Calling patterns in human communication dynamics

Zhi-Qiang Jianga,1, Wen-Jie Xiea, Ming-Xia Lia, Boris Podobnikb,c,d, Wei-Xing Zhoua,1, and H. Eugene Stanleya,b

aSchool of Business, School of Science, and Research Center for Econophysics, East China University of Science and Technology, Shanghai 200237, China; bDepartment of Physics and Center for Polymer Studies, Boston University, Boston, MA 02215; cZagreb School of Economics and Management, 10000 Zagreb, Croatia; and dFaculty of Civil Engineering, University of Rijeka, 51000 Rijeka, Croatia

Contributed by H. Eugene Stanley, November 27, 2012 (sent for review October 14, 2012)

Modern technologies not only provide a variety of communication modes (e.g., texting, cell phone conversation, and online instant messaging), but also detailed electronic traces of these communications between individuals. These electronic traces indicate that the interactions occur in temporal bursts. Here, we study intercall duration of communications of the 100,000 most active cell phone users of a Chinese mobile phone operator. We confirm that the intercall durations follow a power-law distribution with an exponential cutoff at the population level but find differences when focusing on individual users. We apply statistical tests at the individual level and find that the intercall durations follow a power-law distribution for only 3,460 individuals (3.46%). The intercall durations for the majority (73.34%) follow a Weibull distribution. We quantify individual users using three measures: out-degree, percentage of outgoing calls, and communication diversity. We find that the cell phone users with a power-law duration distribution fall into three anomalous clusters: robot-based callers, telecom fraud, and telephone sales. This information is of interest to both academics and practitioners, mobile telecom operators in particular. In contrast, the individual users with a Weibull duration distribution form the fourth cluster of ordinary cell phone users. We also discover more information about the calling patterns of these four clusters (e.g., the probability that a user will call the c-th most contact and the probability distribution of burst sizes). Our findings may enable a more detailed analysis of the huge body of data contained in the logs of massive users.

Materials and Methods

U
nderstanding the temporal patterns of individual human interactions is essential in managing information spreading and in tracking social contagion. Human interactions (e.g., cell phone conversations and e-mails) leave electronic traces that allow the tracking of human interactions from the perspective of either static complex networks (1–6) or human dynamics (7). Because static networks only describe sequences of instantaneous interacting links, temporal networks in which the temporal patterns of interacting activities for each node are recorded have recently received a considerable amount of research interest (8, 9). Investigations of interevent intervals between two consecutive interacting actions, such as e-mail communications (7, 10), short-message correspondences (11–13), cell phone conservations (14, 15), and letter correspondences (16–18), indicate that human interactions have non-Poissonian characteristics. Previous studies were conducted either on aggregate samples (14, 15, 19) or on a small group of selected individuals (7, 10–12, 16–18), but the communication behavior of individuals is not well understood.

We study the complete voice information for cell phone users supplied by a Chinese cell phone operator and study the interevent time between two consecutive outgoing calls (intercall duration). Our studies are performed at both the individual and group levels. To ensure better statistics, the top 100,000 cell phone users with the largest number of outgoing calls are chosen as our data sample, each having more than 997 outgoing calls. We propose a bottom-up approach to investigate individual cell phone communication dynamics: (1) finding the functional form of the distribution of each individual’s intercall durations, (2) grouping individuals with the same distribution, and (3) understanding the calling patterns for each group. We apply an automatic fitting technology to each mobile phone user and filter out two groups of users according to their intercall duration distributions. One group comprises individuals with a power-law duration distribution (3,464 individuals) and the other comprises individuals with a Weibull duration distribution (73,339 individuals). We demonstrate that the two groups exhibit different calling patterns and that the individuals from the power-law group exhibit anomalous communication behaviors (e.g., the group includes individuals sending spam).

Results

Distribution at the Population Level. There are 5,921,696 different individuals in our dataset (see the data description in Materials and Methods). For each individual, we estimate the intraday intercall durations (d seconds) (see definition of intraday intercall durations in Materials and Methods), and we find that 4,635,536 individuals have nonempty intraday durations (d > 0), which we consider one unique sample when we investigate the distribution at the population level. To this end we also analyze the aggregate level where the data comprise only the durations of the top 100,000 individuals.

Fig. L4 shows the empirical distributions of the two samples of aggregate data. Both curves exhibit excellent power-law behaviors in the range of (80, 2,000) s. We apply the least-square method and find that a linear fit gives the power-law exponent γ = 0.873 for all of the individuals and γopt = 0.942 for the top 100,000 individuals, respectively. We compare the empirical distributions obtained from our dataset with the empirical distributions of intercall durations based on a different dataset provided by a European cell phone operator (14) (see also the supplementary information of ref. 20) and note that the empirical distributions of both datasets share very similar patterns for d ≤ 105, where only intraday intercall durations are taken into consideration. The reported power-law exponent γ = 0.9 in ref. 20 is approximately equal to the estimated exponents γd as shown in Fig. L4. A similar functional form with a power-law exponent γ = 0.7 is also reported in ref. 15 for intercall durations smaller than 109. This similarity is further consolidated by fitting the empirical duration distributions by means of a formula of power law with an exponential cutoff.

Fig. L4 shows a clear deviation from the power-law distribution in the tails of both curves, which is usually interpreted as an exponential cutoff. To test which distribution better fits the data, we apply Kolmogorov–Smirnov (KS) statistics by means of which the smaller the value, the better the fit. We set the truncation value at 2,000 s and find that for all individuals the tail is better fit by the Weibull distribution (KS = 0.016) than by the exponential distribution (KS = 0.063). Similarly, for the top 105 individuals, the Weibull distribution (KS = 0.006) also fits the data better than


The authors declare no conflict of interest.

Freely available online through the PNAS open access option.

1To whom correspondence may be addressed. E-mail: zqjiang@ecust.edu.cn, wxzhou@ecust.edu.cn, or hes@bu.edu.
the exponential distribution ($KS = 0.066$). Fig. 1B for varying truncation values shows a plot of KS as a function of $d_{\text{min}}$. The KS statistic displays a more stable behavior for the Weibull fit than for the exponential fit, indicating that the Weibull distribution is better able to capture the tail behavior than the exponential distribution.

We further divide the sequence of individuals according to the number of outgoing calls into 46 groups, sorted in ascending order. The first group comprises 135,536 individuals, and the remaining 45 groups each comprise 100,000 individuals. We calculate the empirical distributions of the aggregate intercall durations for each group and find that all of the distributions share patterns similar to those shown in Fig. 1A. Fig. 1C shows a plot of the estimated power-law exponents with respect to the average number of outgoing calls ($\langle \tilde{\tau}_{\text{call}} \rangle$) for different groups. The solid line stands for the power-law exponent of the whole sample. (D) Probability distribution of mean intercall durations for different samples of individuals.

### Classification of Cell Phone Users

According to the above analysis at the aggregate level, we propose to classify the individuals according to their duration distributions. Motivated by ref. 21, but here for each individual cell phone user, because we are focusing on the tail of the distribution, we assume the candidate duration distributions to be left-truncated and we assign each of them a distribution that is either power law or Weibull. We estimate the truncation value $d_{\text{min}}$ associated with distribution parameters by finding the minimum KS statistic. We then apply statistical tests to check the significance of the fitting parameters (Materials and Methods, Fitting Distributions and Statistical Tests). Finally, based on statistical tests, we find that there are 3,464 individuals whose intraday durations follow a power-law distribution and 73,339 individuals whose intraday durations follow a Weibull distribution (Materials and Methods, Determining the Distribution Form).

Fig. 2 shows the empirical duration distributions for three randomly chosen individuals (2308772, 28012863, and 52701654) whose intraday intercall durations follow a power-law distribution. The solid lines correspond to the power-law fits with power-law exponents $\gamma = 2.15$, $\gamma = 2.03$, and $\gamma = 1.69$ for individuals 2308772, 28012863, and 52701654, respectively. Fig. 2B plots the distribution of the estimated power-law exponents $\gamma$ for all individuals with an intraday intercall duration that follows a power-law distribution and finds that none of the power-law exponents are lower than 1.5. This is in sharp contrast to the power-law exponents lower than 1 that we found for the aggregate durations in Fig. 1C. Note that there is a large fraction of individuals whose power-law exponents are between 1 and 3, which are the characteristic values for the Lévy regime (1.3).

We investigate the distribution of the aggregate intraday intercall durations by treating the individual durations from the power-
Calling Patterns for Power-Law and Weibull Groups. Using three measurements, we quantitatively distinguish the calling patterns of the individuals belonging to two different classified groups.

i) The out-degree $k_i$ describes the number of different callees for a specified cell phone user.

ii) The percentage of outgoing calls $r_{out}$ is defined by dividing the number of outgoing calls by the total number of calls—note that the number sending spams (junk message pusher) is characterized by $r_{out} = 1$.

iii) The communication diversity $\phi_i$. Motivated by the social diversity proposed in ref. 22, we define the communication diversity $\phi_i$ as a function of Shannon entropy to quantify how the cell phone users split the number of calls to their friends:

$$
\phi_i = \frac{-\sum_{j=1}^{k_i} p_{ij} \log(p_{ij})}{\log(k_i)}.
$$

Here $k_i$ is the out-degree and $p_{ij}$ is the probability defined as $p_{ij} = n_{ij} / n_i = n_{ij} / \sum_1^k n_{ij}$, where $n_{ij}$ is the number of outgoing calls

\[ \text{These individuals are excluded from the power-law group. We find that more than 97}\% \text{ of the individuals have Weibull tail distributions, as shown in Fig. 4B. However, the fact that the fitting range is lower than 1.5 orders of magnitude [83}\% \text{ of the individuals are in the range of (1,1.5)] disallows these individuals from being classified in the Weibull group. Fig. 4C shows a very small number of individuals whose intercall durations cannot be described by either power-law or Weibull distributions. Because most of the individuals have Weibull-tail distributions, the distributions of aggregate intercall durations and the mean intercall durations exhibit patterns very similar to the results obtained from the Weibull group (Figs. 4D and E and 3 C and D).} \]

\[ \text{Fig. 3.} \text{ Classified results of individuals with Weibull distributions of intraday intercall durations. (A) Probability distributions of intraday duration for three randomly chosen individuals. The markers of individual 5196860 and 6466665 are shifted vertically by a factor of 10^{-2} and 10^{-4} for better visibility. The solid lines are the best MLE fit to the Weibull distributions, which give the Weibull exponents } \beta = 0.54, \beta = 0.64, \text{ and } \beta = 0.51 \text{ for individual 3263120, 28012863, and 6466665, respectively. (B) Distribution of the Weibull exponents and the solid curve stands for the fits to normal distribution. (C) Probability distribution of collective intercall durations by aggregating the durations of different individuals from the Weibull group as one sample. The solid line is the best fit to the data by means of the least-square method, which gives an estimation of power-law exponent } \gamma = 0.882. (D) Probability distribution of the mean intercall durations for the Weibull group.} \]

\[ \text{Fig. 4.} \text{ Analysis of the remaining individuals. (A) Probability distribution of intercall durations for three individuals, whose durations approach to power-law behaviors without passing the statistical tests. (B) Probability distribution of intercall durations for three individuals, whose duration distributions are like Weibull shape but not confirmed by the statistical tests. (C) Plots of duration distributions for three individuals, whose distribution shapes are uncommon. (D) Probability distribution of collective intercall durations by aggregating the durations of different individuals from the Weibull group as one sample. The solid line is the best fit to the data by means of the least-square method, which gives an estimation of power-law exponent } \gamma = 0.879. (E) Probability distribution of the mean intercall durations for the remaining individuals.} \]

\[ \text{Jiang et al.} \text{ PNAS Early Edition | 3 of 6} \]
from individual $i$ to individual $j$ and $n_j$ is the total number of outgoing calls for individual $i$. A higher $\phi_i$ value indicates that the caller’s outgoing calls are split more evenly to his friends and a smaller $\phi_i$ value implies that most of the caller’s outgoing calls are to only one of his friends. Note that we define $\phi_i = 0$ when $k_i = 1$.

To distinguish between the calling patterns of the power-law group of Fig. 2 and the Weibull group of Fig. 3, in Fig. 5 we plot the distribution of the percentage of outgoing calls $r_{out}$ and the distribution of the communication diversity $\psi$ for power law group. Three ellipses correspond to the three clusters of individuals. (C) Similar as A but for Weibull group. (D) Similar as B but for Weibull group.

Fig. 5. Calling patterns for the individuals from power-law and Weibull group. (A) Distribution of the percentage of outgoing calls $r_{out}$ and the call diversity $\psi$ for power-law group. (B) Plots of out-degree $k$ with respect to communication diversity $\psi$ for power law group. Three ellipses correspond to the three clusters of individuals. (C) Similar as A but for Weibull group. (D) Similar as B but for Weibull group.

$\psi$ value implies that most of the callers are robot-based users. We next see that the individuals in cluster 1 — the ones characterized by a high frequency of call initiation, a small number of callees, or an allocation of almost all outgoing calls to only one callee — are robot-based users. We next see that the individuals in cluster 3 — the ones characterized by high frequency of call initiation, a large number of callees, and an even distribution of outgoing calls among all callees — are associated with telecom frauds and telephone sales. We also note that the individuals in cluster 4 are ordinary cell phone users. We next describe further differences in cell phone communication activities among the four clusters (e.g., the probability that a caller will call the $c_r$-th most contact and the burst size probability during burst periods).

Because most of the calls (mean 99.5% and min 94%) made by individuals in cluster 1 are to only one contact, we now calculate the probability that individuals belonging to the other three clusters will only call the $c_r$-th most contact. To rule out the influence of newly entering cell phone users, we take into account only those individuals listed in the data on the starting date of June 28, 2012. Fig. 6 shows the average calling frequency $f(c_r)$ of the $c_r$-th most contact friends for the individuals with the same degree in cluster 2. There is a linear relationship between $f(c_r)$ and $\ln c_r$ in Fig. 6A, which indicates an exponential distribution in the number of outgoing calls to different contacts (23). We see that the slope obtained between $f(c_r)$ and $\ln c_r$ increases as the out-degree $k$ increases, but the lack of individuals prevents us from finding the functional form between the slopes and the out-degree values $k$. We also observe power-law behavior between $f(c_r)$ and $c_r$ in cluster 3 of Fig. 6B. The least-square linear fits provide an estimate for power-law exponent 0.52 and also show that the behavior of the power-law exponent is not affected by the out-degree $k$.

Fig. 6C plots $f(c_r)^p$ versus $\ln c_r$ for cluster 4, where $b$ is associated with the maximum correlation coefficient of least-square
linear fits to \( f(e_i)^b \) versus \( \ln c_i \) by varying \( b \) from 0.01 to 0.99 with a step of 0.01. The linear relationship between \( f(e_i)^b \) and \( \ln c_i \) suggests that the number of outgoing calls to contacts follows a stretched exponential distribution (23, 24). In Fig. 6D we show the exponent \( b \) plotted with respect to the out-degree \( k \), where we observe a striking linear relationship: \( b = -4.836 \times 10^{-3}k + 0.329 \). Here we report that the probability to call the \( c_i \)-th most contact is in sharp contrast to the results reported in ref. 25, where a Zipf law with a power-law exponent 1.5 is observed when, in contrast to our “microscopic” study, individuals are not grouped according to their distributions of intercall durations.

It was recently proposed that the distribution of burst sizes indicates the presence of memory behaviors in the timing of consecutive events (15), where the deviation from exponential distributions is a hallmark of correlated properties. For a given series of events, a burst period is a cluster of consecutive events following their previous events within a short time interval \( \Delta t \), which is an arbitrarily assigned value in empirical analysis. The burst size \( e_i \) is defined as the number of events in a burst period.

Based on our dataset, Fig. 7 shows the probability distribution of burst sizes \( e_i \) at burst periods for the four clusters by setting \( \Delta t = 100, 300, 600, \) and 1,000 s. In Fig. 7A we find the probability distributions of \( e_i \) for cluster 1. Although we find a very good power-law relationship between \( p(e_i) \) and \( e_i \), with an exponent 2.677 for \( \Delta t = 100 \) s, the distributions deviate from power-law distributions and tend to exponential distributions for \( \Delta t = 300, 600, \) and 1,000 s. Fig. 7B shows that the probability distribution of \( e_i \) exhibits excellent exponential distributions for varying values of \( \Delta t \) for cluster 2. Fig. 7C plots the probability distributions of \( e_i \) for cluster 3. We see the power-law behavior of \( p(e_i) \) with an exponent 2.61 only when \( \Delta t = 300 \) s. When \( \Delta t = 600 \) and 1,000 s, \( p(e_i) \) switches from power-law behavior to a bimodal pattern (with exponential tails). Fig. 7D shows that the probability distributions of \( e_i \) corresponding to different values of \( \Delta t \) for individuals in cluster 4 all display very good power-law behavior, and that the power-law decay exponent is 3.6. Comparing our distribution with the distribution reported in Fig. 2A in ref. 15, we find that the distribution shapes are very similar for cluster 4, the only difference being that the extremely large burst sizes \( e_i \geq 500 \) disappear in the plots for the individuals with very long burst sizes assigned into cluster 1.

Discussion

Contrary to common belief, we find that only 3.46% of callers have intercall durations that follow a power-law distribution. The majority of callers (73.34%) have intercall durations that follow a Weibull distribution. Further examination reveals that callers with a power-law distribution exhibit anomalous and extreme calling patterns often linked to robot-based calls, telecom frauds, or telephone sales—information valuable to both academics and practitioners, especially mobile telecom providers. We note that Weibull distributions are ubiquitous in such routine human activities as intervals for online gamers (26) and intertrade intervals in stock trading (27, 28).

Although most of the individuals exhibit Weibull distributions of the intercall durations, the distribution at the population level is a power law with an exponential cutoff, consistent with other works using mobile phone communication data from other sources (15, 20). We argue that a superposition of individuals’ heterogeneous calling behaviors leads to the exponentially truncated power-law distribution at the population level, showing the importance of different characteristic scales.

Although individual callers exhibit heterogeneities across the entire population and their personal activities are also heterogeneous, individual callers can be grouped into clusters according to their similarities. The findings reported in this article enable us to construct dynamic models at an individual level that agree with empirical collective properties. Every reasonable dynamic model for cell phone use should include the major findings of this article (i.e., that individuals are not identical and do not exhibit identical behavior). Our strategy is to propose models based, not on individuals, but on clusters of individuals. Thus, to accurately model the trigger process in human activity, we need a precise classification of individuals according to the similarities in their activities and also a detailed investigation of the complete activity log for each individual.

Materials and Methods

Data Description. Our data, which are provided by a cell phone provider in China, contain all of the calling records covering two periods. One is from June 28, 2010 to July 24, 2010, and the other is from October 1, 2010 to December 31, 2010. For unknown reasons, the calling logs for a few hours on certain days (October 12, November 5, 6, 13, 21, and 27, and December 6, 8, 21, and 22) are missing, and they are excluded from our analysis, which results in a total of 109 d.

For each entry of record, we have the information of caller number, callee number, call starting time, call length, and call status. The caller and callee number is encrypted to protect personal privacy. The call status indicates whether the call is terminated normally. Note that we only take into account normal calls that begin and end normally. The calls that are not completed or are interrupted are discarded. To better explain our data, Fig. 8A shows the call records for a given individual subscriber, where a call starts at \( t^* \) and ends at \( t^* \). We usually have:

\[
\ldots < t^*_1 < t^*_i < t^*_{i+1} < \ldots
\]

Further examination is made to check whether \( t^*_i < t^*_i \) for each individual. The records that do not obey the equation \( t^*_i < t^*_{i+1} \) can be attributed to the recording errors introduced by the system, and the \( i+1 \)-th call record is discarded.
Definition of Intraday Intercall Durations. As shown in Fig. B4, the intercall duration is defined as the time that elapses between two consecutive calls and it can be calculated via \( d = t_i - t_{i-1} \). To avoid the influence on the results of discontinuous recording days, which produce very large intercall durations, we restrict the durations to a period of 1 d (the typical human circadian rhythm). Although it might seem obvious to separate the days at midnight (00:00 AM), late night calls (made by lonely people, lovers, and friends) are common, so we divide the days at 4:00 AM, which is the time point associating with the lowest call volume in a 24-h period (Fig. B8). This allows us to take into account the people who go out and stay awake later as well. Our restriction is equivalent to excluding intercall durations that span the dividing point (4:00 AM).

Fitting Distributions and Statistical Tests. A simple approach based on MLE fits and KS tests is used to check whether the candidate distributions (power-law or Weibull) can be used to fit the individual intraday intercall durations. Because people are more interested in the distribution form of large durations, we assume that the durations larger than a truncated value \( d_{\text{min}} \) are described by the candidate distributions, such that:

\[
 p(d) = d^{-\alpha} \beta \exp(-\beta d), \quad d \geq d_{\text{min}}. \quad [3]
\]

\[
 p(d) = ap^d \exp(-\gamma d), \quad d \geq d_{\text{min}}. \quad [4]
\]

We also determine the lower boundary \( d_{\text{min}} \) as an additional parameter. Once \( d_{\text{min}} \) is obtained, the distribution parameters can be estimated by means of MLE fits to the left-truncated candidate distribution. Hence, the accuracy of estimated \( d_{\text{min}} \) plays an important role in estimating accurate distribution parameters. Inspired by the method proposed in ref. 21, the best \( d_{\text{min}} \) is associated with the truncated sample with the smallest KS value. The truncated sample is obtained by discarding the durations below \( d_{\text{min}} \) in the original duration sample. After the lowest \( d_{\text{min}} \) and the corresponding distribution parameters are obtained, we use the KS test and CvM test to check the fitting. The null hypothesis \( H_0 \) for our KS test and CvM test is that the data \( (d > d_{\text{min}}) \) are drawn from the candidate distribution (power-law distribution or Weibull distribution).

Determining the Distribution Form. The sample of individual intraday intercall durations, which we assume conforms to a power-law distribution, must (i) pass either of the two tests at the significant level 0.01 and (ii) exhibit a fitting range of no less than 1.5 orders of magnitude. For Weibull distributions, in addition to the two above conditions, the Weibull exponent \( \beta \) of the intraday duration sample must be in the range (0, 1). Because a power-law distribution is a two-parameter model and a Weibull distribution is a three-parameter model, we first filter out the individuals with durations that follow a power-law distribution and then inject the remaining individuals into the Weibull filtering procedure.

ACKNOWLEDGMENTS. Z.-Q.J., W.-J.X., M.-X.L., and W.-X.Z. received support from the National Natural Science Foundation of China Grant 11205075, the Humanities and Social Sciences Fund (Ministry of Education of China Grant 09YJC7040), and Fok Ying Tong Education Foundation Grant 132013. B.P. and H.E.S. received support from the Defense Threat Reduction Agency (DTRA), the Office of Naval Research (ONR), and the National Science Foundation (NSF) Grant CMMI 1125290.
