Voltage induced by Coriolis force: a new sensing scheme for rotation velocity

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We study the motion of the charged particles between a pair of conductor plates in the non-inertial reference frame. It is found that there exists a stable voltage between the two conductor plates, which is proportional to the rotation velocity of the non-inertial system. This effect is similar to the Hall effect as the result of the Lorenz force. As an application, we propose a rotation velocity measurement scheme based on this Coriolis force -induced effect.

I. INTRODUCTION

Rotation velocity sensing is an important part of inertial navigation, the aim of which is to measure the rotation velocity of non-inertial system. And it is realized by a instrument named gyroscope [1, 2]. The earliest gyroscope utilizes the precession of the mechanical rotor, and then the goals of the gyroscope's development are to achieve high precision and miniaturization. With the development of laser and microelectronics, people have made optoelectronic gyroscopes, such as Ring Laser Gyroscopes [3–5] and Micro-Electro-Mechanical System gyroscope [2]. In recent years, the development of nuclear magnetic resonance technology and cold atom technology leads to continuous improvement in accuracy of the quantum gyroscopes [6–11], the new member of the gyroscope family. Different types of quantum gyroscopes are also being proposed, such as gyroscope based on nitrogen-vacancy (NV) color centers [12] and gyroscope [13] that utilize the decoherence of the transverse field Ising moedel (TFIM) in non-initial system.

Interferometric Fiber Optic Gyroscope (IFOG) and Atom Interference Gyroscope (AIG) [6, 14]are based on the Sagnac effect [15, 16]. In 1980 [17], Sakurai gave a approach to derive the Sagnac effect by using the similarity between the Coriolis force and the Lorentz force. With this in mind, a problem arises here that while the Lorentz force of the charged particles in magnetic field leads to the Hall effect, then can the Coriolis force lead to a similar effect? In order to solve this problem, we study the motion of the charged particles between the conductor plates in the non-inertial system. It is found that the Coriolis force would induce a stable voltage between the conductor plates, which is proportional to the rotation velocity of the system. Thus we use this effect to design a new kind gyroscope and we call it charging capacitor gyroscope (CCG).

This paper is organized as follows. The general motion equation of charged particles in non-initial reference frame is given in Sec. II. In Sec. III, we obtain the voltage difference between two conductor plates when the charged particles moving through them, thus the charging capacitor gyroscope is put forward . Then, the resolution of CCG with different structure is discussed in Sec. IV, and conclusions are given in Sec V.

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II. MOTION EQUATION FOR A CHARGED PARTICLE IN NON-INERTIAL SYSTEM

We first consider a charged particle with mass m, change q moves in a non-inertial reference frame with rotation velocity $\overrightarrow{\Omega}$. The equation of motion of such a particle reads

$$m\ddot{\overrightarrow{r}} = q\left(\vec{E} + \overrightarrow{v} \times \overrightarrow{B}\right) + 2m\overrightarrow{v} \times \overrightarrow{\Omega} + m\overrightarrow{\Omega} \times \left(\overrightarrow{\Omega} \times \overrightarrow{r}\right) \tag{1}$$

where $\overrightarrow{r}$ is the particle's coordinate, $\overrightarrow{E}$ and $\overrightarrow{B}$ are the electromagnetic field. Obviously, Eq. (1) can be rewritten in the Cartesian coordinates as

$$\begin{cases} \ddot{x} &= \frac{q}{m}\left(E_x + v_y B_z - v_z B_y\right) + 2\left(v_y \Omega_z - v_z \Omega_y\right) \\ \ddot{y} &= \frac{q}{m}\left(E_y + v_z B_x - v_x B_z\right) + 2\left(v_z \Omega_x - v_x \Omega_z\right) \\ \ddot{z} &= \frac{q}{m}\left(E_z + v_x B_y - v_y B_x\right) + 2\left(v_x \Omega_y - v_y \Omega_x\right) \end{cases}, \tag{2}$$

where the second order terms of Ω have been ignored in Eq. (2) in the limit $v \gg \Omega$.

III. CHARGED PARTICLES MOVING BETWEEN CONDUCTOR PLATES

As shown in Fig. 1, it is obviously that the particles will shift to the upper plate due to the Coriolis force, thereby charging the upper conductor plate. In this case, the motion equation for each particle is

$$\ddot{z} = \frac{q}{m}E_z + 2v_0\Omega_0, \tag{3}$$

to which the steady solution is

$$E_z = \frac{2mv_0\Omega_0}{q}. \tag{4}$$

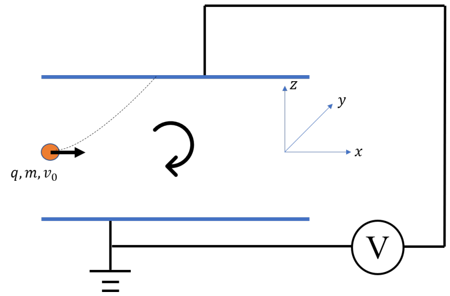

Figure1. Charged particle flows between conductor plates. The particles are now moving in x direction with velocity v_0 between two conductor plates, where the distance between them is d and The lower conductor plate is grounded to maintain its potential at zero. Assuming the system rotates around y-direction with rotation velocity Ω_0, and there is no external electromagnetic field in the system.

For the electric field between the two plates can be approximated evenly, the stable voltage between the two plates is obtained as

$$U_z = E_z d = \frac{2mv_0\Omega_0 d}{q}. \tag{5}$$

This voltage is induced by the Coriolis force of the moving charged particles in the non-inertial reference frame, and formally similar to the Hall effect. Once the voltage between the conductor plates is measured, the rotation velocity of the non-initial system is given by Eq. (5) as

$$\Omega_0 = \frac{qU_z}{2mv_0 d}. \tag{6}$$

Thus, the measurement of rotation velocity is achieved by this system, which can be used as a new type of gyroscope, and we call it charging capacitor gyroscope (CCG).

IV. RESOLUTION OF CCG

It follows from Eq. (6) that the resolution of CCG is

$$\triangle\Omega = \frac{q}{2mv_0 d}\triangle U, \tag{7}$$

where $\triangle U$ is the resolution of the voltmeter used to detect the voltage between the two conductor plates. For $\triangle U \sim \mu$V, $m \sim 10^{-27}$kg, $d \sim 1$m, $q \sim 10^{-19}$C, $v_0 \sim 10^6$m/s, $\triangle\Omega \sim 10^{-4}$rad/s. In order to improve the resolution of CCG, we put forward an arrangement structure which is shown in Fig. 2.

As shown in Fig. 2, we have

$$U_i^u = U_{i+1}^l. \tag{8}$$

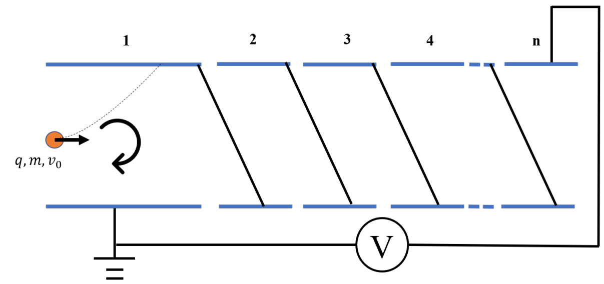

Figure. 2. Charging capacitor gyroscope (CCG) with Linear structure. The upper plate of the ith conductor plates pair is connect to the lower plate of the $i+1$th pair with wire to make them have the same potential.

Here, U_i^u (U_i^l) is the potential of the upper (lower) plate belonging to the ith conductor plates pair. On the other hand, for the the ith pair , the difference in potential of the two conductor plates is the same as that in Sec. III, thus

$$U_i^u - U_i^l = \frac{2mv_0\Omega_0 d}{q}. \tag{9}$$

Combining Eqs. (8) and (9), we have

$$U_n^u - U_1^l = \frac{2nmv_0\Omega_0 d}{q}, \tag{10}$$

where n is the number of the conductor plates pairs. If we measure the potential difference between the lower plate of the first pair of conductor plates and the upper plate of the n th pair, the resolution of Ω_0 will be

$$\triangle\Omega = \frac{q}{2nmv_0 d}\triangle U, \tag{11}$$

which is decreased by a factor $1/n$ compared with the result in Eq. (7). This indicates that the resolution of CCG with this structure is n times better than that of the original one. In the actual production, we can follow the design of cyclotron, then the above linear cascade structure will changes to a spiral structure, which can be seen in Fig. 3.

When $l \ll R$, $d \ll R$, the disc can be loaded with N-layer conductor plates pairs, thus $N = R/d$. So the total length of conductor is

$$L = \sum_{i=1}^{i=R/d} 2\pi i d = 2\pi d\frac{(1+R/d)\,R/d}{2} = \pi R + \frac{S}{d} \approx \frac{S}{d}, \tag{12}$$

where S is the area of the disc. So the number of conductor plates pair in this disc structure is $n = L/l = S/dl$. Thus, the expression of resolution for CCG with the disc structure in Fig. 3 is

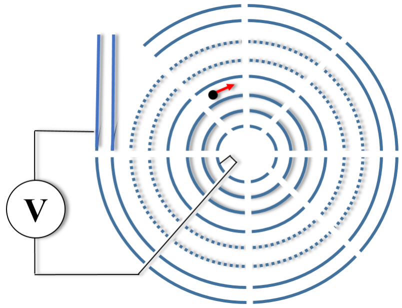

Figure. 3. Carrier changing gyroscope with helical structure.The distance and length of each conductor plates pair are d and l, and the radius of the periphery of the disc structure is R.

$$\triangle\Omega = \frac{ql}{2Smv_0}\triangle U. \quad (13)$$

For $S \sim m^2$,$l \sim \mu m$, $\triangle\Omega \sim 10^{-10}$rad/s, which has theoretically reached the resolution of ultra-high-precision [1] gyroscopes. However, the structure discussed above also basically belongs to a two-dimensional structure. So it is possible to further optimize. By stacking the above-mentioned disc structures in layers, a three-dimensional structure can be formed. For conductor plates with height h, there are $n^{'} = H/h$ layers of the helical structure in the three-dimensional structure with height H. The connection conditions for voltage in the three-dimensional structure CCG is thus

$$\begin{matrix} U^u_{i,j} = U^l_{i+1,j} \\ U^u_{n,j} = U^l_{i,j+1} \end{matrix}, \quad (14)$$

where i and j denote the order of the conductor plates pairs in each layer and the order of the layers of the spiral structure. It follows from Eqs. (9) and (14) that

$$U^u_{n,n'} - U^l_{11} = \frac{2nn^{'}mv_0\Omega_0 d}{q} = \frac{2HSmv_0\Omega_0}{qlh}. \quad (15)$$

Now we obtain the resolution of the CCG with the helical columnar structure as

$$\triangle\Omega = \frac{qs}{2Vp}\triangle U. \quad (16)$$

Here, $s = hl$ is the surface area of each conductor plate, $V = HS$ is the The volume of the entire CCG structure, and $p = mv_0$ is the momentum of each changed particle. Then, for $l, h \sim \mu m$, $s \sim 10^{-12}m^2$, $V \sim m^3$,$\triangle\Omega \sim 10^{-16}rad/s \sim 10^{-11\circ}/h$.

V. CONCLUSION

In summary, we found that the charged particles moving between the conductor plates in the non-inertial system can induce a voltage between the plates, by measuring which we can obtain the rotational velocity of the system. This effect gives a new design of microelectronics gyroscope, which is named as charging capacitor gyroscope (CCG). Inspired by the analogy of the Coriolis force and the Lorentz force, we have proposed a sensing scheme for rotational velocity. This protocol is similar to the magnetic field measurement based on the Hall effect. By optimizing the structure of the CCG, the best accuracy we have achieved in this paper is $\triangle\Omega \sim 10^{-11\circ}/h$.

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