding to each q_j.
4. Calculate the standard normal quantiles, z_j that correspond to each p_j.
5. Plot the pairs $(z_1, q_1), (z_2, q_2), \ldots (z_T, q_T)$.

Chart A.2 shows an example of a Q-Q plot for USD/DEM daily standardized returns for the period January 1988 through September 1996.

Chart A.2
Quantile-quantile plot of USD/DEM
standardized returns



[2] For a complete description of this test see Johnson and Wichern (1992, pp. 153-158).

The straighter the plot, the closer the distribution of returns is to a normal distribution. If all points were to lie on a straight line, then the distribution of returns would be normal. As the chart above shows, there is some deviation from normality in the distribution of daily returns of USD/DEM over the last 7 years.

A good way to measure how much deviation from normality occurs is to calculate the correlation coefficient of the Q-Q plot,

[A.8] $$\rho_Q = \frac{\sum_{j=1}^{T} (q_j - \bar{q})(z_j - \bar{z})}{\sqrt{\sum_{j=1}^{T} (q_j - \bar{q})^2} \sqrt{\sum_{j=1}^{T} (z_j - \bar{z})^2}}$$

For large sample sizes as in the USD/DEM example, ρ_Q needs to be at least 0.999 to pass a test of normality at the 5% significant.[3] In this example, $\rho_Q = 0.987$. The returns are not normal according to this test.

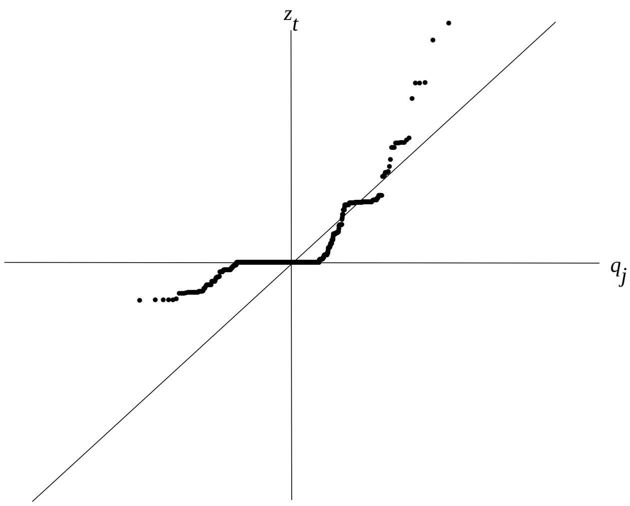
Used across asset classes, ρ_Q can provide useful information as to how good the univariate normality assumption approximates reality. In the example above, while the returns on the USD/DEM exchange rate are not normal, their deviation is slight.

Deviations from normality can be much more significant among other time series, especially money market rates. This is intuitively easy to understand. Short-term interest rates move in a discretionary fashion as a result of actions by central banks. Countries with exchange rate policies that have deviated significantly from economic fundamentals for some period often show money market rate distributions that are clearly not normal. As a result they either change very little when monetary policy remains unchanged (most of the time), or more significantly when central banks change policy, or the markets force them to do so. Therefore, the shape of the distribution results from discrete “jumps” in the underlying returns.

A typical example of this phenomenon can be seen from the Q-Q chart of standardized price returns on the 3-month sterling over the period 3-Jan-91 to 1-Sep-94. The ρ_Q calculated for that particular series is 0.907.

[3] See Johnson and Wichern (1992, p 158) for a table of critical values required to perform this test.

Chart A.3
Quantile-quantile plot of 3-month sterling
standardized returns



The Q-Q charts are useful because they allow the researcher a visual depiction of departures from normality. However, as stated before, there are several other tests for normality. It is important to remember that when applied directly to financial returns, conventional tests of normality should be used with caution. A reason is that the assumptions that underlie these tests (e.g., constant variance, nonautocorrelated returns) are often violated. For example, if a test for normality assumes that the data is not autocorrelated over the sample period when, in fact, the data are autocorrelated, then the test may incorrectly lead one to reject normality (Heuts and Rens, 1986).

The tests presented above are tests for univariate normality and not multivariate normality. In finance, tests of multivariate normality are often most relevant since the focus is on the return distribution of a portfolio that consists of a number of underlying securities. If each return series in a portfolio is found to be univariate normal, then the set of returns taken as a whole are still not necessarily multivariate normal. Conversely, if any one return series is found not to be univariate normal then multivariate normality can be ruled out. Recently, Richardson and Smith (1993) propose a direct test for multivariate normality in stock returns. Also, Looney (1995) describes test for univariate normality that can be used to determine to whether a data sample is multivariate normality.

Appendix B. Relaxing the assumption of conditional normality

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Since its release in October 1994, RiskMetrics has inspired an important discussion on VaR methodologies. A focal point of this discussion has been the assumption that returns follow a conditional normal distribution. Since the distributions of many observed financial return series have tails that are "fatter" than those implied by conditional normality, risk managers may underestimate the risk of their positions if they assume returns follow a conditional normal distribution. In other words, large financial returns are observed to occur more frequently than predicted by the conditional normal distribution. Therefore, it is important to be able to modify the current RiskMetrics model to account for the possibility of such large returns.

The purpose of this appendix is to describe two probability distributions that allow for a more realistic model of financial return tail distributions. It is organized as follows:

- Section B.1 reviews the fundamental assumptions behind the current RiskMetrics calculations, in particular, the assumption that returns follow a conditional normal distribution.

- Section B.2 presents the RiskMetrics model of returns under the assumption that the returns are conditionally normally distributed and two alternative models (distributions) where the probability of observing a return far away from the mean is relatively larger than the probability implied by the conditional normal distribution.

- Section B.3 explains how we estimate each of the three models and then presents results on forecasting the 1st and 99th percentiles of 15 return series representing 9 emerging markets.

B.1 A review of the implications of the conditional normality assumption

In a normal market environment RiskMetrics VaR forecasts are given by the bands of a confidence interval that is symmetric around zero. These bands represent the maximum change in the value of a portfolio with a specified level of probability. For example, the VaR bands associated with a 90% confidence interval are given by $\{-1.65\sigma_p, 1.65\sigma_p\}$ where –/+1.65 are the 5th/95th percentiles of the standardized normal distribution, and σ_p is the portfolio standard deviation which may depend on correlations between returns on individual instruments. The scale factors –/+ 1.65 result from the assumption that standardized returns (i.e., a mean centered return divided by its standard deviation) are normally distributed. When this assumption is true we expect 5% of the (standardized) realized returns to lie below –1.65 and 5% to lie above +1.65.

Often, whether complying with regulatory requirements or internal policy, risk managers compute VaR at different probability levels such as 95% and 98%. Under the assumption that returns are conditionally normal, the scale factors associated with these confidence intervals are –/+1.96 and –/+2.33, respectively. It is our experience that while RiskMetrics VaR estimates provide reasonable results for the 90% confidence interval, the methodology does not do as well at the 95% and 98% confidence levels.[1] Therefore, our goal is to extend the RiskMetrics model to provide better VaR estimates at these larger confidence levels.

Before we can build on the current RiskMetrics methodology, it is important to understand exactly what RiskMetrics assumes about the distribution of financial returns. RiskMetrics assumes that returns follow a conditional normal distribution. This means that while returns themselves are not normal, returns divided by their respective forecasted standard deviations are normally distributed with mean 0 and variance 1. For example, let r_t, denote the time t return, i.e., the return on an asset over a one-day period. Further, let σ_t denote the forecast of the standard deviation of returns for

[1] See Darryl Hendricks, "Evaluation of Value-at-Risk Models Using Historical Data," *FRBNY Economic Policy Review*, April, 1996.

time t based on historical data. It then follows from our assumptions that while r_t is not necessarily normal, the standardized return, r_t/σ_t, is normally distributed.

To summarize, RiskMetrics assumes that financial returns divided by their respective volatility forecasts are normally distributed with mean 0 and variance 1. This assumption is crucial because it recognizes that volatility changes over time.

B.2 Three models to produce daily VaR forecasts

In this section we present three models to forecast the distribution of one-day returns from which a VaR estimate will be derived.

- The first model that is discussed is referred to as standard RiskMetrics. This model is the basis for VaR calculations that are presented in the current *RiskMetrics—Technical Document*.

- The second model that we analyze was introduced in the 2nd quarter 1996 *RiskMetrics Monitor.* It is referred to in this appendix as the normal mixture model. The name "normal mixture" refers to the idea that returns are assumed to be generated from a mixture of two different normal distributions. Each day's return is assumed to be a draw from one of the two normal distributions with a particular probability.

- The third, and most sophisticated model that we present is known as RiskMetrics-GED. This model is the same as standard RiskMetrics except the returns in this model are assumed to follow a conditional generalized error distribution (GED). The GED is a very flexible distribution in that it can take on various shapes, including the normal distribution.

B.2.1 Standard RiskMetrics

The standard RiskMetrics model assumes that returns are generated as follows

[B.1]
$$r_t = \sigma_t \varepsilon_t$$
$$\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1-\lambda) r_{t-1}^2$$

where

ε_t is a normally distributed random variable with mean 0 and variance 1

σ_t and $\left(\sigma_t^2\right)$, respectively, are the time *t* standard deviation and variance of returns (r_t)

λ is a parameter (decay factor) that regulates the weighting on past variances. For one-day variance forecasts, RiskMetrics sets λ =0.94.

In summary, the standard RiskMetrics model assumes that returns follow a conditional normal distribution—conditional on the standard deviation—where the variance of returns is a function of the previous day's variance forecast and squared return.

B.2.2 Normal mixture

In the second quarter 1996 *RiskMetrics Monitor* we introduced the normal mixture model of returns that was found to more effectively measure the tails of selected return distributions. In essence, this model allows for a larger probability of observing very large returns (positive or negative) than the conditional normal distribution.

The normal mixture model assumes that returns are generated as follows

[B.2] $$r_t = \sigma_{1,t} \cdot \varepsilon_{1,t} + \sigma_{1,t} \cdot \delta_t \cdot \varepsilon_{2,t}$$

where

r_t is the time t continuously compounded return

$\varepsilon_{1,t}$ is a normally distributed random variable with mean 0 and variance 1

$\varepsilon_{2,t}$ is a normally distributed random variable with mean $\mu_{2,t}$ and variance $\sigma^2_{2,t}$

δ_t is a 0/1 variable that takes the value 1 with probability *p* and 0 with probability 1–*p*

$\sigma_{1,t}$ is the standard deviation given in the RiskMetrics model

Alternatively stated, the normal mixture model assumes that daily returns standardized by the RiskMetrics volatility forecasts, $\tilde{r}_t$, are generated according to the model

[B.3] $$\tilde{r}_t = \varepsilon_{1,t} + \delta_t \cdot \varepsilon_{2,t}$$

Intuitively, we can think of Eq. [B.3] as representing a model where each day's standardized return is generated from one of two distributions:

1. If $\delta_t = 0$ then the standardized return is generated from a standard normal distribution, that is, a normal distribution with mean 0 and variance 1.
2. If $\delta_t = 1$ then the return is generated from a normal distribution with mean $\mu_{2,t}$ and variance $1 + \sigma^2_{2,t}$.

We can think of δ_t as a variable that signifies whether a return that is inconsistent with the standard normal distribution has occurred. The parameter p is the probability of observing such a return. It is important to remember that although the assumed mixture distribution is composed of normal distributions, the mixture distribution itself is not normal. Also, note that when constructing a VaR forecast, the normal mixture model applies the standard RiskMetrics volatility.

Chart B.1 shows the tails of two normal mixture models (and the standard normal distribution) for different values of $\mu_{2,t}$, and $\sigma_{2,t}$. Mixture(1) is the normal mixture model with parameter values set at $\mu_{2,t}$ =*-4,* $\sigma_{2,t}$ *=1,* p*=2%,* $\mu_{1,t}$ =0 $\sigma_{1,t}$ =1. Mixture(2) is the normal mixture model with the same parameter values as mixture(1) except now $\mu_{2,t}$ *=0,* $\sigma_{2,t}$ *=10*.

Chart B.1
Tails of normal mixture densities
Mixture(1) $\mu_{2,t}=-4$, $\sigma_{2,t}=1$, $p=2\%$, $\mu_{1,t}=0$ $\sigma_{1,t}=1$;
Mixture(2) $\mu_{2,t}=0$, $\sigma_{2,t}=10$, $p=2\%$, $\mu_{1,t}=0$ $\sigma_{1,t}=1$

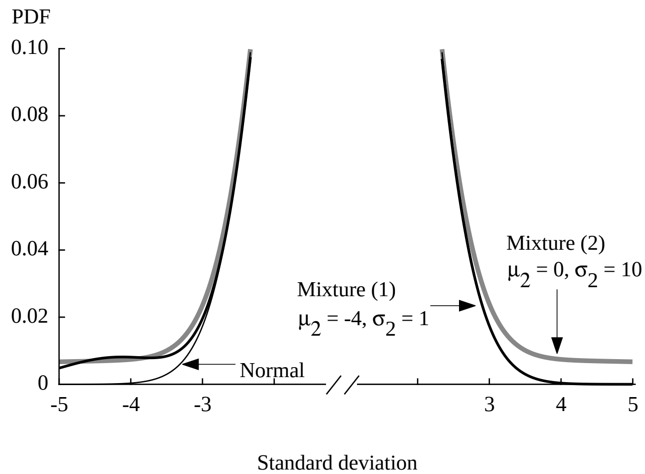


Chart B.1 shows that when there is a large negative mean for one of the normal distributions as in mixture(1), this translates into a larger probability of observing a large negative return relative to the standard normal distribution. Also, as in the case of mixture (2) we can construct a probability distribution with thicker tails than the standard normal distribution by mixing the standard normal with a normal distribution with a large standard deviation.

B.2.3 RiskMetrics-GED

According to this model, returns are generated as follows

[B.4]
$$r_t = \sigma_t \xi_t$$
$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)\, r_{t-1}^2$$

where

r_t is the time t continuously compounded return

ξ_t is a random variable distributed according to the GED (generalized error distribution) with parameter ν. As will be shown below, ν regulates the shape of the GED distribution.

σ_t^2 is the time t variance of returns (r_t)

The random variable (ξ_t) in Eq. [B.4] is assumed to follow a generalized error distribution (GED). This distribution is quite popular among researchers in finance because of the variety of shapes the GED can take. The probability density function for the GED is

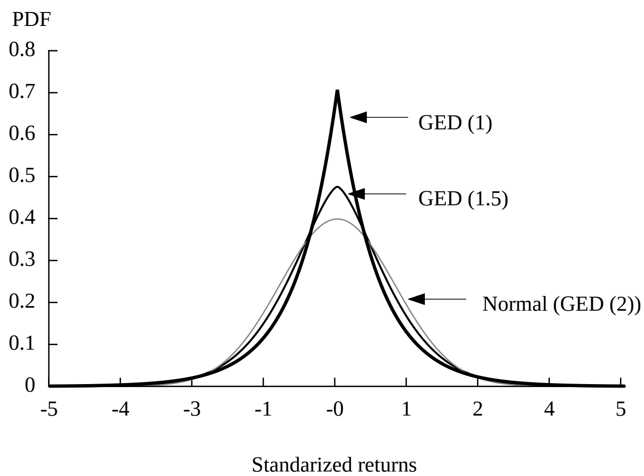
[B.5] $$f(\xi_t) = \frac{\nu \exp\left(-\frac{1}{2}|\xi_t / \lambda|^{\nu}\right)}{\lambda 2^{(1+\nu^{-1})}\Gamma\left(\nu^{-1}\right)}$$

where Γ is the gamma function and

[B.6] $$\lambda = \left[2^{-(2/\nu)}\Gamma(1/\nu)/(3/\nu)\right]^{1/2}$$

When $\nu = 2$ this produces a normal density while $\nu >(<)2$ is more thin (flat) tailed than a normal. Chart B.2 shows the shape of the GED distribution for values of $\nu = 1, 1.5$ and 2.

Chart B.2
GED distribution
ν *=1, 1.5 and 2*



Notice that when the parameter of the GED distribution is below 2 (normal), the result is a distribution with greater likelihood of very small returns (around 0) and a relatively large probability of returns far away from the mean. To better understand the effect that the parameter ν has on the tails of the GED distribution, Chart B.3 plots the left (lower) tail of the GED distribution when ν =1, 1.5 and 2.

Chart B.3
Left tail of GED (ν) distribution
ν = 1, 1.5, and 2

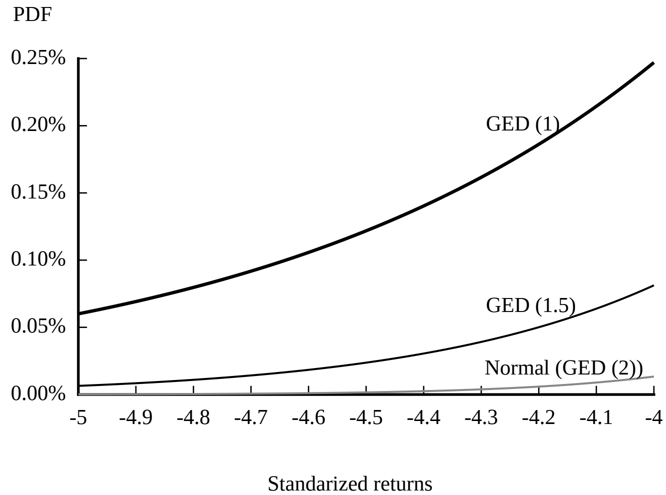


Chart B.3 shows that as ν becomes smaller, away from 2 (normal), there is more probability placed on relatively large negative returns.

B.3 Applying the models to emerging market currencies and equity indices

We applied the three models described above to 15 time series representing 9 emerging market countries to determine how well each model performs at estimating the 1st and 99th percentiles of the return distributions. The time series cover foreign exchange and equity indices. In order to facilitate our exposition of the process by which we fit each of the models and tabulate the results on forecasting the percentiles, we focus on one specific time series, the South African rand.

B.3.1 Model estimation and assessment

We first fit each model to 1152 returns on each of the 15 time series for the period May 25, 1992 through October 23, 1996. Table B.1 shows the parameter estimates from each of the three models for the South African rand.

Table B.1
Parameter estimates for the South African rand

Normal Mixture		**Standard RiskMetrics**		**RiskMetrics-GED**	
Parameter	Estimate	Parameter	Estimate	Parameter	Estimate
$\mu_{2,t}$	−5.086	λ	0.94	ν	0.927
$\sigma_{2,t}$	9.087				
p	0.010				
$\sigma_{1,t}$	1.288				

Table B.1 points to some interesting results:

- In the RiskMetrics-GED model, the estimate of ν implies that the distribution of returns on the rand are much thicker than the normal distribution (recall that ν =2 is a normal distribution). In other words, we are much more likely to observe a return that is far away from the mean return than is implied by the normal distribution.

- In the normal mixture model there is a 1% chance of observing a normally distributed return with a mean –5 and standard deviation 9 and a 99% chance of observing a normally distributed return with mean 0 and standard deviation 1.288.

- The RiskMetrics optimal decay factor for the South African rand is 0.940. This decay factor was found by minimizing the root mean squared error of volatility forecasts. Coincidentally, this happens to be the same decay factor applied to all times series in RiskMetrics when estimating one-day volatility.

If a volatility model such as RiskMetrics fits the data well its standardized returns (i.e., the returns divided by their volatility forecast) should have a volatility of 1. Table B.2 presents four sample statistics—mean, standard deviation, skewness and kurtosis—for the standard RiskMetrics model and estimates of ν for the RiskMetrics-GED model. Recall that skewness is a measure of a distribution's symmetry. A value of 0 implies that the distribution is symmetric. Kurtosis measures a distribution's "tail thickness". For example, since the kurtosis for a normal distribution is 3, values of kurtosis greater than 3 indicate that there is a greater likelihood of observing returns that are far away from the mean return than implied by the normal distribution.

Table B.2
Sample statistics on standardized returns
Standard RiskMetrics model

Instrument type	Source	Mean	Std dev	Skewness	Kurtosis	GED parameter, ν
Foreign exchange	Mexico	0.033	3.520	–21.744	553.035	0.749
	Philippines	–0.061	1.725	–13.865	327.377	0.368
	Taiwan	0.069	1.720	8.200	162.234	0.492
	Argentina	0.028	1.177	5.672	112.230	0.219
	Indonesia	–0.013	1.081	–1.410	12.314	0.460
	Korea	–0.013	1.106	–1.142	10.188	0.778
	Malaysia	0.029	1.210	–0.589	12.488	0.908
	South Africa	0.040	1.291	–6.514	116.452	0.927
	Thailand	–0.004	1.003	0.168	4.865	1.101
Equity	Argentina	0.043	1.007	–0.376	3.817	1.221
	Indonesia	0.020	1.085	1.069	12.436	0.868
	Malaysia	0.002	1.130	0.346	5.966	1.023
	Mexico	0.007	1.046	0.042	4.389	0.798
	South Africa	0.027	1.023	0.081	5.412	1.136
	Thailand	–0.019	1.056	–0.008	5.014	0.999

Under the maintained assumption of the RiskMetrics model the statistics of the standardized returns should be as follows; mean = 0, standard deviation = 1, skewness = 0, kurtosis = 3. Table B.2 shows that except for Mexico, Philippines and Taiwan foreign exchange, standard RiskMetrics does a good job at recovering the standard deviation. The fact that kurtosis for many of the time series are well above three signifies that the tails of these return distributions are much larger than the normal distribution.

Also, note the estimates of ν produced from RiskMetrics-GED. Remember that if the distribution of the standardized returns is normal, $\nu = 2$ and values of $\nu < 2$ signify that the distribution has thicker tails than that implied by the normal distribution. The fact that all of the estimates of ν are well below 2 indicate that these series contain a relatively large number of returns (negative and positive).

B.3.2 VaR analysis

In this section we report the results of an experiment to determine how well each of the models described above can predict the 1st and 99th percentiles of the 15 return distribution. These results are provided in Table B.3.

Our analysis consisted of the following steps:

- First, we estimate the parameters in each of the three models using price data from May, 25, 1992 through October 23, 1996. This sample consists of 1152 historical returns on each of the 15 time series.
- Second, we construct one-day volatility estimates for each of the three models using the most recent 952 returns.
- Third, we use the 952 volatility estimates and the three probability distributions (normal, mixture normal and GED) evaluated at the parameter estimates to construct VaR forecasts at the 1st and 99th percentiles.
- Fourth, we count the number of times the next day realized return exceeds each of the VaR forecasts. This number is then converted to a percentage by dividing it by the total number of trials—952 in this experiment. The "ideal" model would yield percentages of 1%.

Table B.3 presents these percentages for the three models.

Table B.3
VaR statistics (in %) for the 1st and 99th percentiles
RGD = RiskMetrics-GED; RM = RiskMetrics; MX = Normal mixture

		1st percentile (1%)			99th percentile (99%)		
Instrument type	**Source**	**RGD**	**RM**	**MX**	**RGD**	**RM**	**MX**
Foreign exchange	Mexico	1.477	2.346	0.434	1.043	1.998	0.434
	Philippines	1.390	2.520	1.043	1.216	1.998	1.043
	Taiwan	0.956	1.651	0.782	1.043	1.911	0.782
	Argentina	1.998	1.998	1.303	2.172	2.172	1.39
	Indonesia	1.651	3.562	1.651	1.129	1.998	1.411
	Korea	1.216	2.433	1.303	0.521	1.303	0.956
	Malaysia	1.564	2.433	1.477	1.911	3.215	1.911
	South Africa	1.390	1.998	1.216	1.129	1.998	1.303
	Thailand	0.695	1.129	1.043	1.651	2.172	1.072
Equity	Argentina	1.825	2.520	1.646	0.869	1.129	1.129
	Indonesia	0.608	1.998	1.564	1.564	2.520	1.564
	Malaysia	1.651	2.433	1.698	1.651	2.693	1.738
	Mexico	0.956	2.259	1.738	1.043	1.651	1.303
	South Africa	1.216	2.172	1.651	1.477	2.085	1.738
	Thailand	1.129	1.564	1.190	0.869	2.346	1.611
	Column average	1.315	2.201	1.396	1.286	2.060	1.270

Table B.3 shows that for the RiskMetrics-GED model the VaR forecasts at the 1st percentile are exceeded 1.315 percent of the time whereas the VaR forecasts at the 99th percentile are exceeded 1.286% of the time. Similarly, the VaR forecasts produced from the mixture model are exceeded at the 1st and 99th percentiles by 1.396% and 1.270% of the realized returns, respectively. Both models are marked improvements over the standard RiskMetrics model that assumes conditional normality.

Appendix C. Methods for determining the optimal decay factor

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In this appendix we present alternative measures to assess forecast accuracy of volatility and correlation forecasts.

C.1 Normal likelihood (LKHD) criterion

Under the assumption that returns are conditionally normal, the objective here is to specify the joint probability density of returns given a value of the decay factor. For the return on day *t* this can be written as:

[C.1] $$f(r_t|\lambda) = \left(\frac{1}{\sqrt{2\pi}\sigma_{t|t-1}(\lambda)}\right)\exp-\left[\frac{1}{2}\left(\frac{r_t^2}{\sigma^2_{t|t-1}(\lambda)}\right)\right]$$

Combining the conditional distributions from all the days in history for which we have data, we get:

[C.2] $$f(r_1, \ldots, r_T|\lambda) = \left\{\prod_{t=1}^{T}\left(\frac{1}{\sqrt{2\pi}\sigma_{t|t-1}(\lambda)}\right)\exp\left[-\frac{1}{2}\left(\frac{r_t^2}{\sigma^2_{t|t-1}(\lambda)}\right)\right]\right\}$$

Equation [C.2] is known as the normal likelihood function. Its value depends on λ. In practice, it is often easier to work with the log-likelihood function which is simply the natural logarithm of the likelihood function.

The maximum likelihood (ML) principle stipulates that the optimal value of the decay factor λ is one which maximizes the likelihood function Eq. [C.2]. With some algebra, it can be shown that this is equivalent to finding the value of λ that minimizes the following function:

[C.3] $$LKLHD_v = \sum_{t=1}^{T}\left\{\ln\left[\sigma_{t|t-1}(\lambda)\right] + \frac{1}{2}\left(\frac{r_t^2}{\sigma^2_{t|t-1}(\lambda)}\right)\right\}$$

Notice that the criterion Eq. [C.3] imposes the assumption that returns are distributed conditionally normal when determining the optimal value of λ. The RMSE criterion, on the other hand, does not impose any probability assumptions in the determination of the optimal value of λ.

C.2 Other measures

In addition to the RMSE and Normal likelihood measures alternative measures could also be applied such as the mean absolute error measure for the variance

[C.4] $$MAE_v = \frac{1}{T}\sum_{t=1}^{T}\left|r_{t+1}^2 - \hat{\sigma}^2_{t+1|t}\right|$$

For individual cashflows, RiskMetrics VaR forecasts are based on standard deviations. Therefore, we may wish to measure the error in the standard deviation forecast rather than the variance forecast. If we take as a proxy for the one period ahead standard deviation, $|r_t|$, then we can define the RMSE of the standard deviation forecast as

[C.5] $$RMSE_{\sigma} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(\left|r_{t+1}\right| - \hat{\sigma}_{t+1|t}\right)^2}$$

Notice in Eq. [C.5] that $E_t\left|r_{t+1}\right| \neq \sigma_{t+1}$. In fact for the normal distribution, the following equation holds: $E_t\left|r_{t+1}\right| = (2/\pi)^{-1/2}\sigma_{t+1}$.

Other ways of choosing optimal λ include the Q-statistic described by Crnkovic and Drachman (RISK, September, 1996) and, under the assumption that returns are normally distributed, a likelihood ratio test that is based on the normal probability density likelihood function.

C.3 Measures for choosing an optimal decay factor for multiple time series.

In Chapter 5, we explained how an optimal decay factor for the 480 RiskMetrics time series was chosen. This method involved finding optimal decay factors for each series, and then taking a weighted average of these factors, with those factors which provided superior performance in forecasting volatility receiving the greatest weight. In this section, we briefly describe some alternative methods which account for the performance of the correlation forecasts as well.

The first such method is an extension of the likelihood criterion to a multivariate setting. If we consider a collection of n assets whose returns on day t are represented by the vector $\vec{r}_t$, then the joint probability density for these returns is

[C.6] $$f(\vec{r}_t|\lambda) = \left(\frac{1}{(2\pi)^{\frac{n}{2}}\left|\Sigma_{t|t-1}(\lambda)\right|^{\frac{1}{2}}}\right)\exp-\left[\frac{1}{2}\left(\vec{r}^{T}{}_{t}\Sigma_{t|t-1}(\lambda)^{-1}\vec{r}_t\right)\right],$$

where $\Sigma_{t|t-1}(\lambda)$ is the matrix representing the forecasted covariance of returns on day t using decay factor λ. The likelihood for the returns for all of the days in our data set may be constructed analogously to Eq. [C.2]. Using the same reasoning as above, it can be shown that the value of λ which maximizes this likelihood is the one which also maximizes

[C.7] $$LKLHD_v = \sum_{t=1}^{T}\left\{\ln\left[\left|\Sigma_{t|t-1}(\lambda)\right|\right] + \vec{r}^{T}{}_{t}\Sigma_{t|t-1}(\lambda)^{-1}\vec{r}_t\right\}.$$

As noted before, choosing the decay factor according to this criterion imposes the assumption of conditional normality. In addition, to evaluate the likelihood function in Eq. [C.7], it is necessary at each time to invert the estimated covariance matrix $\Sigma_{t|t-1}(\lambda)$. In theory, this matrix will always be invertible, although in practice, due to limited precision calculations, there will likely be cases where the inversion is impossible, and the likelihood function cannot be computed.

A second approach is a generalization of the RMSE criterion for the covariance forecasts. Recall from Chapter 5 that the covariance forecast error on day t for the ith and jth returns is

[C.8] $$\varepsilon_{ij,t|t-1}(\lambda) = r_{i,t}r_{j,t} - \Sigma_{ij,t|t-1}(\lambda).$$

(Recall also that under the RiskMetrics assumptions, $E_{t-1}[\varepsilon_{ij,t|t-1}] = 0$.) The total squared error for day t is then obtained by summing the above over all pairs (i, j), and the mean total squared error (MTSE) for the entire data set is then

[C.9] $$MTSE = \frac{1}{T}\sum_{t=1}^{T}\sum_{i,j}\varepsilon_{ij,\,t|t-1}(\lambda)^2 .$$

The value of λ which minimizes the MTSE above can be thought of as the decay factor which historically has given the best covariance forecasts across all of the data series.

The above description presents a myriad of choices faced by the researcher when determining "optimal λ". The simple answer is that there is no clear-cut, simple way of choosing the optimal prediction criterion. There has been an extensive discussion among academics and practitioners on what error measure to use when assessing post-sample prediction.[1] Ultimately, the forecasting criterion should be motivated by the modeler's objective. For example, West, Edison and Cho (1993) note "an appropriate measure of performance depends on the use to which one puts the estimates of volatility…." Recently, Diebold and Mariano (1995) remind us, "of great importance and almost always ignored, is the fact that the economic loss associated with a forecast may be poorly assessed by the usual statistical measures. That is, forecasts are used to guide decisions, and the loss associated with a forecast error of a particular sign and size induced directly by the nature of the decision problem at hand." In fact, Leitch and Tanner (1991) use profitability rather than size of the forecast error or its squared value as a test of forecast accuracy.

[1] For a comprehensive discussion on various statistical error measures (including the RMSE) to assess forecasting methods, see the following:

Ahlburg, D.
Armstrong, J. S., and Collopy, F.
Fildes, R.

in the *International Journal of Forecasting*, 8, 1992, pp. 69–111.

Appendix D. Assessing the accuracy of the delta-gamma approach

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In this appendix we compare the VaR forecasts of the delta-gamma approach to those produced by full simulation. Before doing so, however, we investigate briefly when the delta-gamma approach is expected to perform poorly in relation to full simulation.

The accuracy of the delta-gamma approach depends on the accuracy of the approximation used to derive the return on the option. The expression for the option's return is derived using what is known as a "Taylor series expansion." We now present the derivation.

[D.1] $$V_{t+n} \approx V_t + \delta \cdot (P_{t+n} - P_t) + 0.5 \cdot \Gamma \cdot (P_{t+n} - P_t)^2 + \theta \cdot (\tau_{t+n} - \tau_t)$$

This expression can be rewritten as follows:

[D.2] $$V_{t+n} - V_t \approx \delta \cdot (P_{t+n} - P_t) + 0.5 \cdot \Gamma \cdot (P_{t+n} - P_t)^2 + \theta \cdot (\tau_{t+n} - \tau_t)$$

We now express the changes in the value of the option and the underlying in relative terms:

[D.3] $$V_t \cdot \left(\frac{V_{t+n} - V_t}{V_t}\right) = \delta \cdot P_t \cdot \left(\frac{P_{t+n} - P_t}{P_t}\right) + 0.5 \cdot \Gamma \cdot P_t^2 \cdot \left(\frac{P_{t+n} - P_t}{P_t}\right)^2 + \theta \cdot (\tau_{t+n} - \tau_t)$$

Dividing Eq. [D.3] by P_t, we get

[D.4] $$\left(\frac{V_t}{P_t}\right) \cdot \left(\frac{V_{t+n} - V_t}{V_t}\right) = \delta \cdot \left(\frac{P_{t+n} - P_t}{P_t}\right) + 0.5 \cdot \Gamma \cdot P_t \cdot \left(\frac{P_{t+n} - P_t}{P_t}\right)^2 + \left(\frac{\theta}{P_t}\right) \cdot (\tau_{t+n} - \tau_t)$$

and define the following terms:

$$R_V = \left(\frac{V_{t+n} - V_t}{V_t}\right),\ R_P = \left(\frac{P_{t+n} - P_t}{P_t}\right),\ n = (\tau_{t+n} - \tau_t)\text{ , and }\eta = \left(\frac{P_t}{V_t}\right)$$

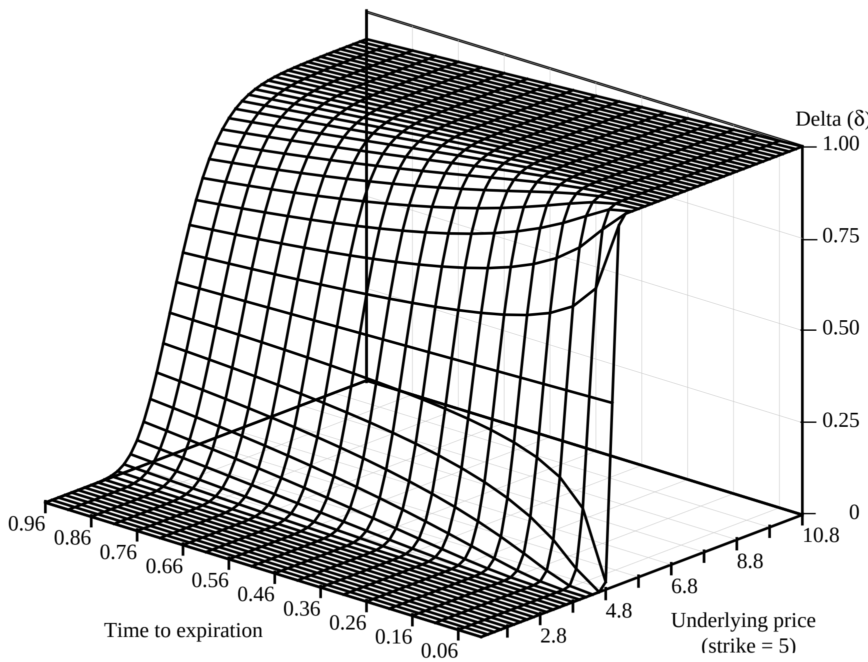
We can now write the return on the option as follows:

[D.5] $$\begin{aligned} R_V &= \eta\delta R_P + 0.5\,(\alpha\Gamma P_t)\,(R_P)^2 + \left(\frac{\theta}{V_t}\right)n \\ &= \tilde{\delta} R_P + 0.5\tilde{\Gamma}\,(R_P)^2 + \tilde{\theta}\,(\tau_{t+n} - \tau_t) \end{aligned}$$

This expansion is a reasonable approximation when the "greeks" δ and Γ are stable as the underlying price changes. In our example, the underlying price is the US dollar/deutschemark exchange rate. If changes in the underlying price causes large changes in these parameters then we should not expect the delta-gamma approach to perform well.

Chart D.1 shows the changes in the value of delta (δ) when the underlying price and the time to the option's expiry both change. This example assumes that the option has a strike price of 5.

Chart D.1
Delta vs. time to expiration and underlying price

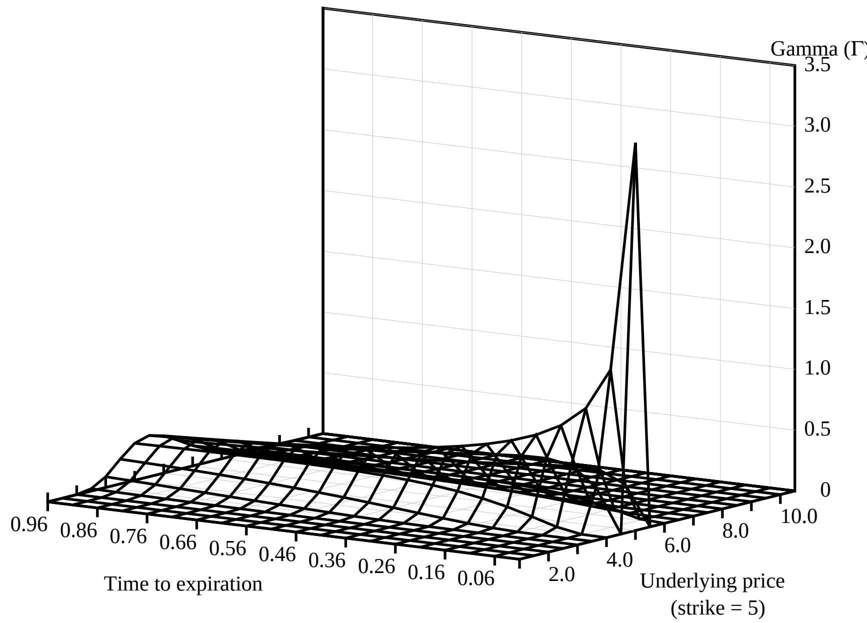


Notice that large changes in delta occur when the current price in the underlying instrument is near the strike. In other words, we should expect to see large changes in delta for small changes in the underlying price when the option is exactly, or close to being, an at-the-money option.

Since the delta and gamma components of an option are closely related, we should expect a similar relationship between the current underlying price and the gamma of the option. For the same option, Chart D.2 presents values of gamma as the underlying price and the time to expiry both change.

The chart shows that gamma changes abruptly when the option is near to being an at-the-money option and the time to expiry is close to zero.

Chart D.2
Gamma vs. time to expiration and underlying price



Together, Charts D.1 and D.2 demonstrate that we should expect the delta-gamma method to do most poorly when portfolios contain options that are close to being at-the-money and the time to expiry is short (about one week or less).

D.1 Comparing full simulation and delta-gamma: A Monte Carlo study

In this section we describe an experiment undertaken to determine the difference in VaR forecasts produced by the full simulation and delta-gamma methodologies. The study focuses on one call option. (For more complete results, see the third quarter 1996 *RiskMetrics Monitor.*) VaR forecasts, defined as the 5th percentile of the distribution of future changes in the value of the option, were made over horizons of one day. The Black-Scholes formula was used to both revalue the option and to derive the "greeks."

We set the parameters used to value the option, determine the "greeks", and generate future prices (for full simulation) as shown in Table D.1.

Table D.1
Parameters used in option valuation

Parameter	Value
Strike price (K)	5.0
Standard deviation (annualized)	23.0%
Risk-free interest rate	8.0%

Given these parameter settings we generate a series of underlying spot prices, P_t, with values 4.5, 4.6, 4.7,..., 5.6. Here the time *t* subscript denotes the time the VaR forecast is made. These spot prices imply a set of ratios of spot-to-strike price, P_t/K, that define the "moneyness" of the option. The values of P_t/K are 0.90, 0.92,0.94,...,1.12. In addition, we generate a set of time to expirations, τ, (expressed in years) for the option. Values of τ range from 1 day (0.004) to 1 year (1.0).

In full simulation, we are required to simulate future prices of the underlying instrument. Denote the future price of the underlying instrument by P_{t+n} where n denotes the VaR forecast horizon (i.e., n = 1 day, 1 week, 1 month and 3 months). We simulate underlying prices at time $t+n$, P_{t+n}, according to the density for a lognormal random variable

[D.6] $$P_{t+n} = P_t e^{((r - \sigma^2/2) + z\sigma\sqrt{n})}$$

where z is a standard unit normal random variable.

In full simulation, VaR is defined as the difference between the value of the option at time $t + n$ (the forecast horizon) and today, time t. This means that all instruments are revalued.

[D.7] $$\text{Exact} = BS(P_{t+n}, t+n) - BS(P_t, t),$$

where *BS()* stands for the Black-Scholes formula.

We use the term “Exact” to represent the fact that the option is being revalued using its exact option pricing formula. In the delta-gamma approach, VaR is approximated in terms of the Taylor series expansion discussed earlier:

[D.8] $$\text{Approx} = \delta \cdot (P_{t+n} - P_t) + 0.5 \cdot \Gamma \cdot (P_{t+n} - P_t)^2 + \theta \cdot n$$

Here, the term “Approx” denotes the approximation involved in using only the delta, gamma and theta components of the option. To compare VaR forecasts we define the statistics VaR_E and VaR_A as follows:

VaR_E = the 5th percentile of the Exact distribution which represents full simulation.

VaR_A = the 5th percentile of the Approx distribution which represents delta-gamma.

For a given spot price, P_t, time to expiration, τ, and VaR forecast horizon, n, we generate 5,000 future prices, P_{t+n}, and calculate VaR_E and VaR_A. This experiment is then repeated 50 times to produce 50 VaR_E’s and VaR_A’s. We then measure the difference in these VaR forecasts by computing two metrics:

[D.9] $$MAPE = \frac{1}{50}\sum_{i=1}^{50}\left|\frac{\left(VaR_A^i - VaR_E^i\right)}{VaR_E^i}\right| \quad \text{(Mean Absolute Percentage Error)}$$

[D.10] $$ME = \frac{1}{50}\sum_{i=1}^{50}\left(VaR_A^i - VaR_E^i\right) \quad \text{(Mean Error)}$$

Tables D.2 and D.3 report the results of this experiment. Specifically, Tables D.2 and D.3 show, respectively, the mean absolute percentage error (MAPE) and the mean error (ME) for a call option for a one-day forecast horizon. Each row of a table corresponds to a different time to expiration (maturity). Time to expiration is measured as a fraction of a year (e.g., 1 day = 1/250 or 0.004) and cannot be less than the VaR forecast horizon which is one day. Each column represents a ratio of the price of the underlying when the VaR forecast was made (spot) to the option’s strike price, P_t/K. This ratio represents the option’s “moneyness” at the time VaR was computed. All entries greater than or equal to 10 percent are reported without decimal places.

Table D.2
MAPE (%) for call, 1-day forecast horizon

Time to maturity, (years)	Spot/Strike											
	0.90	**0.92**	**0.94**	**0.96**	**0.98**	**1.00**	**1.02**	**1.04**	**1.06**	**1.08**	**1.10**	**1.12**
0.004	3838	2455	1350	633	187	28	21	1.7463	0.004	0.004	0.004	0.0036
0.054	11	7.12	3.960	1.561	0.042	0.762	0.946	0.733	0.372	0.082	0.052	0.069
0.104	3.226	2.061	1.180	0.486	0.021	0.271	0.395	0.399	0.330	0.229	0.133	0.061
0.154	1.592	1.039	0.615	0.272	0.028	0.130	0.216	0.245	0.232	0.195	0.148	0.102
0.204	0.983	0.655	0.400	0.190	0.036	0.069	0.133	0.163	0.168	0.155	0.131	0.104
0.254	0.685	0.465	0.293	0.148	0.040	0.037	0.087	0.115	0.125	0.122	0.111	0.095
0.304	0.515	0.355	0.230	0.123	0.041	0.018	0.059	0.084	0.096	0.098	0.093	0.084
0.354	0.407	0.285	0.189	0.106	0.042	0.007	0.041	0.063	0.075	0.079	0.078	0.073
0.404	0.333	0.237	0.160	0.094	0.041	0.003	0.028	0.047	0.059	0.065	0.066	0.063
0.454	0.28	0.202	0.139	0.084	0.041	0.007	0.019	0.036	0.048	0.054	0.056	0.055
0.504	0.241	0.175	0.123	0.077	0.040	0.010	0.012	0.028	0.038	0.045	0.048	0.048
0.554	0.211	0.155	0.111	0.071	0.039	0.013	0.007	0.021	0.031	0.038	0.041	0.042
0.604	0.187	0.139	0.101	0.066	0.038	0.015	0.003	0.016	0.025	0.032	0.035	0.037
0.654	0.167	0.125	0.092	0.062	0.037	0.016	0.001	0.012	0.021	0.027	0.031	0.033
0.704	0.151	0.114	0.085	0.058	0.035	0.017	0.003	0.008	0.017	0.023	0.026	0.029
0.754	0.138	0.105	0.079	0.055	0.034	0.018	0.005	0.006	0.013	0.019	0.023	0.025
0.804	0.126	0.097	0.074	0.052	0.033	0.018	0.006	0.003	0.011	0.016	0.020	0.022
0.854	0.117	0.09	0.069	0.049	0.033	0.019	0.007	0.002	0.008	0.014	0.017	0.020
0.904	0.108	0.084	0.065	0.047	0.032	0.019	0.008	0.001	0.006	0.011	0.015	0.018

Table D.3
ME (%) for call, 1-day forecast horizons

Time to maturity, (years)	Spot/Strike											
	0.90	**0.92**	**0.94**	**0.96**	**0.98**	**1.00**	**1.02**	**1.04**	**1.06**	**1.08**	**1.10**	**1.12**
0.004	0.000	0.000	0.000	–0.003	–0.186	0.569	–0.180	–0.010	0.000	0.000	0.000	0.000
0.054	–0.004	–0.006	–0.008	–0.005	0.000	0.005	0.007	0.005	0.003	0.001	0.000	0.000
0.104	–0.003	–0.004	–0.003	–0.002	0.000	0.001	0.002	0.003	0.002	0.002	0.001	0.000
0.154	–0.002	–0.002	–0.002	–0.001	0.000	0.001	0.001	0.002	0.002	0.001	0.001	0.001
0.204	–0.002	–0.002	–0.001	–0.001	0.000	0.000	0.001	0.001	0.001	0.001	0.001	0.001
0.254	–0.001	–0.001	–0.001	–0.001	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001
0.304	–0.001	–0.001	–0.001	–0.001	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001
0.354	–0.001	–0.001	–0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
0.404	–0.001	–0.001	–0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.454	–0.001	–0.001	–0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.504	–0.001	–0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.554	–0.001	–0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.604	–0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.654	–0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.704	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.754	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.804	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.854	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.904	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

D.2 Conclusions

The results reported in this appendix show that the relative error between delta-gamma and full simulation is reasonably low, but becomes large as the option nears expiration and is at-the-money. Note that the extremely large errors in the case where the option is out-of-the-money reflects the fact that the option is valueless. Refer to Tables C.10 and C.19 in the *RiskMetrics Monitor* (third quarter, 1996) to see the value of the option at various spot prices and time to expirations. Therefore, aside from the case where the option is near expiration and at-the-money, the delta-gamma methodology seems to perform well in comparison to full simulation.

Overall, the usefulness of the delta-gamma method depends on how users view the trade-off between computational speed and accuracy. For risk managers seeking a quick, efficient means of computing VaR that measures gamma risk, delta-gamma offers an attractive method for doing so.

Appendix E. Routines to simulate correlated normal random variables

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In Section E.1 of this appendix we briefly introduce three algorithms for simulating correlated normal random variables from a specified covariance matrix Σ (Σ is square and symmetric). In Section E.2 we present the details of the Cholesky decomposition.

E.1 Three algorithms to simulate correlated normal random variables

This section describes the Cholesky decomposition (CD), eigenvalue decomposition (ED) and the singular value decomposition (SVD). CD is efficient when Σ is positive definite. However, CD is not applicable for positive semi-definite matrices. ED and SVD, while computationally more intensive, are useful when Σ is positive semi-definite.

- Cholesky decomposition

  We begin by decomposing the covariance matrix as follows:

  [E.1] $$\Sigma = P^T P$$

  where P is an upper triangular matrix. To simulate random variables from a multivariate normal distribution with covariance matrix Σ we would perform the following steps:

  1. Find the upper triangular matrix P.
  2. Compute a vector of standard normal random variables denoted ε. In other words, ε has a covariance matrix I (identity matrix).
  3. Compute the vector $y = P^T\varepsilon$. The random vector y has a multivariate normal distribution with a covariance matrix Σ.

  Step 3 follows from the fact that

  [E.2] $$V(y) = P^T E\left(\varepsilon\varepsilon^T\right)P = P^T I P = P^T P = \Sigma$$

  where $V(\ )$ and $E(\ )$ represent the variance and mathematical expectation, respectively.

- Eigenvalue decomposition

  Applying spectral decomposition to Σ yields

  [E.3] $$\Sigma = C\Delta C^T = Q^T Q$$

  where C is an NxN orthogonal matrix of eigenvectors, i.e., $C^T C = I$

  Δ is an NxN matrix with the N-eigenvalues of X along its diagonal and zeros elsewhere

  [E.4] $$Q = \Delta^{1/2} C^T$$

To simulate random variables from a multivariate normal distribution with covariance matrix Σ we would perform the following steps:

1. Find the eigenvectors and eigenvalues of Σ.
2. Compute a vector of standard normal random variables denoted ε. In other words, ε has a covariance matrix I (identity matrix).
3. Compute the vector $y = Q^T\varepsilon$. The random vector y has a multivariate normal distribution with a covariance matrix Σ.

Step 3 follows from the fact that

[E.5] $$V(y) = Q^T E\left(\varepsilon\varepsilon^T\right)Q = Q^T IQ = Q^T Q = C\Delta^{1/2}\Delta^{1/2}C^T = C\Delta C^T = \Sigma$$

The final algorithm that is proposed is known as the singular value decomposition.

- Singular Value decomposition

We begin with the following representation of the covariance matrix

[E.6] $$\Sigma = UDV^T$$

where *U* and *V* are *N*x*N* orthogonal matrices, i.e., $V^T V = U^T U = I$, and *D* is an *N*x*N* matrix with the *N* singular values of Σ along its diagonal and zeros elsewhere.

It follows directly from Takagi's decomposition that for any square, symmetric matrix, $\Sigma = VDV^T$. Therefore, to simulate random variables from a multivariate normal distribution with covariance matrix Σ we would perform the following steps:

1. Apply the singular value decomposition to Σ to get *V* and *D*.
2. Compute a vector of standard normal random variables denoted ε. In other words, ε has a covariance matrix *I* (identity matrix).
3. Compute the vector $y = Q^T\varepsilon$ where $Q = D^{1/2}V^T$. The random vector y has a multivariate normal distribution with a covariance matrix Σ.

Step 3 follows from the fact that

[E.7] $$V(y) = Q^T E\left(\varepsilon\varepsilon^T\right)Q = Q^T IQ = Q^T Q = VD^{1/2}D^{1/2}V^T = VDV^T = \Sigma$$

E.2 Applying the Cholesky decomposition

In this section we explain exactly how to create the A matrix which is necessary for simulating multivariate normal random variables from the covariance matrix Σ. In particular, Σ can be decomposed as:

[E.8] $$\Sigma = A^T A$$

If we simulate a vector of independent normal random variables X then we can create a vector of normal random variables with covariance matrix Σ by using the transformation Y=A'X. To show how to obtain the elements of the matrix A, we describe the Cholesky decomposition when the dimension of the covariance matrix is 3 x 3. After, we give the general recursive equations used to derive the elements of A from Σ.

Consider the following definitions:

[E.9] $$\Sigma = \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} \qquad A^T = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad A = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ 0 & a_{22} & a_{32} \\ 0 & 0 & a_{33} \end{bmatrix}$$

Then we have

[E.10] $$\begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ 0 & a_{22} & a_{32} \\ 0 & 0 & a_{33} \end{bmatrix}$$

equivalent to

[E.11] $$\begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} = \begin{bmatrix} a_{11}^2 & a_{11}a_{21} & a_{11}a_{31} \\ a_{11}a_{21} & a_{21}^2 + a_{22}^2 & a_{21}a_{31} + a_{32}a_{22} \\ a_{11}a_{31} & a_{21}a_{31} + a_{32}a_{22} & a_{31}^2 + a_{32}^2 + a_{33}^2 \end{bmatrix}$$

Now we can use the elements of Σ to solve for the $a_{i,j}$'s – the elements of A. This is done recursively as follows:

[E.12]
$$\begin{aligned} s_{11} &= a_{11}^2 \Rightarrow a_{11} = \sqrt{s_{11}} \\ s_{21} &= a_{11}a_{21} \Rightarrow a_{21} = \frac{s_{21}}{a_{11}} \\ s_{22} &= a_{21}^2 + a_{22}^2 \Rightarrow a_{22} = \sqrt{s_{22} - a_{21}^2} \\ s_{31} &= a_{11}a_{31} \Rightarrow a_{31} = \frac{s_{31}}{a_{11}} \\ s_{32} &= a_{21}a_{31} + a_{32}a_{22} \Rightarrow a_{32} = \frac{1}{a_{22}}(s_{32} - a_{21}a_{31}) \\ s_{33} &= a_{11}^2 + a_{22}^2 + a_{33}^2 \Rightarrow a_{33} = \sqrt{a_{33} - a_{11}^2 - a_{22}^2} \end{aligned}$$

Having shown how to solve recursively for the elements in A we now give a more general result. Let *i* and *j* index the row and column of an *N* x *N* matrix. Then the elements of A can be solved for using

[E.13] $$a_{ii} = \left(s_{ii} - \sum_{k=1}^{i-1} a_{ik}^2 \right)^{1/2}$$

and

[E.14] $$a_{ij} = \frac{1}{a_{ii}}\left(s_{ij} - \sum_{k=1}^{i-1} a_{ik}a_{jk} \right)^{1/2} \qquad j = i+1, i+2, \ldots, N$$

Appendix F. BIS regulatory requirements

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The Basel Committee on Banking Supervision under the auspices of BIS issued in January 1996 a final Amendment to the 1988 Capital Accord that requires capital charges to cover market risks in addition to the existing framework covering credit risk. The framework covers risks of losses in on- and off- balance sheet positions arising from movements in market prices.

Banks' minimum capital charges will be calculated as the sum of credit risk requirements under the 1988 Capital Accord, excluding debt and equity securities in the trading book and all positions in commodities, but including the credit counterparty risk on all OTC derivatives, and capital charges for market risks. The proposal sets forth guidelines for the measurement of market risks and the calculation of a capital charge for market risks.

I. Measurement of market risk

Market risk may be measured using banks' internal models (subject to approval by the national supervisor) and incorporates the following:

1. Market risk in the trading account (i.e., debt and equity securities and derivatives):

   Standardized method — uses a "building block" approach where charges for general risk and issuer specific risk for debt and equities risks are calculated separately.

   Internal model — must include a set of risk factors corresponding to interest rates in each currency in which the bank has interest sensitive on- and off-balance sheet positions and corresponding to each of the equity markets in which the bank holds significant positions.

2. Foreign exchange risk across the firm (including gold:

   Standardized method — uses the shorthand method of calculating the capital requirement.

   Internal model — must include FX risk factors of the bank's exposures.

3. Commodities risk across the firm (including precious metals but excluding gold)

   Standardized method — risk can be measured using the standardized approach or the simplified approach.

   Internal model — must include commodity risk factors of the bank's exposures.

4. Options risk across the firm:

   Standardized method — banks using only purchased options should use a simplified approach and banks using written options should, at a minimum, use one of the intermediate approaches ("delta plus" or simulation method).

   Internal model — must include risk factors (interest rate, equity, FX, commodity) of the bank's exposures.

II. Capital charge for market risk

Standardized method —simple sum of measured risk for all factors (i.e., debt/equity/FX/commodities/options)

Internal model —

- Higher of the previous day's VaR (calculated in accordance with specific quantitative standards) or average of daily VaR on each of the preceding 60 days times a multiplication factor, subject to a minimum of 3.

- A separate capital charge to cover the specific risk of traded debt and equity securities if not incorporated in model.

- A "plus" will be added that is directly related to the ex-post performance of the model (derived from "back-testing" outcome)

- Among other qualitative factors, stress testing should be in place as a supplement to the risk-analysis based on the day-to-day output of the model.

III. Methods of measuring market risks

A choice between a Standardized Methodology and an Alternative Methodology (i.e., use of banks' internal models) will be permitted for the measurement of market risks subject to the approval of the national supervisor.

1. The standardized methodology

This method uses a "building block" approach for debt and equity positions, where issuer-specific risk and general risk are calculated separately. The capital charge under the standardized method will be the arithmetic sum of the measures of each market risk (i.e., debt/equity/foreign exchange/commodities/options).

Debt securities
Instruments covered include: debt securities (and instruments that behave like them including non-convertible preferred shares) and interest rate derivatives in the trading account. Matched positions in identical issues (e.g., same issuer, coupon rates, liquidity, call features) as well as closely matched swaps, forwards, futures and FRAs which meet additional conditions are permitted to be offset. The capital charge for debt securities is the sum of the specific risk charge and general risk charge.

- Specific risk

  The specific risk charge is designed to protect against an adverse movement in the price of an individual security owing to factors related to the individual issuer. Debt securities and derivatives are classified into broad categories (government, qualified, and other) with a varying capital charge applied to gross long positions in each category. Capital charges range from 0% for the government category to 8% for the Other category.

- General market risk

  The general risk charge is designed to capture the risk of loss arising from changes in market interest rates. A general risk charge would be calculated separately for each currency in which the bank has a significant position. There are two principal methods to choose from:

1. **Maturity method —** long and short positions in debt securities and derivatives are slotted into a maturity ladder with 13 time bands (15 for deep discount securities). The net position in each time band is risk weighted by a factor designed to reflect the price sensitivity of the positions to changes in interest rates.
2. **Duration method** — achieves a more accurate measure of general market risk by calculating the price sensitivity of each position separately.

The general risk charge is the sum of the risk-weighted net short or long position in the whole trading book, a small proportion of the matched positions in each time-band (vertical disallowance 10% for maturity method; 5% for duration method), and a larger proportion of the matched positions across different time bands (horizontal disallowance).

Equities
Instruments covered include: common stocks, convertible securities that behave like equities, commitments to buy or sell equities, and equity derivatives. Matched positions in each identical equity in each market may be fully offset, resulting in a single net short or long position to which the specific and general market risk charges apply. The capital charge for equities is the sum of the specific risk charge and general risk charge.

- Specific risk

  Specific risk is the risk of holding a long or short position in an individual equity, i.e., the bank's absolute equity positions (the sum of all long and short equity positions). The specific risk charge is 8% (or 4% if the portfolio is liquid and diversified). A specific risk charge of 2% will apply to the net long or net short position in an index comprising a diversified portfolio of equities.

- General market risk

  General market risk is the risk of holding a long or short position in the market as a whole, i.e., the difference between the sum of the longs and the sum of the shorts (the overall net position in an equity market). The general market risk charge is 8% and is calculated on a market by market basis.

Foreign exchange risk (including gold)
The shorthand method of calculating the capital requirement for foreign exchange risk is performed by measuring the net position in each foreign currency and gold at the spot rate and applying an 8% capital charge to the net open position (i.e., the higher of net long or net short positions in foreign currency and 8% of the net position in gold).

Commodities risk
Commodities risk including precious metals, but excluding gold, can be measured using the standardized approach or the simplified approach for banks which conduct only a limited amount of commodities business. Under the standardized approach, net long and short spot and forward positions in each commodity will be entered into a maturity ladder. The capital charge will be calculated by applying a 1.5% spread rate to matched positions (to capture maturity mismatches) and a capital charge applied to the net position in each bucket. Under the simplified method, a 15% capital charge will be applied to the net position in each commodity.

Treatment of options
Banks that solely use purchased options are permitted to use a simplified approach; however, banks that also write options will be expected to use one of the intermediate approaches or a comprehensive risk management model. Under the standardized approach, options should be “carved out” and become subject to separately calculated capital charges on particular trades to be added to other capital charges assessed. Intermediate approaches are the “delta plus approach” and scenario

analysis. Under the delta plus approach, delta-weighted options would be included in the standardized methodology for each risk type.

2. Alternative methodology: internal models

This method allows banks to use risk measures derived from their own internal risk management models, subject to a general set of standards and conditions. Approval by the supervisory authority will only be granted if there are sufficient numbers of staff (including trading, risk control, audit and back office areas) skilled in using the models, the model has a proven track record of accuracy in predicting losses, and the bank regularly conducts stress tests.

- Calculation of capital charge under the internal model approach
  - Each bank must meet on a daily basis a capital requirement expressed as the higher of its previous day's value at risk number measured according to the parameters specified or an average of the daily value at risk measures on each of the preceding sixty business days, multiplied by a multiplication factor.
  - The multiplication factor will be set by supervisors on the basis of their assessment of the quality of the bank's risk management system, subject to a minimum of 3. The plus factor will range from 0 to 1 based on backtesting results and that banks that meet all of the qualitative standards with satisfactory backtesting results will have a plus factor of zero.The extent to which banks meet the qualitative criteria may influence the level at which supervisors will set the multiplication factor.
  - Banks using models will be subject to a separate capital charge to cover the specific risk of traded debt and equity securities to the extent that this risk is not incorporated into their models. However, for banks using models, the specific risk charge applied to debt securities or equities should not be less than half the specific risk charge calculated under the standardized methodology.
  - Any elements of market risk not captured by the internal model will remain subject to the standardized measurement framework.
  - Capital charges assessed under the standardized approach and the internal model approach will be aggregated according to the simple sum method.
- Requirements for the use of internal models:

Qualitative standards

- Existence of an independent risk control unit with active involvement of senior management
- Model must be closely integrated into day-to-day risk management and should be used in conjunction with internal trading and exposure limits
- Routine and rigorous programs of stress testing and back-testing should be in place
- A routine for ensuring compliance and an independent review of both risk management and risk measurement should be carried out at regular intervals
- Procedures are prescribed for internal and external validation of the risk measurement process

Specification of market risk factors
The risk factors contained in a risk measurement system should be sufficient to capture the risk inherent in the bank's portfolio, i.e., interest rates, exchange rates, equity prices, commodity prices.

Quantitative standards

- Value at risk should be computed daily using a 99th percentile, one-tailed confidence interval and a minimum holding period of 10 trading days. Banks are allowed to scale up their 1-day VaR measure for options by the square root of 10 for a certain period of time after the internal models approach takes effect at the end of 1997.

- Historical observation period will be subject to a minimum length of one year. For banks that use a weighting scheme or other methods for the historical observation period, the “effective” observation period must be at least one year.

- Banks will have discretion to recognize empirical correlations within broad risk categories. Use of correlation estimates across broad risk categories is subject to regulatory approval of the estimation methodology used.

- Banks should update their data sets no less frequently than once every three months and should reassess them whenever market prices are subject to material change

- Models must accurately capture the unique risks associated with options within the broad risk categories (using delta/gamma factors if analytical approach is chosen)

IV. Calculation of the capital ratio

- The minimum capital ratio representing capital available to meet credit and market risks is 8%.

- The denominator of the ratio is calculated by multiplying the measure of market risk by 12.5 (reciprocal of the 8% ratio) and adding the results to credit risk-weighted assets. The numerator is eligible capital, i.e., sum of the bank's Tier 1 capital, Tier 2 capital under the limits permitted by the 1988 Accord, and Tier 3 capital, consisting of short-term subordinated debt. Tier 3 capital is permitted to be used for the sole purpose of meeting capital requirements for market risks and is subject to certain quantitative limitations.

- Although regular reporting will in principle take place only at intervals (in most countries quarterly), banks are expected to manage the market risk in their trading portfolios in such a way that the capital requirements are being met on a continuous basis, i.e., at the close of each business day.

V. Supervisory framework for the use of backtesting

Backtesting represents the comparison of daily profits and losses with model-generated risk measures to gauge the quality and accuracy of banks' risk measurement systems. The backtests to be applied compare whether the observed percentage of outcomes covered by the risk measure is consistent with a 99% level of confidence. The backtesting framework should use risk measures calibrated to a 1-day holding period. The Committee urges banks to develop the capability to perform backtests using both hypothetical (based on the changes in portfolio value that would occur were end-of-day positions to remain unchanged) and actual trading outcomes.

The framework adopted by the Committee calculates the number of times that the trading outcomes are not covered by the risk measures (exceptions) on a quarterly basis using the most recent 12 months of data. The framework encompasses a range of possible responses which are classified into 3 zones. The boundaries are based on a sample of 250 observations.

- **Green zone —** the backtesting results do not suggest a problem with the quality or accuracy of a bank's model (only four exceptions are allowed here).

- **Yellow zone —** the backtesting results do raise questions, but such a conclusion is not definitive (only 9 exceptions are allowed here). Outcomes in this range are plausible for both accurate and inaccurate models. The number of exceptions will guide the size of potential supervisory increases in a firm's capital requirement. The purpose of the increase in the multiplication factor is to return the model to a 99th percentile standard. Backtesting results in the yellow zone will generally be presumed to imply an increase in the multiplication factor unless the bank can demonstrate that such an increase is not warranted. The burden of proof in these situations should not be on the supervisor to prove that a problem exists, but rather should be on the bank to prove that their model is fundamentally sound.

- **Red zone** — the backtesting results almost certainly indicate a problem with a bank's risk model (10 or more exceptions). If a bank's model falls here, the supervisor will automatically increase the multiplication factor by one and begin investigation.

Appendix G. Using the RiskMetrics examples diskette

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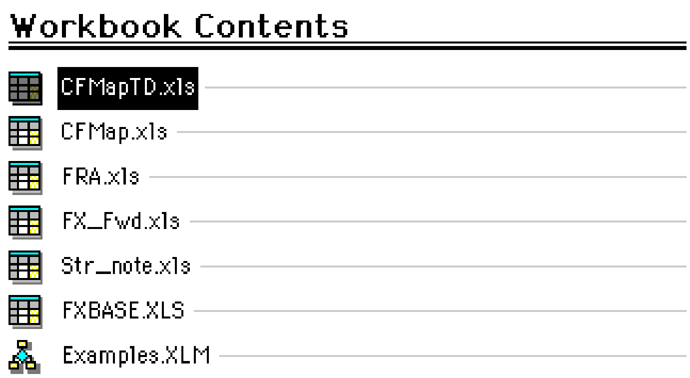
A number of the examples in this *Technical Document*, are included on the enclosed examples diskette. This diskette contains a Microsoft Excel workbook file containing six spreadsheets and one macro file. The workbook can be used under Excel Version 4.0 or higher.

Some of the spreadsheets allow the user to modify inputs in order to investigate different scenarios. Other spreadsheets are non-interactive. In the latter case, the objective is to provide the user with a detailed illustration of the calculations. This workbook and user guide is presented to the experienced user of Microsoft Excel, although we hope the material is meaningful to less experienced users. Please make a duplicate of the Examples.XLW workbook and save at least one copy on your hard drive and at least one copy on a floppy disk. This will allow you to manipulate the enclosed spreadsheets without sacrificing their original format.

Opening the Examples.XLW workbook will show the following file structure:



The files listed above are described as follows:

File	Section, page	Description
CFMapTD.xls	Section 6.4, page 134	Decomposition of the 10-year benchmark OAT into RiskMetrics vertices
CFMap.xls	Section 6.4, page 135	Generic Excel cash flow mapping spreadsheet (users are given flexibility to map standard bullet bonds)
FRA.xls	Section 6.4, page 136	Mapping and VaR calculation of a 6x12 French franc FRA
FX_Fwd.xls	Section 6.4, page 143	Mapping and VaR calculation of a DEM/USD 1-year forward
Str_note.xls	Section 6.4, page 139	Mapping and VaR calculation of a 1-year Note indexed to 2-year DEM swap rates
FXBase.xls	Section 8.4, page 183	Generic calculator to convert U.S. dollar based volatilities and correlations to another base currency
Examples.XLM		Macro sheet that links to buttons on the various spreadsheets

CFMapTD.xls & CFMap.xls

These two spreadsheets are similar, although CFMap.xls allows the user to change more of the inputs in order to investigate different scenarios, or to perform sensitivity analysis. CFMap.xls allows provides more vertices to which to map the cash flows. Note that only data in red is changeable on all spreadsheets.

In CFMapTD.xls, Example Part 1 illustrates the mapping of a single cash flow, while Example Part II illustrates the mapping of the entire bond.

To begin mapping on either spreadsheet, enter your chosen data in all cells that display red font. Then click the “Create cash flows” button. Wait for the macro to execute, then click the “Map the cash flows to vertices” button to initiate the second macro, which executes for the final output of Diversified Value at Risk, Market Value, and Percentage of market value. If you wish to print the cash flow mapping output, simply click the “Print Mapping” macro button.

FRA.xls

This Forward Rate Agreement example is for illustrative purposes only. We encourage the user, however, to manipulate the spreadsheet is such a way as to increase it’s functionality. Changing any spreadsheet, of course, should be done after creating a duplicate workbook.

In this spreadsheet, cells are named so that formulae show the inputs to their respective calculations. This naming convention, we hope, increases user friendliness. For example, looking at cell C21 shows the calculation for the FRA rate utilizing the data in 1. Basic Contract Data and data under the Maturity column under 2. FRA Mapping and VaR on 6-Jan-95.

Cells are named according to the heading under which they fall, or the cell to their left that best describes the data. For example, cell B30 is named Maturity_1, while cell K32 is named Divers_VaR_1. Also note that the RiskMetrics Correlations are named in two-dimensional arrays: cells L30:M31 are named Corr_Matrx_1, while cells L40:N42 are named Corr_Matrx_2.

If you have any confusion about the naming convention, simply go to the *Formula Define name…* command. The *Define Name* dialogue box will appear, where the cell names are listed in alphabetical order along with their respective cell references.

The cells containing the individual VaR calculations (K30, K31, K40, K41, K42) contain the absolute value of the value at risk. In order to calculate the Diversified VaR, however, in cells K32 and K43, we have placed the actual VaR values to the right of the correlation matrices. If you go to cell K32, you will see that the formula makes use of VaR_Array_1, which refers to cells O31:O32. This VaR array contains the actual values of VaR_1 and VaR_2, which are essential to calculating the Divers_VaR_1. Cells O31 and O32 are formatted in white font in order to maintain the clarity of the spreadsheet. Similarly, the calculation in cell K43 utilizes VaR_Array_2, found in cells O40:O42.

FX_Fwd.xls

This spreadsheet offers some interaction whereby the user can enter data in all red cells.

Before examining this spreadsheet, please review the names of the cells in the 1. Basic contract data section in order to better understand the essential calculations. If you have any confusion about the naming convention, simply go to the *Formula Define name…* command. The *Define Name* dialogue box will appear, where the cell names are listed in alphabetical order along with their respective cell references.

Please note that the Diversified Value at Risk calculation utilizes the var_array input, which refers to cells I33:I35. These cells are formatted in white font in order to maintain the clarity of the worksheet.

Str_note.xls

This spreadsheet is for illustrative purposes only. Again, we encourage the user to format the spreadsheet for custom use.

Please notice that the Diversified VaR calculations make use of VaR_Array1 and VaR_Array2. VaR_Array1 references cells N26:N28, while VaR_Array2 references cells O37:O40. These two arrays are formatted in white font in order to maintain the clarity of the worksheet.

If you have any confusion about the naming convention, simply go to the *Formula Define name…* command. The *Define Name* dialogue box will appear, where the cell names are listed in alphabetical order along with their respective cell references.

Appendix H. RiskMetrics on the Internet

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RiskMetrics home pages on the Internet are currently located at

http://www.jpmorgan.com/RiskManagement/RiskMetrics/RiskMetrics.html

and

http://www.riskmetrics.reuters.com

The RiskMetrics home page on the Reuters Web is located at:

http://riskmetrics.session.rservices.com

The Internet can be accessed through such services as CompuServe®, Prodigy®, or America® Online, or through service providers by using browsers such as Netscape™ Navigator, Microsoft® Internet Explorer, Mosaic or their equivalents. The Reuters Web is available with the Reuters 3000 series.

RiskMetrics data sets can be downloaded from the Internet and from the Reuters Web. RiskMetrics documentation and a listing of third parties, both consultants and software developers who incorporate RiskMetrics methodology and/or data sets, are also freely available from these sites or from local Reuters offices. Users can receive e-mail notification of new publications or other information relevant to RiskMetrics by registering at the following address:

http://www.jpmorgan.com/RiskManagement/RiskMetrics/rmform.html

Note that URL addresses are subject to change.

H.1 Data sets

RiskMetrics data sets are updated daily on the Internet at *http://www.riskmetrics.reuters.com* and on the Reuters Web at *http://riskmetrics.session.rservices.com.*

The data sets are available by 10:30 a.m. U. S. Eastern Standard Time. They are based on the previous day’s data through close of business, and provide the latest estimates of volatilities and correlations for daily and monthly horizons, as well as for regulatory requirements.

The data sets are not updated on official U.S. holidays. For these holidays, foreign market data is included in the following business day’s data sets; U.S. market data is adjusted according to the Expectation Maximization (EM) algorithm described in Section 8.25. EM is also used in a consistent fashion for filling in missing data in other markets.

The data sets are supplied in compressed file format for DOS, Macintosh, and UNIX platforms. The DOS and Macintosh files are auto-extracting, i.e., the decompression software is enclosed in the file.

On the same page as the data sets is the Excel add-in, which enables users to perform DEaR/VaR calculations on other than a US dollar currency basis. The add-in allows users full access to the data sets when building customized spreadsheets. Current rate, price volatility, and correlation of specified pairs can be returned. The add-in was compiled in 16 bit and runs under Excel 4.0 and 5.0. It does not run under Windows NT. The name of the add-in is JPMVAR for the Mac and JPMVAR.XLL for the PC. It has an expiration date of November 1, 1997.

H.2 Publications

http://www.jpmorgan.com/RiskManagement/riskMetrics/pubs.html

The annual *RiskMetrics—Technical Document*, the quarterly *Monitor* and all other RiskMetrics documents are available for downloading in Adobe Acrobat pdf file format. Adobe Acrobat Reader is required to view these files. It can be downloaded from *http://www.adobe.com*.

RiskMetrics documents are also available from your local Reuters office.

H.3 Third parties

http://www.jpmorgan.com/RiskManagement/RiskMetrics/Third_party_directory.html

Setting up a risk management framework within an organization requires more than a quantitative methodology. A listing of several consulting firms who have capital advisory practices to help ensure the implementation of effective risk management and system developers who have integrated RiskMetrics methodology and/or data sets is available for viewing or saving as a file.

Users should be able to choose from a number of applications that will achieve different goals, offer various levels of performance, and run on a number of different platforms. Clients should review the capabilities of these systems thoroughly before committing to their implementation. **J. P. Morgan and Reuters do not endorse the products of these third parties nor do they warrant their accuracy in the application of the RiskMetrics methodology** and in the use of the underlying data accompanying it.

Reference

Glossary of terms

absolute market risk. Risk associated with the change in value of a position or a portfolio resulting from changes in market conditions i.e., yield levels or prices.

adverse move X. Defined in RiskMetrics as 1.65 times the standard error of returns. It is a measure of the most the return will move over a specified time period.

ARCH. Autoregressive Conditional Heteroskedascticity. A time series process which models volatility as dependent on past returns. GARCH—Generalized ARCH, models volatility as a function of past returns and past values of volatility. EGARCH—Exponential GARCH, IGARCH—Integrated GARCH. SWARCH—Switching Regime ARCH.

autocorrelation (serial correlation). When observations are correlated over time. In other words, the covariance between data recorded on the same series sequentially in time is non-zero.

beta. A volatility measure relating the rate of return on a security with that of its market over time. It is defined as the covariance between a security's return and the return on the market portfolio divided by the variance of the return of the market portfolio.

bootstrapping. A method to generate random samples from the observed data's underlying, possibly unknown, distribution by randomly resampling the observed data. The generated samples can be used to compute summary statistics such as the median. In this document, bootstrapping is used to show monthly returns can be generated from data which are sampled daily.

CAPM. Capital Asset Pricing Model. A model which relates the expected return on an asset to the expected return on the market portfolio.

Cholesky factorization/decomposition. A method to simulation of multivariate normal returns based on the assumption that the covariance matrix is symmetric and positive-definite.

constant maturity. The process of inducing fixed maturities on a time series of bonds. This is done to account for bonds "rolling down" the yield curve.

decision horizon. The time period between entering and unwinding or revaluing a position. Currently, RiskMetrics offers statistics for 1-day and 1-month horizons.

decay factor. See lambda.

delta equivalent cash flow. In situations when the underlying cash flows are uncertain (e.g. option), the delta equivalent cash flow is defined as the change in an instrument's fair market value when its respective discount factor changes. These cash flows are used to find the net present value of an instrument.

delta neutral cash flows. These are cash flows that exactly replicate a callable bond's sensitivity to shifts in the yield curve. A single delta neutral cash flow is the change in the price of the callable bond divided by the change in the value of the discount factor.

duration (Macaulay). The weighted average term of a security's cash flow.

EM algorithm. A statistical algorithm that can estimate parameters of a function in the presence of incomplete data (e.g. missing data). EM stands for Expectation Maximization. Simply put, the missing values are replaced by their expected values given the observed data.

exponential moving average. Applying weights to a set of data points with the weights declining exponentially over time. In a time series context, this results in weighing recent data more than the distant past.

GAAP. Generally Accepted Accounting Principles.

historical simulation. A non-parametric method of using past data to make inferences about the future. One application of this technique is to take today's portfolio and revalue it using past historical price and rates data.

kurtosis. Characterizes relative peakedness or flatness of a given distribution compared to a normal distribution.[1]

$$K_x = \left\{ \frac{N^2 - 2N + 3}{(N-1)(N-2)(N-3)} \sum_{i=1}^{N} \left(\frac{X_i - \bar{x}}{\sigma_x} \right)^4 \right\} - 3\frac{(N-1)(2N-3)}{N(N-2)(N-3)}$$

Since the unconditional normal distribution has a kurtosis of 3, excess kurtosis is defined as K_x-3.

λ **lambda (decay factor).** The weight applied in the exponential moving average. It takes a value between 0 and 1. In the RiskMetrics lambda is 0.94 in the calculation of volatilities and correlations for a 1-day horizons and 0.97 for 1-month horizon.

leptokurtosis (fat tails). The situation where there are more occurrences far away from the mean than predicted by a standard normal distribution.

linear risk (nonlinear). For a given portfolio, when the underlying prices/rates change, the incremental change in the payoff of the portfolio remains constant for all values of the underlying prices/rates. When this does not occur, the risk is said to be nonlinear.

log vs. change returns. For any price or rate P_t, log return is defined as ln (P_t/P_{t-1}) whereas the change return is defined by $(P_t - P_{t-1})/P_{t-1}$. For small values of $(P_t - P_{t-1})$, these two types of returns give very similar results. Also, both expressions can be converted to percentage returns/changes by simply multiplying them by 100.

mapping. The process of translating the cash flow of actual positions into standardized position (vertices). Duration, Principal, and cash flow.

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} X_i$$

mean. A Measure of central tendency. Sum of daily rate changes divided by count

mean reversion. When short rates will tend over time return to a long-run value.

modified duration. An indication of price sensitivity. It is equal to a security's Macaulay duration divided by one plus the yield.

outliers. Sudden, unexpectedly large rate or price returns.

[1] We would like to thank Steven Hellinger of the New York State Banking Department for pointing this formula out for us.

overlapping data. Consecutive returns that share common data points. An example would be monthly returns (25-day horizon) computed on a daily basis. In this instance adjacent returns share 24 data points.

nonparametric. Potential market movements are described by assumed scenarios, not statistical parameters.

parametric. When a functional form for the distribution a set of data points is assumed. For example, when the normal distribution is used to characterize a set of returns.

principle of expected return. The expected total change in market value of the portfolio over the evaluation period.

relative market risk. Risk measured relative to an index or benchmark

residual risk. The risk in a position that is issue specific.

skewness. Characterizes the degree of asymmetry of the distribution around its mean. Positive skews indicate asymmetric tail extending toward positive values (right-hand side). Negative skewness implies asymmetry toward negative values (left-hand side).

$$S_x = \frac{N}{(N-1)(N-2)} \sum_{i=1}^{N} \left(\frac{X_i - \bar{x}^2}{\sigma_x} \right)$$

speed of adjustment. A parameter used in modelling forward rates. It is estimated from past data on short rates. A fast speed of adjustment will result in a forward curve that approaches the long-run rate at a relatively short maturity.

stochastic volatility. Applied in time series models that take volatility as an unobservable random process. Volatility is often modeled as a first order autoregressive process.

standard deviation. Indication of the width of the distribution of changes around the mean.

$$\sigma_x = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (X_i - \bar{x})^2}$$

Structured Monte Carlo. Using the RiskMetrics covariance matrix to generate random normal variates to simulate future price scenarios.

total variance. The variance of the market portfolio plus the variance of the return on an individual asset.

zero mean. When computing sample statistics such as a variance or covariance, setting the mean to a prior value of zero. This is often done because it is difficult to get a good estimate of the true mean.

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