Calculating the Potato Radius of Asteroids using the Height of Mt. Everest

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Abstract

At approximate radii of 200-300 km, asteroids transition from oblong 'potato' shapes to spheres. This limit is known as the Potato Radius, and has been proposed as a classification for separating asteroids from dwarf planets. The Potato Radius can be calculated from first principles based on the elastic properties and gravity of the asteroid. Similarly, the tallest mountain that a planet can support is also known to be based on the elastic properties and gravity. In this work, a simple novel method of calculating the Potato Radius is presented using what is known about the maximum height of mountains and Newtonian gravity for a spherical body. This method does not assume any knowledge beyond high school level mechanics, and may be appropriate for students interested in applications of physics to astronomy.

I. INTRODUCTION

Spacecraft are currently exploring asteroids and dwarf planets, such as the Near Earth Asteroid Rendezvous mission (NEAR) landing on Eros,¹ the Dawn mission orbiting Ceres and Vesta,² and the New Horizons flyby of Pluto and Charon.³ Additionally, the Mars Reconnaissance Orbiter (MRO) has observed the Martian moons Phobos and Deimos.⁴ These missions observe a remarkable variety of shapes for these bodies, shown in Fig. 1. Smaller asteroids have irregular shapes while dwarf planets (large asteroids) are nearly spherical. This follows from some simple physics.



FIG. 1. Five solar system minor bodies imaged by visiting probes. Clockwise from top left: (a) Eros as seen by the NEAR mission, (b) Phobos as seen by the MRO, (c) Vesta and (d) Ceres as seen by the Dawn mission, and (e) Charon seen by New Horizons. Approximate radii from Ref.⁵.

A well known result of Newtonian gravity is that a material with uniform density has the minimum gravitational potential energy when shaped into a sphere. If a sufficiently deep hole is dug in a spherical body, material will fall in from the edges. If matter is stacked into a sufficiently high mountain it will eventually fall down under its own weight.

For a sufficiently large surface deviation from spherical the gravitational force will be able to overcome the material's yield strength and deform it back to some maximum allowable deviation, determined by the strength of the force of gravity and the material's elastic properties. Thus, planets are spherical with small surface deviations.

In reality, all large isolated bodies that have been observed are found to be nearly spher-

ical, or at least oblate spheroidal with an equatorial bulge due to rotation. However, many asteroids and moons in the solar system with radii less than 200-300 km are known to have oblong and asymmetric geometries.⁶ This is because these asteroids and moons do not have sufficient gravity to overcome their intrinsic rigidity, and can thus maintain their nonspherical shapes. This radius range associated with the transition between oblong and spherical geometries has been dubbed 'The Potato Radius.'

Additionally, the highest possible mountain that a planet can support has been studied from first principles, where it has been found that the maximum height of a mountain is dependent on the surface gravity and elastic properties of the mountain.⁷

As a simple illustrative argument, the maximum height of a mountain on a body of uniform density is limited by the yield strength S of the mountain. The pressure at the base of the mountain is $\sim \rho gh$, where ρ is the density, g the surface gravity, and h the height of the mountain. If the height is such that $\rho gh \gtrsim S$ the base will break, causing the mountain to crumble back to the maximum allowable height.⁸ Rearranging terms, we can find that $hg \sim S/\rho$, implying that the product of the tallest possible mountain and the surface gravity is constant. If a pair of bodies are made of the same material an approximate relation arises which we call the Height-Gravity relation:

$$h_1 g_1 = h_2 g_2 = C \tag{1}$$

Where bodies 1 and 2 have respective surface gravities g_1 and g_2 , and maximum mountain heights of h_1 and h_2 , and $C \sim S/\rho$ is taken to be a constant dependent on composition which we call the Rock Constant in this work.⁹ For two bodies with similar compositions, Eq. 1 implies larger planets will have a smaller tallest possible mountain while smaller planets can support larger mountains.

This relation is obeyed quite well in the inner solar system. For example, the surface gravity of Mars is $\approx (2/5)g_{Earth}$ and the height of Olympus Mons (the tallest mountain on Mars) is nearly $(5/2)h_{Everest}$. For rocky bodies in the inner solar system, the heights of the tallest mountains are given in Tab. I.

The product hg gives a value for the Rock Constant that is constant to ~ 10% for Venus, Earth, and Mars. The exception is Mercury, whose tallest mountain falls well below the limit. Recall that the Height-Gravity relation provides an upper-bound - the tallest mountain on Mercury is simply not the tallest possible. Similarly, the tallest mountain on the Moon

Planet	Tallest Mountain	Height above mean	Surface gravity ¹⁰ $(\frac{m}{s^2})$	Rock Constant $\left(\frac{m^2}{s^2}\right)$
		planetary radius (m)		
Mercury	Caloris Montes	>3000 ¹¹	3.7	11100
Venus	Maxwell Montes	10670^{12}	8.9	94963
Earth	Mt Everest	8850	9.8	86730
Mars	Olympus Mons	21900^{13}	3.7	81030

TABLE I. List of the tallest mountains on each planet, taken to be the highest point above mean surface elevation. The Rock Constant of the Height-Gravity relation is calculated from the height of the tallest mountain and surface gravity of each planet.

and other rocky bodies in the solar system are found to be well below the limit implied by Height-Gravity relation. This could be because Mercury and the Moon are not geologically active. Caloris Montes on Mercury is the rim of an impact crater,¹¹ while Maxwell Montes, Mt. Everest, and Olympus Mons were all produced by volcanism or tectonic activity.^{12,13}

Because the constant in the Height-Gravity relation and the Potato Radius are both dependent on material specific constants, the Height-Gravity relation can be used to derive the Potato Radius. This derivation could be of interest to introductory physics students with an interest in applications of physics to astronomy, particularly because it does not assume an understanding of advanced calculus or material science (e.g. Young's Modulus, Stress, Yield Strength, etc) that are required to understand previous derivations of the Potato Radius.

II. CALCULATING THE POTATO RADIUS

Recall that the Potato Radius is the radius where there is a transition from oblong asteroids to spherical dwarf planets. If we consider an asteroid as a small sphere with large surface deviations we can apply the Height-Gravity relation to find the tallest possible mountain. As the radius of this sphere increases the surface gravity increases, and thus the maximum mountain height eventually decreases below the radius, and the asteroid becomes nearly spherical. The radius where the maximum height of a mountain is equal to the radius of the asteroid should therefore be the Potato Radius.

Consider an ellipsoidal asteroid with a semi-minor axis R and a semi-major axis 1.5R. This asteroid could be approximated by a sphere of radius R with a mountain of height R covering one hemisphere, as shown in Fig. 2.



FIG. 2. Oblong asteroid approximated as a sphere of radius R with a hemisphere-spanning mountain of height R.

The Height-Gravity relation can be applied

$$h_{asteroid} g_{asteroid} = C \tag{2}$$

and the Rock Constant C can be taken to be the product of the height of Everest with earth surface gravity - this value was nearly the mean of the possible values calculated in Tab. I. The height $h_{asteroid}$ of the mountain is already taken to be the radius R, and the surface gravity of the sphere can be found by Newton's Law of Gravitation

$$g = \frac{GM}{R^2} \tag{3}$$

and by taking the asteroid to have a constant density ρ this can be expressed purely in terms of the radius,

$$g = \frac{G(\frac{4}{3}\pi\rho R^3)}{R^2} = \frac{4}{3} \ G\rho R.$$
 (4)

At this point, the Height-Gravity relation can be applied:

$$h_{asteroid}g_{asteroid} = h_{Earth}g_{Earth}$$

$$R(\frac{4}{3}\pi G\rho R) = h_{Earth}g_{Earth}$$

$$R = \sqrt{\frac{3h_{Earth}g_{Earth}}{4\pi G\rho}}.$$
(5)

Using $h_{Earth} = 8850 \text{ m}$, $g_{Earth} = 9.8 \text{ m/s}^2$, and $\rho = 5.5 \text{ g/cm}^3$ ($\approx \rho_{Earth}$), this equation gives a value of R = 238 km. This is within the range of 200-300 km calculated directly by Lineweaver and Norman, and in agreement with observation of the oblong shapes of Eros, Phobos, and Vesta are irregular while larger bodies like Ceres and Charon are nearly spherical.

Slightly different values of the Potato Radius can be obtained by using a more realistic density for the asteroid (there doesn't seem to be any reason it should be the same density as the earth), though even the most well constrained asteroid densities still vary between $\sim 2 - 10 \text{ g/cm}^3$.¹⁴ Furthermore, on a large planet $R \gg h$ so surface gravity is approximately constant over the span of the mountain. In contrast, when $h \sim R$ the surface gravity may vary considerably over the mountain, thus requiring a more rigorous treatment. Lastly, the choice of Everest and earth gravity for the Rock Constant applies for rocky bodies of earth like composition, while objects of different composition will have a different constant.

III. CONCLUSION

Objects larger than the Potato Radius must be nearly spherical, while objects smaller objects can be asymmetric. In this work, a novel method was presented for calculating the Potato Radius using the maximum height of mountains on planets. Our result for the Potato Radius $R \approx 240$ km agrees well with spacecraft observation. This method assumes no knowledge beyond introductory mechanics, and may be of interest to students and teachers interested in practical applications of physics to astronomy. Unlike previous methods for calculating the Potato Radius and the maximum height of mountains, this method does not require extensive knowledge of calculus or materials physics, such as the Young's modulus, stress, and yield strength.

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- ¹ "NASA GSFC NEAR," http://nssdc.gsfc.nasa.gov/planetary/mission/near/near_ eros.html (2001), accessed: 2015-11-05.
- ² "NASA JPL Dawn Mission," http://dawn.jpl.nasa.gov/ (2015), accessed: 2015-11-05.
- ³ "Johns Hopkins University Applied Physics Laboratory New Horizons," http://pluto. jhuapl.edu/ (2015), accessed: 2015-11-05.
- ⁴ "NASA Mars Reconnaissance Orbiter," http://www.nasa.gov/mission_pages/MRO/main/ index.html (2015), accessed: 2015-11-05.
- ⁵ "NASA GSFC Asteroid Fact Sheet," http://nssdc.gsfc.nasa.gov/planetary/factsheet/ asteroidfact.html (2001), accessed: 2015-11-05.
- ⁶ C. H. Lineweaver and M. Norman, Australian Space Science Conference Series: Proceedings of the 9th Australian Space Science Conference, 67 (2009).
- ⁷ P. Scheuer, Journal of Astrophysics and Astronomy **2**, 165 (1981).
- ⁸ G. Cole and M. Woolfson, Planetary Science: The Science of Planets around Stars, Second Edition (CRC Press, 2013).
- ⁹ A similar constant could be derived for ice which could be used, for example, to determine the maximum height of cryovolcanoes on various moons. Similarly, a constant could be derived using the shear modulus of nuclear matter to find the maximum height of mountains on neutron stars.
- ¹⁰ "NASA GSFC Planetary Fact Sheet," http://nssdc.gsfc.nasa.gov/planetary/ factsheet/ (2015), accessed: 2015-11-04.
- ¹¹ J. Oberst, F. Preusker, R. J. Phillips, T. R. Watters, J. W. Head, M. T. Zuber, and S. C. Solomon, Icarus **209**, 230 (2010), Mercury after Two MESSENGER Flybys.
- ¹² K. B. Klose, J. A. Wood, and A. Hashimoto, Journal of Geophysical Research **97**, 16353 (1992).
- ¹³ J. B. Plescia, Journal of Geophysical Research (Planets) **109**, E03003 (2004).
- ¹⁴ B. Carry, Planetary and Space Science **73**, 98 (2012).