Modelling Some Properties of Stock Markets in Transition Economics

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Abstract

In contrast to predominant behaviour of financial series of developed markets (no or very short serial correlations), financial series of emerging markets exhibit different behaviour. We investigate financial series of index returns for ten European transition economies. The results suggest the presence of long-range correlations. Additionally, all series seem to be asymmetric distributed and exhibit magnitude long-range correlations, as commonly found for developed markets. We model these properties with a process, which is presented in Section II. To support some of these model findings, we employ wavelet estimates of the Hurst exponent, the Geweke and Porter-Hudak method, and detrended fluctuation analysis.

Keywords: efficient-market hypothesis, fractionally integrated process, power-law correlations, phase-randomization procedure, nonlinearity

JEL Classification: C13, C22, G14, G15

1. Introduction

Currently, most transition countries have formal capital markets. While banks have slowly evolved into the main source of finance for businesses, there is in-
creasing focus on the crucial role these markets can play in supporting economic development. It is more and more evident that capital markets extend choice and competition in the provision of financial services, provide longer-term finance, help improve corporate governance, diversify risks and, ultimately, contribute to a sound financial system.

Significant improvements in the financial sector have been made in the last decade in many areas, including establishment of the formal exchanges, the development of legal frameworks and regulatory institutions, the establishment of internationally compatible accounting standards, and improvements in transparency and corporate governance. However, most markets are still in their infancy. Many of the small markets remain illiquid or exist only on paper. Even in the advanced countries, there remains considerable room for improving market depth and liquidity, as well as regulations and institutions.

Due to these developments it would be interesting to provide some deeper insight into the properties of the markets in transition economies. First and probably most interesting question would be whether the observed markets differ in their behaviour from developed capital markets. One way to analyse the differences in behaviour is to test the week form of market efficiency.

In contrast to predominant behaviour of financial series of developed markets (no or very short serial correlations), financial series of emerging markets exhibit different behaviour (Wright, 2001). For example, it is shown that weekly returns of large number of Greek stocks exhibit statistically significant long memory (Panas, 2001). Recently, for six transition economies in East and Central Europe it has been shown that series of index returns daily recorded exhibit long memory (Jagric et al., 2005).

Here we expand our analysis and investigate financial series of index returns for ten European transition economies. The results suggest the presence of long-range correlations. Additionally, all series seem to be asymmetric distributed and exhibit magnitude long-range correlations, as commonly found for developed markets. We model these properties with a process, which is presented in Section 2. To support some of these model findings, we also employ wavelet estimates of the Hurst exponent and the Geweke and Porter-Hudak method, and detrended fluctuation analysis.

2. Methodological Framework

In order to test long memory, we will first employ wavelet estimates of the Hurst exponent. The process of calculating the estimates is based on the estima-
tion of the wavelet coefficient \( |d_x(j,k)|^2 \) which measures the amount of energy in the analysed signal about the time instant \( 2^j k \) and frequency \( 2^{-\nu} \), where \( \nu_0 \) is an arbitrary reference frequency selected by the choice of mother wavelet \( \psi_0 \). It has been suggested (Abry et al., 1993) that a useful spectral estimator can be designed by performing a time average of \( |d_x(j,k)|^2 \) at a give scale, that is

\[
\hat{\Gamma}_x(2^{-\nu_0}) = \frac{1}{n_j} \sum_k |d_x(j,k)|^2
\]

where \( n_j \) is the available number of wavelet coefficients at octave \( j \). Essentially \( n_j = 2^{-j} n \), where \( n \) is the length of the data. \( \hat{\Gamma}_x(\nu) \) is therefore a measure of the amount of energy that lies within a give bandwidth around the frequency \( \nu \) and can be regarded as a statistical estimator for the spectrum \( \Gamma_x(\nu) \) of \( x \). It is possible to design an estimator \( \hat{H} \) of the parameter \( H \) from a simple linear regression of \( \log_2(\hat{\Gamma}_x(2^{-j}\nu_0)) \) on \( j \). Under Gaussian and quasi-decorrelation of the wavelet coefficient hypotheses and in the asymptotic limit, a closed-form for the variance of the estimate of \( H \) can be obtained and given by

\[
\sigma_H^2 = \text{var} \hat{H}(j_1, j_2) = \frac{2}{n_{j_1} \ln^2 2} \frac{1 - 2^J}{1 - 2^{-J+1}(J+4) + 2^{-2J}}
\]

where \( J = j_2 - j_1 \) is the number of octaves involved in the linear fit and \( n_{j_1} = 2^{-j_1} n \) is the number of available coefficients at scale \( j_1 \). It can be shown that this variance is the smallest possible, that is equal to the Cramer-Rao bound, for a given \( J \). For further details on this estimator see Abry et al. (1995).

From the above closed-form for the variance estimation, one can derive a confidence interval

\[
\hat{H} - \sigma_{\hat{H}} z_\beta \leq H \leq \hat{H} + \sigma_{\hat{H}} z_\beta
\]

where \( z_\beta \) is the \( 1 - \beta \) quantile of the standard Gaussian distribution \( P(z \geq z_\beta) = \beta \). All the results presented below, both in numerical simulations and actual data analysis, are computed with \( \beta = 0.025 \) (95% confidence intervals), based on the above hypothesis.

With the goal of constructing a stochastic process that can simulate time series of changes (\( \delta \tau_j \)) with power-law correlations in \( \delta \tau \) and power-law correla-
tions in magnitudes \( |\delta \tau_k| \), we use a combination of two fractionally integrated processes, namely, the ARFIMA process (Hosking, 1981) with the FIARCH process (Granger and Ding, 1996) and define process (Podobnik et al. 2005a, 2006):

\[
\delta \tau_i = \sum_{n=1}^{\infty} a_n(\rho_1) (\delta \tau_{i-\alpha} - \lambda \delta \tau_{i-\alpha-1}) + \sigma_i \eta_i \quad (4a)
\]

\[
\sigma_i = \sum_{n=1}^{\infty} a_n(\rho_2) \frac{|\delta \tau_{i-\alpha}|}{|\delta \tau_i|} \quad (4b)
\]

\[
a_n(\rho) = \rho \frac{\Gamma(n-\rho)}{\Gamma(1-\rho)\Gamma(1+n)} \quad (4c)
\]

Here, \( \lambda \) is an asymmetric parameter, \( \rho_1 \in [0,0.5) \) and \( \rho_2 \in [0, 0.5) \) are free parameters, \( \Gamma \) denotes the Gamma function, and \( \eta_i \) denotes independently and identically distributed Gaussian variables with expectation value \( \langle \eta_i \rangle = 0 \) and variance \( \langle \eta_i^2 \rangle = \sigma^2 \) (in numerical simulations we do not use for the variance the value 1 but we use the variance as a free parameter – that is a parameter needed to obtain the signal of appropriate variance). For \( \rho \in (0,0.5) \), the weights \( a_n(\rho) \) satisfy the constraint \( \sum_{n=1}^{\infty} a_n(\rho) = 1 \), and by using the Stirling formula it can be shown that the weights scale as \( a_n(\rho) \propto n^{1-\rho} \) for asymptotically large values of \( n \) (Hosking, 1981). If \( \lambda = 0 \), the process of equations (4a) - (4c) we denote \( A(\rho_1, \rho_2) \).

To eliminate trends in empirical data, one commonly takes first-order differences \( x_i - x_{i-1} \), or calculates higher-order integer differences. Fractional processes (Hosking, 1981) are obtained by allowing the order \( \rho \) in the fractionally differencing operator \( (1-L)^\rho \) to take fractional values. After expanding \( (1-L)^\rho \) as an infinite binomial series in powers of \( L \), one can show that equations (4a) – (4c) can be expressed as \( (1-L)^\rho x_i = \sigma_i \eta_i \).

For \( \rho_2 \to 0 \), \( \sigma_i \) becomes 1, and process \( A(\rho_1, \rho_2) \) reduces to the ARFIMA process \( A(\rho_1, 0) \); for \( \rho_1 \to 0 \), process \( A(\rho_1, \rho_2) \) reduces to the FIARCH process \( A(0, \rho_2) \); and for positive \( \rho_1 = \rho_2 \), process \( A(\rho_1, \rho_2) \) reduces to process \( B(\rho, \lambda = 0) \) proposed in Podobnik et al. (2005a). Process \( B(\rho, \lambda) \) was proposed with the goal of modelling time series with power-law correlations in \( \delta \tau_i \) and an
asymmetric distribution of $\delta \tau$, and for $\lambda = 0$ process $B(\rho, \lambda)$ generates time series with a symmetric distribution of $\delta \tau$ and power-law correlations in $\delta \tau$, similar to time series generated by the ARFIMA process $A(\rho, 0)$. Since equations (4a) – (4c) are invariant under the transformation $x_i \rightarrow -x_i$ and $\eta_i \rightarrow -\eta_i$, process $A(\rho_1, \rho_2)$ generates symmetric probability distributions, i.e., $P(x) = P(-x)$, for arbitrary $\rho_1 \in (-0.5, 0.5)$ and $\rho_2 \in [0, 0.5)$.

3. Data

Since the main objective of this paper is the analysis of a selected capital markets in transition economies, we need aggregate time series, which will represent the activity in the observed markets. As already indicated in the introduction, we collect the data for ten transition economies of central and east Europe: Czech Republic, Hungary, Poland, Russia, Slovakia, Slovenia, Croatia, Lithuania, Estonia and Latvia. We decided to use major stock market indices (PX50, BUX, WIG20, RTS, SAX, SBI, CROEMI, VILSE, TALSE and RICI). In order to capture the real dynamics of the market we use daily close values.

Most financial models do not attempt to model close values, but instead deal with returns on the instrument. This is heavily reflected in the literature of finance and economics (see, for example, Mantegna and Stanley, 2000). The return is the profit or loss in buying a financial instrument, holding it for some period of time and then selling it. The most common way to calculate a return is the log-return:

$$ R_t = \log S(t + \Delta t) - \log S(t) $$

where $S(t)$ is an index and $\Delta t = 1$ day. Basic statistics for the indices are presented in Table 1. Let us first look at the skewness. The skewness of a symmetric distribution, such as the normal (Gaussian) distribution, is zero. However none of the series seems to be symmetric. Among them, five series (PX50, SBI, CROEMI, TALSE and RICI) are with positive skewness (distributions with pronounced right tails), while other series have negative skewness (distributions with pronounced left tails).

Next statistic, which is important for our analysis is the kurtosis. The kurtosis of the normal distribution is three. In our case all series have a kurtosis which exceeds a value of three. This means that the distributions are peaked (leptokurtic) relative to the normal distribution.

Since both descriptive statistics (skewness and kurtosis) indicate deviations from the values characterized for normal distribution, we can expect that the observed distributions are not normally distributed. Therefore we calculate the
Jarque-Bera statistic, which is a test statistic for testing whether the series is normally distributed. The reported probability is the probability that a Jarque-Bera statistic exceeds (in absolute value) the observed value under the null hypothesis – a small probability value leads to the rejection of the null hypothesis of a normal distribution. In our case the results for all series indicate a rejection of the null hypothesis.

Procedures for the estimations of the properties of the stock markets are very sensible to the statistical properties of the sample data. The testing of the stationarity hypothesis is particularly difficult in the presence of LRD (Labour Research Department) where many classical statistical approaches cease to hold.
Table 1
Even without LRD however, there is the fundamental problem that there are an infinity of ways in which a process can be non-stationary. Normally we must choose a particular model framework and test for stationarity only against the types of non-stationarity encompassed by it (an example is the ADF test). To assist in the process it is important to include a priori information concerning the known physics of the problem (Jagric, 2003).

In contrast to standard procedures, the wavelet based estimator seems to have a substantial robustness against an important class of non-stationarity, namely the addition of deterministic trends (Jagric and Ovin, 2004). This is a particularly important advantage in a LRD context where it is very difficult in theory and in the practice to distinguish between real trends and long term sample path variations due to LRD.

We perform a simple unit root test in order to test the stationarity of the selected series. We use the Augmented Dickey-Fuller (ADF) test. The results for the ADF test statistic are compared with MacKinnon critical values for rejection of hypothesis of a unit root at 1% significant level. In our case all series seem to be stationary – at least in the form, which is incorporated in the ADF test.

4. Results

After the examination of basic statistical properties of the selected time series, we can now perform the estimations of the Hurst exponent. The exponent is estimated for every series for ten different numbers of vanishing moments. This enables us to test if the estimations are stable and can be treated as reliable. Additionally we can also make some conclusions about the statistical properties based on the relation between the number of vanishing moments and the Hurst exponent.

The results for the estimated Hurst exponent are presented in Figure 1. The weighted average estimation of the Hurst exponent gives a surprising picture of the stock markets. The results indicate that we can make two groups of markets. In the first group are the markets where quite strong long-range dependence can be detected. This is the case for Czech Republic ($\bar{H}_{PX50} = 0.65 \pm 0.03$), Hungary ($\bar{H}_{BUX} = 0.63 \pm 0.03$), Russia ($\bar{H}_{RTSH} = 0.65 \pm 0.04$), Slovenia ($\bar{H}_{SBIH} = 0.66 \pm 0.03$), Croatia ($\bar{H}_{CROEMI} = 0.62 \pm 0.05$), Lithuania ($\bar{H}_{VILSE} = 0.63 \pm 0.04$), Latvia ($\bar{H}_{RICI} = 0.62 \pm 0.04$), and Estonia ($\bar{H}_{TALSE} = 0.65 \pm 0.04$). The second group consists of markets where no or only extremely week form of long-range de-
dependence could be detected. This is the case for Poland ($\widetilde{H}_{WIG20} = 0.57 \pm 0.04$) and Slovakia ($\widetilde{H}_{SAX} = 0.53 \pm 0.04$).

Figure 1
Estimated Hurst Exponent for the Selected Stock Markets
Note: H-LRD – wavelet estimation of the Hurst exponent for different number of vanishing moments with 95% confidence interval, \( D \) – number of vanishing moments (type of the selected wavelet), \( j_1 - j_1 \) – number of octaves included in the estimation.

Sources: authors calculations.

Based on the weighted average estimates of the exponent, the strongest evidence for long-range dependence is detected in Slovenia. Since Slovenian stock market is small in comparison to other markets in the group, this could be the reason for market inefficiency. This is also supported by the results for Poland, which is the largest stock market in the group and exhibits only extremely weak form of long-range dependence. However, the results for other countries do not support this hypothesis. We try to find the explanation for these results in different measures of the market size (market capitalization, number of domestic firms listed, number of foreign firms listed...), however, none of the statistical measures could successfully explain the results. We think that the explanation of different behaviour of the stock markets is more complex and involves the historical development, institutional regulations and the properties of the economic system.

To additionally analyse correlations properties, we employ the Geweke and Porter-Hudak (1983) method (GPH) for estimating differencing parameter \( d \) and detrended fluctuation analysis (DFA) (Peng et al., 1994). For \( d = (0, 0.5) \), the process is said to exhibit long memory, or long-range positive dependence. The process is said to exhibit intermediate memory (anti-persistence), or long-range negative dependence, for \( d = (-0.5, 0) \). The process exhibits short memory for \( d = 0 \), corresponding to stationary and invertible ARMA process. For \( d = [0.5, 1) \) the process is mean reverting, even though it is not covariance stationary, as there is no long-run impact of an innovation on future values of the process.

Table 1 reports the empirical estimates for the fractional differencing parameter \( d \) as well as the test results regarding their statistical significance. To test the statistical significance of the \( d \) estimates, two-sided \( (d = 0 \text{ versus } d \neq 0) \) test is performed. The known theoretical variance of the spectral regression is imposed in the construction of the \( t \)-statistic for \( d \). As Table 1 indicates, there is some evidence supporting the long-memory hypothesis for the returns series of most stock indices. As in the case of wavelet estimates of Hurst exponent, the only exceptions are Slovakia and Poland, where it is impossible to reject the null hypothesis of \( d = 0 \). When we consider the absolute returns series, evidence of long memory is found for all stock markets. Additionally, it is interesting that for all markets the estimated parameter \( d_{gh} \) is lower than \( d_{pq} \), implying that magnitude correlations are generally stronger than serial correlations.

Similar results are obtained by using detrended fluctuations analysis (DFA) that is an analogue of variance growth proposed for signals in the presence of nonstationarity (Peng et al., 1994). The scaling function \( F(n) \) follows scaling law
$F(n) \propto n^\alpha$ if power-law correlations exist. Exponent $\alpha = 0.5$ corresponds to series with no correlations. The results are presented in Table 1. The results indicate that we can group markets in at least two groups. First group, characterized by strong or medium long-range correlations includes indices of Estonia ($\alpha = 0.70$), Lithuania ($\alpha = 0.63$), Czech Republic ($\alpha = 0.63$), Slovenia ($\alpha = 0.62$), Russia ($\alpha = 0.60$), Hungary ($\alpha = 0.59$), Latvia ($\alpha = 0.58$), and Croatia ($\alpha = 0.58$). The second group comprises markets specified by no or very weak form of long-range dependence. This is the case for Poland ($\alpha = 0.56$) and Slovakia ($\alpha = 0.53$).

Now we calculate the DFA scaling exponents for time series. From Table 1 we see that for each index series (i) $|\hat{R}_t|$ is specified by long-range correlations, as found for almost all well-known market indices, (ii) the DFA exponent calculated for $|\hat{R}_t|$ is larger than the one calculated for $R_t$, and (iii) the DFA exponent $\alpha_R$ and fractional differencing parameter $d$ calculated by GHP method roughly follow the relation $\alpha_R = 0.5 + d$.

Next, we investigate to which degree the time series of indices exhibit linear and nonlinear properties (Ashkenazy et al., 2001). It is known that randomizing of the phases of the Fourier transform of signal $x_t$ does not change the power spectrum or autocorrelations of $x_t$, but it may change the power-spectrum of the time series $|x_t|$. Specifically, we perform a Fourier transformation of the sequence of $x_t$ and randomly permute the phases. Then we perform an inverse Fourier transformation and obtain a new surrogate signal $\tilde{x}_t$. For nonlinear (linear) process, magnitude correlations are (not) changed by a phase-randomization of the original time series (Podobnik et al., 2005b). We find the same behaviour for all other indices studied – after the phase-randomization procedure, correlations in phase-randomized surrogate series of magnitudes $|\tilde{R}_t|$ vanish, while correlations in phase-randomized surrogate series $\tilde{R}_t$ remain unchanged compared to the original series.

To model time series $R_t$ with power-law correlations in $R_t$ and $|\hat{R}_t|$ and asymmetry in the distributions $P(R_t)$, we employ the process of equations (4a) – (4c). We present the results for PX50 index, as representative index among European transition economies, characterized by significant nonvanishing values calculated for skewness (approx 1.3) and kurtosis (approx 17). First, from the scaling relations $\alpha_R = 0.5 + \rho_1$ and $\alpha_{|R|} = 0.5 + \rho_2$ between DFA exponents and model parameters, we set the two parameters $\rho_1 = 0.13$ and $\rho_2 = 0.36$ to model scaling exponents $\alpha_R = 0.63$ and $\alpha_{|R|} = 0.86$ found in the data (Table 1).
by numerical simulations we obtain the time series $R_t$ and $|R_t|$ and in Figure 2 show that the scaling exponents $\alpha_R = 0.63$ and $\alpha_{\bar{R}} = 0.85$ calculated for the scaling function $F(n)$ for both time series $R_t$ and $|R_t|$ are in good agreement with the values found in the data.

**Figure 2**

**Time Series of Rate of Return PX50 Index**

Note: a) Time series for the process with $\rho_1 = 0.13$, $\rho_2 = 0.36$, and $\lambda = 0.15$. DFA curves calculated for time series $R_t$ and that obtained after phase randomization procedure $\tilde{R}_t$, and for two magnitudes time series $|R_t|$ and $|\tilde{R}_t|$. We see that after phase randomization series $|\tilde{R}_t|$ exhibits no correlations.

**Sources:** authors calculations.
Note: b) Linear-log plot of distributions. After rescale we show probability distribution $P(R)$ together with the distribution for the empirical data. Differences in the tails are due to different length of empirical and model time scales.

Sources: authors calculations.

In Figure 2(b) we show the probability distributions $P(R)$ calculated for both process and empirical data, where to account for skewness in empirical data, we set $\lambda = 0.15$. For the time series of the process of equation 4, for skewness and kurtosis we calculate the following values, 0.9 and 18, respectively, in a good agreement with the corresponding values 1.3 and 17 found in the data. For the process of equation 4 we find not just that (i) $\alpha_{\text{PR}} > \alpha_x$ as previously found in the data, but also that (ii) after phase randomization procedure, magnitudes correlations $|\tilde{R}_x|$ and $|\tilde{R}_t|$ completely vanish, while correlations in $\tilde{R}_t$ practically remain unchanged compared to the original time series $R_t$, that is a behaviour we also found in empirical data. In Table 1 we see that the same scaling behaviour exists for all indices analysed.

Conclusion

We analysed the long-range dependence in the capital markets of transition economies in Central and East Europe. For the selected data of ten major stock indices, we first employed a wavelet analysis and find (i) the long-range serial-dependence. The results in this study are mixed. The first group represents the stock markets where quite strong long-range dependence is detected (Estonia Lithuania, Czech Republic, Slovenia, Russia, Hungary, Latvia, and Croatia).
second group represents the stock market, where no or only a weak form of long-range dependence was detected (Poland and Slovakia). These findings are also supported by the results of GHP and DFA, where the same two groups can be identified.

Additionally to these findings, the results also indicate (ii) long-range dependence in the magnitudes for all indices. The probability distributions exhibit (iii) asymmetric behaviour. These properties should be included in modelling of trading systems. For that reason, we use a stochastic process specified by three parameters with properties as found in the empirical data. For data analysed model parameters are easy related with scaling exponents of empirical time series.

Analysed properties of selected stock markets indicate different behaviour in comparison to developed markets. We think that there are several reasons, which could explain such behaviour. First, the stock markets in the region have not yet reached a level that corresponds to the size of its population and economy. Market liquidity has generally been increasing in many of the transition economies but is still modest by comparison to other emerging markets. Furthermore, concentration of the markets is substantial since stock markets in the region are dominated by a small number of large firms – typically those in the banking, electric power, natural resource and telecommunications sectors.

Second, all analysed markets are in the process of developing and harmonizing the corresponding legal systems. However, the length of the law’s reach varies widely from country to country. For example, minority shareholders’ rights and corporate governance standards in Russia have been weak over the decade and firms have tended to be controlled by insiders rather than shareholders. Senior managers often used their powers for personal gain. Even in the most advanced economies, the implementation and enforcement of established laws and regulations remains problematic as regulatory institutions lack sufficient empowerment, as judges often hesitate to apply newly introduced laws, and as the necessary information is not widely distributed. Thus, the legal protection of investors has been more effective on paper than in reality.

Third, there is still lack of sound corporate governance and business practices. In recent years, significant progress has been made in this area in a number of countries. A new commercial code came into effect in Poland in 2001, which permits small investors to band together in order to influence a company’s activities. In Hungary, a new comprehensive securities law has been enacted as of January 1, 2002. In the Czech Republic, a comprehensive commercial code of 1992 was amended effective January 1, 2001, replacing approximately 80 assorted codes and regulations and establishing a sound legal framework for most
business-to-business activities. The legislation should improve the protection of minority shareholders' rights, clarify the responsibilities of the board of directors, improve disclosure and make takeovers more transparent.

We believe that stock markets will eventually resemble the properties of developed markets if the emphasis in development of the markets will be on opening exchanges, introduction of internationally recognised accounting systems and audit standards, strengthening the institutional capabilities of regulator, establishment of good corporate governance, introduction of foreign portfolio investment, broadening the asset base of institutional investors, and demutualisation and merger/alliance of stock exchanges. Many of these are inter-related and complementary and therefore represent a though challenge for the future development of the Central and Eastern European stock markets.

References

